The Distributional Consequences of a Central Bank's Price Index Choice

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Abstract

This paper examines the aggregate and distributional consequences of an inflation-targeting central bank's choice of price index. I develop a two-agent, two-sector New Keynesian model featuring heterogeneous consumption baskets between agent types and a flexible and sticky-price sector. A simulation of a negative economy-wide supply shock suggests that the central bank's price index choice does not have significant implications for aggregates except for consumption inequality, where a central bank that targets CPI or utilitarian inflation rates increases consumption inequality between agent types. This is driven by a feed-back effect of a kink in the Bank Rate caused by the volatile flexible-sector price adjustment, which raises output and wages in a sector that favours richer agents. On the other hand, a simulation of a negative supply shock to the flexible-price sector generates a strong differential in dynamics between a central bank that targets core inflation as opposed to CPI or a utilitarian inflation rate, with the former generating less economic volatility while reducing consumption inequality. In the latter cases, the central bank's choice of price index creates an adverse consumption differential because of its effect on real profits. Thus, the model provides preliminary evidence that targeting core inflation may be optimal following an aggregate or volatile supply shock.

[0] I would like to thank Stephen Millard for helpful discussions and encouragement along the way.

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1 Introduction

When a central bank chooses a price index, does it create relative winners and losers of inflation targeting? If a central bank targets an inflation rate based on the consumer price index (CPI) – which weighs price changes by the share of each good in the basket of the *average* consumer – households with a different consumption basket might lose out relative to those closer to the average. For instance, it is well-documented that necessities make up a larger share of lower-income households' expenditures; if these goods see a relative price change that is under-represented in the average CPI inflation rate, then the policy decisions of a CPI-targeting central bank may not be appropriate for lower-income households, rendering them worse-off than others. This logic may suggest that a utilitarian central bank ought to assign a bigger weight to the CPI rate faced by lower-income households following a shock as a means of mitigating inflation inequality among households. However, it is also true that the prices of necessities are more flexible than those of luxuries, and thus also subject to more volatile pricing which temporarily influences 'headline' CPI inflation without reflecting underlying inflationary pressures. Placing higher weight on these prices therefore risks increased economic volatility. This paper thus seeks to evaluate the aggregate and distributional consequences of an inflation-targeting central bank's choice of price index following supply shocks of different origins in the context of household and sectoral heterogeneity.

This exercise is relevant given that the current global inflation shock largely originated from sectoral price shocks to food and energy following the Russian invasion of Ukraine and that there is historical precedent for considering alternative price indices for a central bank to target. The past year's inflation shock has, among other things, caused a gap between measures of 'headline' and 'core' inflation – which exclude extreme or volatile price movements to capture underlying inflationary pressures in an economy – which had been on par while inflation was low. This difference has rendered core inflation measures informative to policymaking, especially in helping guide judgement on the persistence of the inflation shock. Under these circumstances, there is scope to question whether a central bank ought to be targeting a core price index (Dixon et al., 2014). Separately, it is not just the case that different central banks utilise different price indices (e.g. the Federal Reserve targets inflation based on the the Personal Consumption Expenditures index, while the Bank of England relies on the CPI), but also there is precedent for central banks changing their preferred index. For example, the Bank of England formerly relied on the Retail Price Index excluding mortgage interest (RPIX) inflation to set its policy rate. Moreover, if we were to consider alternative metrics by which to judge the best price index for inflation targeting, research suggests that CPI inflation is not necessarily the most optimal measure

(Mankiw and Reis, 2003). The choice to target CPI inflation is, after all, a choice - one which potentially has relative aggregate and distributional consequences.

Assessing the distributional effects of monetary policy, conventional or otherwise, has been a growing focus of academic and institutional research in the aftermath of the Financial Crisis, despite residing outside the latter's remit and the long-standing tradition of viewing monetary policy as neutral in the long-run[1]. This literature has been pushed to the frontier of macroeconomics following the mainstreaming of Heterogeneous Agent New Keynesian (HANK) models, in particular following the seminal work of Kaplan et al. (2018) which displays how modelling household heterogeneity alters both our understanding of the transmission of monetary policy to the real economy, as well as the distributional effects it can engender. This paper aims to provide a complementary analysis to this literature by modelling heterogeneities in the production side of the economy alongside household heterogeneities, giving rise to a relatively unexplored channel through which central banks can aggravate existing inequalities: their price index choice.

The stylised model is thus one of heterogeneous demand and production sides. On the production side, we consider both a flexible price sector and a sticky price sector. The former is modelled via a standard representative firm; the latter is generated via a Calvo (1983) price-setting rigidity. On the demand side, a share $(1-\mu)$ of households are Ricardian consumption-smoothers with unconstrained access to financial markets. The remaining share (μ) of households are excluded from financial markets and consume their entire disposable income each period, matching the portfolio composition and consumption behaviour of the 40% of the UK population denoted as 'hand-to-mouth' (HtM) (Kaplan et al., 2014). Beyond the asymmetry in asset market participation, we also make the assumption that only Ricardian households own firms - creating an environment of wealth inequality that transforms into income inequality as bonds mature and profits are issued as dividends. The two-agent structure in this paper departs from Bilbiie's (2008) model by introducing heterogeneous wage incomes and consumption baskets among agents, giving rise to idiosyncratic consumption and wage risk following a negative shock. The central bank controls the short-term interest rate governed by a rule in the spirit of Taylor (1993); this authority chooses a price index to target before the realisation of any shocks. Fiscal policy is kept simple to focus on monetary policy.

The main simulated experiment investigates the consequences of targeting inflation based on the following price indices after a temporary shock: CPI inflation, a 'utilitarian' price index that places a higher weight on lower-income budget composition, and core inflation. A simulation of a negative economy-wide supply shock suggests that the central bank's price index choice does not have significant implications for aggregates except for

[1] See, for instance: (Bernanke, 2015); (Haldane, 2014); Mersch14.

consumption inequality, where a central bank that targets CPI or utilitarian inflation rates increases consumption inequality between agent types. This is driven by a feed-back effect of a kink in the Bank Rate caused by the volatile flexible-sector price adjustment, which raises output and wages in a sector that favours richer agents. On the other hand, a simulation of a negative supply shock to the flexible-price sector generates a strong differential in dynamics between a central bank that targets core inflation as opposed to CPI or a utilitarian inflation rate, with the former generating less economic volatility while reducing consumption inequality. In the latter cases, the central bank's choice of price index creates an adverse consumption differential because of its effect on real profits. Thus, the model provides preliminary evidence that targeting core inflation may be optimal following an aggregate or volatile supply shock.

While past studies have analysed whether central banks ought to target measures of underlying inflation following relative price movements (Dixon et al., 2014, Mankiw and Reis, 2003), these studies abstract from distributional analysis. At the same time, while recent work has investigated the distributional implications of conducting conventional monetary policy under consumption basket heterogeneity (Lee, 2022, Neyer and Stempel, 2022) and under different sectoral origins of shocks (Chan et al., 2022), this paper complements the existing literature by combining both.

2 Related Literature

This work relates to three strands of literature: analysis of monetary policy under alternative price index measures; New Keynesian modelling featuring heterogeneous agents; and evaluation of the distributional effects of monetary policy.

The paper firstly departs from the workhorse New Keynesian model by modelling sectoral heterogeneity in price stickiness, based on ample cross-country empirical evidence (e.g. Bils and Klenow (2004), Bunn and Ellis (2011), Cravino et al. (2020), Dixon et al. (2014)). Models that have incorporated heterogeneous price rigidities among firms in DSGE models have found, for instance, that it leads monetary shocks to have more persistent real effects than identical-firms models (Carvalho, 2006, Nakamura and Steinsson, 2010) and that it is a more significant driver of the monetary transmission mechanism than other production heterogeneities, like input-output structure (Pasten et al., 2020). Following from work like Aoki (2001), Dixon et al. (2014), and Mankiw and Reis (2003), this paper models different degrees of price stickiness among sectors to create an environment in which the central bank can target alternative measures of inflation, including core inflation. For simplification we consider only two sectors, one sticky and one flexible, in the spirit of Aoki's work.

In adopting a two-agent New Keynesian (TANK) structure, this paper relates to the growing body of literature incorporating heterogeneous households into DSGE modelling. Bibiie's (2008) model, which this paper extends upon, introduces HtM agents alongisde Ricardian agents that hold assets and own firms to an analysis of monetary policy. Bilbiie finds that, under certain calibrations for which the slope of the IS curve does not change sign, profits and their redistribution can drive an amplification of monetary policy in a TANK model relative to its representative agent counterpart, RANK.

More recently, the frontier of this heterogeneous agent literature has focused on models that can account for idiosyncratic shocks and the full distribution of wealth and income, also known as heterogeneous-agent New Keynesian (HANK) models after the work of Kaplan et al. (2018). While recent advances have made significant efforts in reducing the computational barriers to solving HANK models (Auclert et al., 2021, Bayer and Luetticke, 2020), barriers remain, making simpler modelling appealing. It is true that HANK models outperform others by accounting for inequality *within* agent groups as well as changes in the shares of these groups (Debortoli and Galí, 2018). Nevertheless, simpler TANK models provide robust approximations of fluctuations in consumption heterogeneity *between* unconstrained and constrained households in response to aggregate shocks (Debortoli and Galí, 2018). Insofar as this measure of heterogeneity is relevant from a theoretical and policy perspective, it is useful to continue developing TANK models.

There has been a recent surge in studies analysing the distributional effects of conventional monetary policy. Empirical work has identified several channels through which monetary policy shocks can transmit distinctly to different households: 1) the income composition channel, resulting from the fact that different households have different compositions of income such as wages or transfers that are affected heterogeneously by monetary policy shocks; 2) the financial segmentation channel, driven by households connected to financial markets responding quicker to such shocks; 3) the portfolio channel, generated by households holding different asset portfolios; 4) the savings redistribution channel, whereby shocks affect borrowers and lenders distinctly; and 5) the earnings heterogeneity channel, caused by distinct exposures to wage or unemployment risks that are affected by monetary policy (Coibion et al., 2017, Hohberger et al., 2020).

Given that there are multiple - at times counteracting- channels through which monetary policy can impact inequality, it is useful to conduct such analyis through DSGE modelling. In combining the previous two strands of literature, this paper has a similar flavour to Lee (2022) and Neyer and Stempel (2022). Lee's work incorporates consumption basket heterogeneity in a TANK model, revealing a new distributive channel of monetary policy whereby a central bank that does not target a utilitarian-weighted inflation measure sub-optimally benefits rich households at the expense of HtM households. Neyer and Stempel also build a two-agent model, differentiating Ricardian households as lower-income or higher-income, finding that following a negative supply shock, a utilitarian central bank ought to target the CPI level faced by the lower-income household. This paper provides a complementary analysis to such work by modelling a fully-flexible sector alongside a sticky price sector, with the incorporation of the former matching the 27% of prices in the UK economy that change at least once a quarter (Bunn and Ellis, 2011). Not only is this distinction thus empirically motivated, but quantitatively important as it allows the model to explore how placing higher weight on lower-income households' inflation rates can risk increased economic volatility. Further, this paper's focus is distinct in analysing the consequences of the source of a price shock to investigate the distributional effects of central bank use of alternative price index measures.

3 Model

The model extends from the work of Bilbiie (2008) and Aoki (2001). The derivation of the non-standard model equations is detailed in Appendix A.

3.1 Households

There is a continuum of measure one of infinitely-lived households in the economy. A fraction $(1-\mu)$ of these households have unconstrained access to financial markets and exhibit Ricardian consumption-smoothing behaviour, while the remaining share are excluded from financial markets and behave in a hand-to-mouth (HtM) manner. This section refers to these households as Ricardian and HtM and indexes them by r and h, respectively. The TANK structure in this paper departs from Bilbiie's (2008) model by introducing heterogeneous wage incomes and consumption baskets among agents.

Households of type $i \in \{r, h\}$ consume heterogeneous baskets composed of two consumption goods, $C_{i,f,t}$ and $C_{i,s,t}$, indexed by f and s to trace their production to a flexible price and sticky price sector, respectively. A household's total consumption basket therefore takes the following form:

$$C_{i,t} = \frac{(C_{i,s,t})^{\gamma_i}(C_{i,f,t})^{1-\gamma_i}}{\gamma_i^{\gamma_i}(1-\gamma_i)^{1-\gamma_i}} \tag{1}$$

where γ_i is a type-dependent Cobb-Douglas parameter and the sticky price good is a Dixit-Stiglitz aggregate of a continuum of differentiated goods j, defined as

$$C_{i,s,t} = \left[\int_0^1 C_{i,s,t}(j)^{\frac{\eta-1}{\eta}}\, dj\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where η is the elasticity of substitution between differentiated goods in this sector.

Heterogeneity in consumption baskets induces heterogeneity in the consumer price index (CPI) faced by each household. A household's CPI is given by:

$$P_{i,t} = P_{s,t}^{\gamma_i} P_{f,t}^{1-\gamma_i} \tag{3}$$

where the sticky price index is

$$P_{s,t} = \Big[\int_0^1 P_{s,t}(j)^{1-\eta} dj\Big]^{\frac{1}{1-\eta}} \tag{4}$$

3.1.1 Ricardian Households

There is a mass $1-\mu$ of Ricardian households, who produce the sticky price goods and own sticky price firms. Ricardian households obtain utility from consumption and disutility from labour. The optimisation problem of this financially unconstrained household is:

$$\max_{C_{r,t}, N_{r,t}, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{r,t}^{1-\sigma}}{1-\sigma} - \frac{N_{r,t}^{1+\psi}}{1+\psi}\right] \tag{5}$$

s.t

$$P_{r,t}C_{r,t} + B_t = R_{t-1}B_{t-1} + P_t W_{r,t} N_{r,t} + \frac{P_t D_t}{(1-\mu)} - P_t T_{r,t} \tag{6}$$

where β is the discount factor, σ is the coefficient of relative risk aversion, ψ is the inverse Frisch elasticity of labour supply, B_t denotes short-term savings (bank deposits), R_t is the interest rate paid on short-term savings, D_t is dividends paid to Ricaridan households as they are the owners of the sticky price firms, and $T_{r,t}$ is a tax paid by Ricardian households. The wage, dividends and tax are measured in units of the numeraire, P_t which is the aggregate price level in the economy, determined as the price level the central bank chooses to target. Solution to the Ricardian household's optimisation problem yields an Euler equation and labour supply condition,

$$C_{r,t}^{-\sigma} = \beta\Big[\frac{R_t}{\Pi_{r,t+1}} C_{r,t+1}^{-\sigma}\Big] \tag{7}$$

$$\frac{N_{r,t}^{\psi}}{C_{r,t}^{-\sigma}} = \frac{P_t W_{r,t}}{P_{r,t}} \tag{8}$$

where $\Pi_{r,t} = \frac{P_{r,t}}{P_{r,t-1}}$ is the Ricardian agent's CPI inflation rate.

Given a chosen consumption path $C_{r,t}$, households optimally allocate expenditure on sectoral goods $C_{r,s,t}$ and $C_{r,f,t}$ by minimizing total nominal consumption subject to (1), yielding the following demand relations:

$$C_{r,s,t} = \gamma_r \Big(\frac{P_{s,t}}{P_{r,t}}\Big)^{-1} C_{r,t} \tag{9}$$

$$C_{r,f,t} = (1-\gamma_r)\Big(\frac{P_{f,t}}{P_{r,t}}\Big)^{-1} C_{r,t} \tag{10}$$

Given an optimal allocation of $C_{r,s,t}$, a Ricardian household's demand for a differentiated sticky-price good is given by:

$$\begin{aligned} C_{r,s,t}(j) &= \Big(\frac{P_{s,t}(j)}{P_{s,t}}\Big)^{-\eta} C_{r,s,t} \\ &= \gamma_r \Big(\frac{P_{s,t}(j)}{P_{s,t}}\Big)^{-\eta} \Big(\frac{P_{s,t}}{P_{r,t}}\Big)^{-1} C_{r,t} \end{aligned} \tag{11}$$

3.1.2 Hand-to-Mouth Households

There is a mass of size μ of HtM households that are excluded from financial markets and consume all of their disposable income each period. The optimisation problem of a HtM household is:

$$\max_{C_{h,t},N_{h,t}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{h,t}^{1-\sigma}}{1-\sigma} - \frac{N_{h,t}^{1+\psi}}{1+\psi} \right] \tag{12}$$

s.t

$$P_{h,t} C_{h,t} = P_t W_{h,t} N_{h,t} + P_t T_{h,t} \tag{13}$$

The HtM household's labour supply condition is given by:

$$\frac{N_{h,t}^{\psi}}{C_{h,t}^{-\sigma}} = \frac{P_t W_{h,t}}{P_{h,t}} \tag{14}$$

HtM households' consumption allocation problem yields the following demand functions for the flexible price good and the composite sticky price good:

$$C_{h,f,t} = (1-\gamma_h) \Big(\frac{P_{f,t}}{P_{h,t}}\Big)^{-1} C_{h,t} \tag{15}$$

$$C_{h,s,t} = \gamma_h \Big(\frac{P_{s,t}}{P_{h,t}}\Big)^{-1} C_{h,t} \tag{16}$$

Given the above decisions, HtM households allocate consumption over differentiated goods according to:

$$\begin{aligned} C_{h,s,t}(j) &= \Big(\frac{P_{s,t}(j)}{P_{s,t}}\Big)^{-\eta} C_{h,s,t} \\ &= \gamma_h \Big(\frac{P_{s,t}(j)}{P_{s,t}}\Big)^{-\eta} \Big(\frac{P_{s,t}}{P_{h,t}}\Big)^{-1} C_{h,t} \end{aligned} \tag{17}$$

3.2 Production

There are two sectors in the production side of the economy. In the flexible price sector, a representative firm produces $Y_{f,t}$ in a perfectly competitive market. The market price

$P_{f,t}$ can adjust every period. In the sticky price sector, there are a continuum of monopolistically competitive firms indexed by $j \in (0,1)$ that face a Calvo (1983) price-setting rigidity and produce differentiated goods $Y_{s,t}(j)$. For tractability, I assume labour market segmentation such that HtM agents produce the flexible price good and Ricardian agents produce the sticky price good.

3.2.1 Flexible Price Sector

The flexible price firm operates in a perfectly competitive market and produces according to the following production technology:

$$Y_{f,t} = A_t A_{f,t} N_{h,t} \tag{18}$$

where A_t is an exogenous economy-wide productivity shock while $A_{f,t}$ is an exogenous productivity shock that is specific to the flexible price sector.

Given the market price of the flexible price good, $P_{f,t}$, sellers set this price equal to their marginal costs, $P_{f,t} = P_t MC_{f,t}$. Dividing both sides by P_t, defining $Q_{f,t} \equiv \frac{P_{f,t}}{P_t}$ as the relative price of the flexible good to the aggregate price level, and noting that real marginal costs are given by the real wage divided by the marginal product of labour, we get:

$$Q_{f,t} = MC_{f,t} = \frac{W_{h,t}}{A_t A_{f,t}} \tag{19}$$

3.2.2 Sticky Price Sector

There is a continuum of measure one of monopolistically competitive firms indexed by $j \in (0,1)$ that produce differentiated goods according to the following technology:

$$Y_{s,t}(j) = A_t A_{s,t} N_{r,t} \tag{20}$$

These firms face a Calvo (1983) price-setting rigidity whereby each period there is a history-independent probability that a firm cannot adjust its prices, α. Monopolistically competitive firms seek to maximise profits subject to demand. The profit maximisation problem for a firm that can adjust its price at time t is:

$$\max_{P^*_{s,t}} \mathbb{E}_0 \sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} \left[\left(\frac{P^*_{s,t}}{P_{s,t+k}} \right) Y_{s,t+k}(j) - MC_{s,t+k} Y_{s,t+k}(j) \right] \tag{21}$$

s.t

$$Y_{s,t+k}(j) = \mu \left[\gamma_h \left(\frac{P^*_{s,t}}{P_{s,t+k}} \right)^{-\eta} \left(\frac{P_{s,t+k}}{P_{h,t+k}} \right)^{-1} C_{h,t+k} \right] + (1-\mu) \left[\gamma_r \left(\frac{P^*_{s,t}}{P_{s,t+k}} \right)^{-\eta} \left(\frac{P_{s,t+k}}{P_{r,t+k}} \right)^{-1} C_{r,t+k} \right] \tag{22}$$

where $P^*_{s,t}$ is a price-reoptimising firm's choice of optimal price, $Y_{s,t+k}(j)$ is the demand for good j at time $t+k$ if firm j last reset its price at time t, and $\Lambda_{t,t+k}$ is the Ricardian household's stochastic discount factor for nominal payoffs between time t and $t+k$, such that

$$\Lambda_{t,t+k} \equiv \beta^k \left[\frac{C_{r,t+k}}{C_{r,t}}\right]^{-\sigma} \left[\frac{P_{r,t}}{P_{r,t+k}}\right]$$

This maximisation yields the following first order condition (FOC):

$$0 = \mathbb{E}_0 \sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} \Bigg[\mu\gamma_h(1-\eta) \left(\frac{P^*_{s,t}}{P_{s,t+k}}\right)^{-\eta} Q^{-1}_{h,s,t+k} C_{h,t+k} + (1-\mu)\gamma_r(1-\eta) \left(\frac{P^*_{s,t}}{P_{s,t+k}}\right)^{-\eta} Q^{-1}_{r,s,t+k} C_{r,t+k}$$
$$+ \mu\gamma_h\eta P_{t+k} MC_{s,t+k} \left(\frac{P^*_{s,t}}{P_{s,t+k}}\right)^{-\eta} Q^{-1}_{h,s,t+k} C_{h,t+k} \frac{1}{P^*_{s,t}} + (1-\mu)\gamma_r\eta P_{t+k} MC_{s,t+k} \left(\frac{P^*_{s,t}}{P_{s,t+k}}\right)^{-\eta} Q^{-1}_{r,s,t+k} C_{r,t+k} \frac{1}{P^*_{s,t}} \Bigg]$$

where the index j is suppressed under the assumption of symmetry among producers in the sticky price sector. The sectoral price level is determined by:

$$P_{s,t}^{1-\eta} = (1-\alpha) P^{*\,1-\eta}_{s,t} + \alpha P^{1-\eta}_{s,t-1} \tag{23}$$

3.3 Government Policies

3.3.1 Fiscal Policy

Fiscal policy is kept simple to focus on the role of monetary policy. The nominal government budget constraint is:

$$B_t + (1-\mu) P_t T_{r,t} = R_{t-1} B_{t-1} + \mu P_t T_{h,t} \tag{24}$$

For further simplicity, we can assume that the real tax on Ricardian agents finances respective transfers to constrained agents, and is a constant proportion $\tau \in (0,1)$ of real dividends,

$$\mu T_{h,t} = (1-\mu) T_{r,t} = \tau D_t \tag{25}$$

3.3.2 Monetary Policy

The central bank controls one monetary instrument, the short-term interest rate, and chooses a price index to determine what inflation rate it will target before any shocks are realised. The central bank's policy rate is always governed by a simple linear rule in the spirit of Taylor (1993):

$$\hat{R}_t = \phi_\pi \hat{\Pi}^z_t + \phi_y \hat{Y}_t \tag{26}$$

where the 'hat' notation denotes log-linearised variables and $\phi_\pi > 1$ and $\phi_y > 0$ govern the policy rule's response to log-deviations of inflation and output from their steady-states. The inflation rate can take 3 different forms, $\hat{\Pi}_t^z \in \{\hat{\Pi}_t^{CPI}, \hat{\Pi}_t^c, \hat{\Pi}_t^u\}$, representing the CPI, core, and utilitarian inflation rates, respectively, where:

$$\hat{\Pi}_t^{CPI} = \mu\hat{\Pi}_{h,t} + (1-\mu)\hat{\Pi}_{r,t} \quad (27)$$

$$\hat{\Pi}_t^u = 0.5\hat{\Pi}_{h,t} + 0.5\hat{\Pi}_{r,t} \quad (28)$$

$$\hat{\Pi}_t^c = \hat{\Pi}_{s,t} \quad (29)$$

3.4 Market Clearing

Market clearing for each good is given by:

$$Y_{f,t} = (1-\mu)C_{r,f,t} + \mu C_{h,f,t} \quad (30)$$

$$Y_{s,t} = (1-\mu)C_{r,s,t} + \mu C_{h,s,t} \quad (31)$$

The aggregate resource constraint is:

$$Y_t = \mu C_{h,t} + (1-\mu)C_{u,t} \quad (32)$$

3.5 Exogenous Processes

Firms may be subject to the following economy-wide or sector-specific productivity shocks which follow a standard autoregressive process:

$$A_t = \rho_a A_{t-1} + \varepsilon_t^a \quad (33)$$

$$A_{x,t} = \rho_{a_x} A_{x,t-1} + \varepsilon_t^{a_x} \quad (34)$$

where $x \in \{f, s\}$, ρ_a and ρ_{a_x} govern the autocorrelation of the economy-wide and sector-specific productivity, and $\varepsilon_t^a \sim N(0, \sigma_a^2)$, $\varepsilon_t^{a_x} \sim N(0, \sigma_{a_x}^2)$.

3.6 Equilibrium and Solution

Equilibrium in this model is characterised by a sequence of processes for all aforementioned prices and quantities such that households and firms meet their respective optimality conditions and all markets clear. This model is solved by taking a first-order log-linear approximation around steady state. The derivation of the log-linearised model can be found in Appendix (A).

Table 1: Parameter Values

Parameter	Description	Value
σ	Coefficient of relative risk aversion	1
ψ	Inverse Frisch elasticity of labour supply	1
β	Discount factor	0.99
τ	Tax rate on dividends	0.15
α	Calvo probability of not changing price	0.75
ϕ_π	Taylor coefficient on inflation	1.5
ϕ_y	Taylor coefficient on output	0.125
μ	Share of constrained households	0.4
ρ_a	Autocorrelation of economy-wide productivity	0.8
ρ_{a_s}	Autocorrelation of sticky-sector productivity	0.8
ρ_{a_f}	Autocorrelation of flexible-sector productivity	0.8
γ_h	HtM Cobb-Douglas parameter	0.6
γ_r	Ricardian Cobb-Douglas parameter	0.8

3.7 Calibration

Table (1) details the parameter values used in the baseline calibration of the experiment.

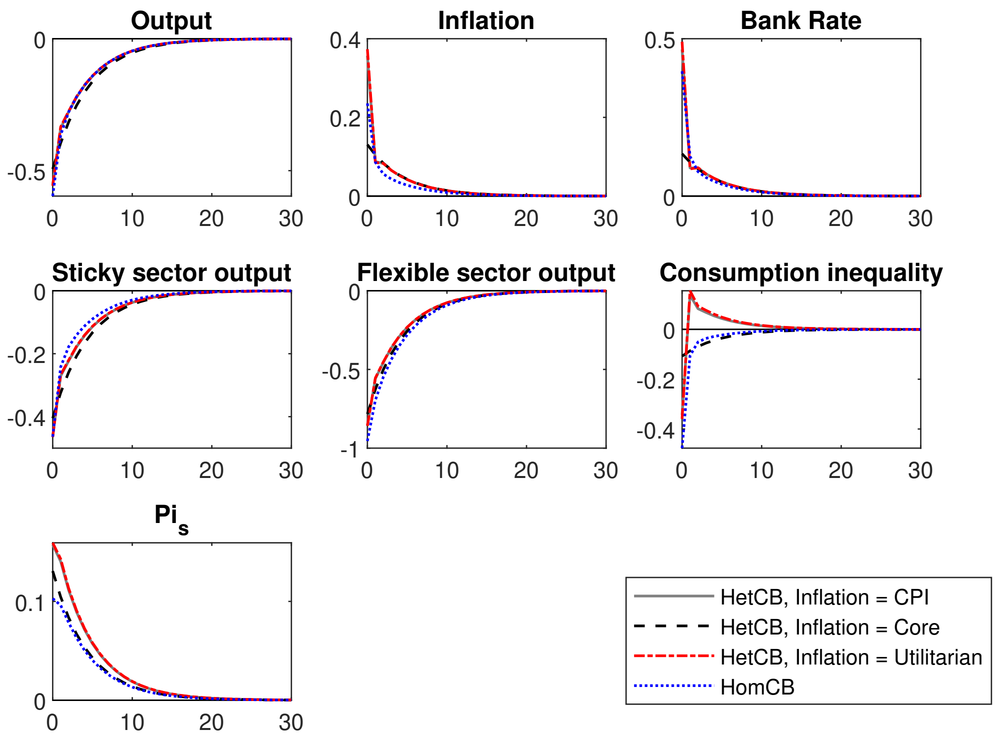
On the household side, the calibrations for $\sigma, \psi, \beta, \alpha, \phi_\pi, \phi_y$ and β are all standard convention in the literature. The tax rate on dividends is chosen to fit evenly within the bounds of the possible tax rates on dividends in the UK prior to 2016, which were between 0 and 31%. The share of HtM agents is set to match the findings in Kaplan et al. (2014) that stem from analysis of the UK Wealth and Assets Survey. The autocorrelation rates are all set to the same parameter to be generate comparable impulse response functions; the value of 0.8 was chosen to induce a persistent enough shock to make the analysis relevant to the current (highly persistent) inflation shock we are currently undergoing. Cravino et al. (2020) find that households in the median of the income distribution spend 22% of their budget on goods that face frequent price changes, while for those at the top income percentile, this figure falls to 17%. I thus calibrate the Ricardian Cobb-Douglas parameter to fall between these figures at 0.8. This is relatively higher than the economy-wide share of sticky-priced goods, estimated by Bunn and Ellis (2011) to be 73%. Given a lack of data on the HtM budget share of flexible-price goods, I set γ_h slightly lower than the economy-wide level, to match the stylised fact that HtM agents face inflation rates that are higher than other agents' household inflation rates as well as the aggregate inflation rate in response to an aggregate shock (Kaplan and Schulhofer-Wohl, 2017).

4 Results

4.1 Economy-wide supply shock

I now consider a negative economy-wide supply shock in the form of a shock to aggregate productivity, A_t. Figure (4.2) compares the impulse reponse functions of such a shock under the following scenarios: agents have heterogeneous consumption baskests (HetCB), as in the baseline model, and the central bank targets one of three possible price indices' inflation rates (CPI, core or utilitarian); or agents face homogeneous consumption baskets (HomCB) and the central bank targets the aggregate price index.

Figure 1: Shock to A_t



Responses to a negative 1 standard deviation shock. The y-axis corresponds to percentage deviations from steady-state. The x-axis is time in quarters.

Following an aggregate negative productivity shock, aggregate and sectoral output decrease in all four scenarios by similar magnitudes. Inflation (and correspondingly, the Bank Rate) rise in all four scenarios, though there is some heterogeneity here: when aggregate inflation is measured as core inflation, it is unsurprisingly much lower initially, reflecting only price rises in the sticky sector. In the second and third quarters, the aggregate inflation rate under a CPI or utilitarian price index dips slightly below that of a core price index as the flexible price sector experiences a steep fall in prices following its initial upwards jolt. From the fourth quarter on, when prices have adjusted in the flexible price sector, all four inflation rates converge to a very similar path. In the case of CPI/utilitarian inflation rates, inflation in the sticky price sector is slightly higher than in the other two cases: this is due to higher sticky-price inflation expectations in these cases. Overall, aggregate dynamics do not seem to be much affected by whether the model considered homogeneous or heterogeneous consumption baskets.

It thus appears that, of the variables graphed above, the central bank's price index choice generates the most heterogeneity in the dynamics of consumption inequality in response to this shock. In the case of HetCB, both under CPI inflation and utilitarian inflation, consumption inequality initially shoots down relative to HetCB under core inflation, as the Bank Rate differential induces Ricardian agents to substitute away from consumption to savings in the first quarter following the shock. From the second quarter onwards, however, consumption inequality rises in the cases of HetCB under CPI and utilitarian inflation, despite the Bank Rate being at a relatively similar level. This differential is driven by the effect of including the volatile price adjustment from the flexible price sector in the central bank's inflation target. Under a CPI/utilitarian inflation target, there is a slight kink in the path of inflation and the Bank Rate (reflecting the effect of the flexible price-level adjustment on the central bank's level of aggregate inflation), which increases Ricardian consumption relative to the core inflation case. Since Ricardian agents consume a disproportionate share of sticky sector output, this raises Y_s relative to the core inflation case. Given labour market segmentation, this allows W_r to recover quicker than W_h, widening the consumption gap between agents even once the Bank Rate has returned to a relatively similar path as under the core inflation case.

The savings redistribution channel

As noted above, the transmission of conventional monetary policy to consumption inequality in this scenario is partly driven by the savings redistribution channel. To elaborate: since Ricardian agents are savers, they will substitute away from consumption and towards savings when the Bank Rate increases, while the HtM agents will not respond directly to this change. This is evident from the Ricardian agent's log-linearised Euler Equation, derived in Appendix (A), given by:

$$\hat{C}_{r,t} = \mathbb{E}_t\hat{C}_{r,t+1} - \frac{1}{\sigma}[\hat{R}_t - \mathbb{E}_t\hat{\Pi}_{r,t+1}] \tag{35}$$

Notice that there will be two variables affected by the central bank's price index choice: the Bank Rate, as well as Ricardian inflation expectations.

When the central Bank targets core inflation, the Bank Rate is initially lower than Ricardian inflation expectations as agents factor in the volatile price rise from the flexible price sector. However, from the second quarter on, the Bank Rate remains persistently higher than Ricardian inflation expectations (which have now factored in the flexible price adjustment), suppressing Ricardian consumption throughout the post-shock period.

When the aggregate price index is measured as CPI or utilitarian, the aggregate inflation rate is higher than under a core price index. This causes both the Bank Rate and Ricardian inflation expectations to to rise relatively more than their counterparts under a core inflation target. In the first quarter following the shock, the Bank Rate is more elevated than Ricardian inflation expectations, suppressing $\hat{C}_{r,t}$. However, under a CPI/utilitarian aggregate price index, Ricardian inflation expectations remain more elevated than the Bank Rate from $t = 2$ onwards as they spend a greater amount of their budget on sticky sector goods, leading Ricardian agents to substitute back from savings towards consumption, mitigating their drop in consumption relative to a core inflation case.

The earnings heterogeneity channel

Monetary policy also generates heterogeneous consumption responses through the earnings heterogeneity channel, as labour market segmentation creates idiosyncratic wage risk for agents in this model.

To explain: take the case of a central bank targeting a price index based on CPI or a utilitarian measure. As noted above, Ricardian agents face less of a hit to consumption in this case relative to the core inflation case. Given that Ricardian agents spend a greater proportion of their budget on sticky priced goods, this raises demand for sticky goods, $\hat{Y}_{s,t}$, from $t = 2$ onwards. Since the labour market is segmented and production technology in the stickty sector is given by $\hat{Y}_{s,t} = \hat{A}_t + \hat{A}_{s,t} + \hat{N}_{r,t}$, this demand boost results in an increase in Ricaridan agents' hours worked that overtakes the effects of the negative productivity shock in this sector. The Ricardian agents' labour supply relation is given by:

$$\psi \hat{N}_{r,t} + \sigma \hat{C}_{r,t} = \frac{(\gamma_r - \gamma)}{\gamma_r} \hat{Q}_{r,f,t} + \hat{W}_{r,t} \tag{36}$$

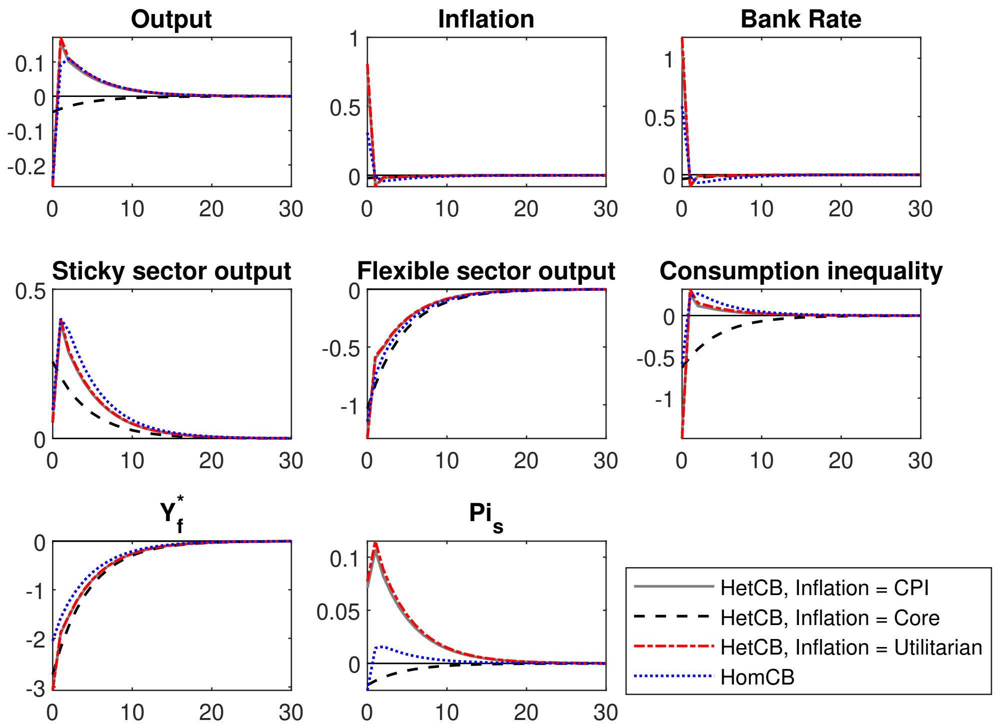
where $Q_{r,f,t} = \frac{P_{f,t}}{P_{r,t}}$ is the relative price of the flexible price good to the Ricardian CPI. Clearly, given that hours worked and consumption increase in this case relative to the core inflation case, the real wage paid to Ricardian agents will also be higher, contributing further to the increase in consumption inequality between agent types.

4.2 Supply shock in the flexible sector

Following a negative productivity shock in the flexible price sector, potential output in that sector, $Y^*_{f,t}$ falls in all four scenarios, causing a fall in the flexible sector's output.

In the cases of the central bank targeting CPI or utilitarian inflation rates, the Bank Rate tightens comparatively aggressively in the first quarter, as the inflation rate takes into account both the initial upwards jolt in flexible prices and a slight increase in sticky-price inflation (due to altered inflation expectation). Though the price in the flexible sector will readjust in the second quarter, this initial tightening of the Bank rate suppresses demand enough to cause an initial decrease in aggregate output.

Figure 2: Shock to $A_{f,t}$



Responses to a negative 1 standard deviation shock. The y-axis corresponds to percentage deviations from steady-state. The x-axis is time in quarters.

Similarly to the previous experiment, consumption inequality initially decreases as the savings redistribution channel generates a fall in Ricardian consumption. When a central bank targets a CPI/utilitarian inflation rate, Ricardian agents begin to substitute back towards consumption in the second quarter as the Bank Rate dips marginally below zero, causing sticky sector output to rise as these agents spend a substantial share of their budget in this sector. This causes Ricardian consumption to increase further from the feed-back effect of the earnings heterogeneity channel, contributing to the jump up in consumption inequality from $t = 2$ onwards. Unlike the previous experiment, there is a further mechanism at play here that causes a divergence in consumption inequality between a central bank that targets CPI/utilitarian or core inflation rates: the effect of the central bank's price level on profits.

The income composition channel

The savings redistribution and earnings heterogeneity channels contribute to increased consumption inequality in the CPI/utilitarian inflation scenarios in the same way as described in the section above. However, following a shock to the flexible-price sector, monetary policy also contributes to increased consumption inequality via the income composition channel as it not only affects wages but also profits, which are issued as dividends, and transfers.

When the central bank targets a CPI/utilitarian price index, aggregate inflation rises and then marginally dips below zero following a shock to $A_{f,t}$. Since the numeraire is initially elevated, real profits are lower at $t = 1$ relative to a core inflation target. As the price level adjusts from $t = 2$ onwards due to the nature of pricing in the flexible sector, sticky sector output peaks at the point at which its real profits peak, also being the point at which the Ricardian gents receive the most dividends, raising consumption inequality. Though HtM agents do receive a fraction of profits in the form of transfers, this redistribution is not sufficient to offset all three channels driving an increase in Ricardian consumption.

5 Conclusion

This paper examines the aggregate and distributional consequences of a central bank's price index choice following negative supply shocks of different origins. I extend the work of Aoki (2001) and Bilbiie (2008) to develop a two-agent, two-sector New Keynesian model to examine the impact of demand and supply-side heterogeneity in the transmission of conventional monetary policy to consumption inequality. The simulated experiments investigate the consequences of targeting inflation based on the three possible price indices after a temporary shock, either aggregate or sector-specific.

A simulation of a negative economy-wide supply shock suggests that the central bank's price index choice does not have significant implications for aggregates except for consumption inequality, where a central bank that targets CPI or utilitarian inflation rates increases consumption inequality between agent types. This is driven by the savings redistribution and earnings heterogeneity channels. On the other hand, a simulation of a negative supply shock to the flexible-price sector generates a strong differential in dynamics between a central bank that targets core inflation as opposed to CPI or a utilitarian inflation rate, with the former generating less economic volatility while reducing consumption inequality. In the latter cases, the central bank's choice of price index transmits to consumption inequality via the savings redistribution, earnings heterogeneity and income composition channels. Thus, the model provides complementary analysis to the existing literature by putting forth preliminary evidence that targeting core inflation may be optimal following an aggregate or volatile supply shock.

References

Aoki, K. (2001), 'Optimal Monetary Policy Responses to Relative-price Changes', *Journal of monetary economics* **48**(1), 55–80.

Auclert, A., Bardóczy, B., Rognlie, M. and Straub, L. (2021), 'Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models', *Econometrica* **89**(5), 2375–2408.

Bayer, C. and Luetticke, R. (2020), 'Solving Discrete Time Heterogeneous Agent Models with Aggregate Risk and Many Idiosyncratic States by Perturbation', *Quantitative Economics* **11**(4), 1253–1288.

Bernanke, B. (2015), 'Monetary Policy and Inequality', Brookings Institution Blog Post.

Bilbiie, F. O. (2008), 'Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic', *Journal of Economic Theory* **140**(1), 162–196.

Bils, M. and Klenow, P. J. (2004), 'Some Evidence on the Importance of Sticky Prices', *Journal of Political Economy* **112**(5), 947–985.

Bunn, P. and Ellis, C. (2011), 'How Do Individual UK Consumer Prices Behave?'.

Calvo, G. A. (1983), 'Staggered Prices in a Utility-Maximizing Framework', *Journal of Monetary Economics* **12**(3), 383–398.

Carvalho, C. (2006), 'Heterogeneity in Price Stickiness and the Real Effects of Monetary Policy Shocks', *Frontiers in Macroeconomics* **2**(1).

Chan, J., Diz, S. and Kanngiesser, D. (2022), 'Energy Prices and Household Heterogeneity: Monetary Policy in a Gas-TANK', *Available at SSRN 4255158* .

Coibion, O., Gorodnichenko, Y., Kueng, L. and Silvia, J. (2017), 'Innocent Bystanders? Monetary Policy and Inequality', *Journal of Monetary Economics* **88**, 70–89.

Cravino, J., Lan, T. and Levchenko, A. A. (2020), 'Price Stickiness along the Income Distribution and the Effects of Monetary Policy', *Journal of Monetary Economics* **110**, 19–32.

Debortoli, D. and Galí, J. (2018), 'Monetary Policy with Heterogeneous Agents: Insights from TANK Models', *UPF Economics Working Paper Series, No. 1686* .

Dixon, H. D., Franklin, J. and Millard, S. (2014), 'Sectoral Shocks and Monetary Policy in the United Kingdom'.

Haldane, A. (2014), 'Unfair Shares', Speech at the Bristol Festival of Ideas, 29 May 2014.

Hohberger, S., Priftis, R. and Vogel, L. (2020), 'The Distributional Effects of Conventional Monetary Policy and Quantitative Easing: Evidence from an Estimated DSGE Model', *Journal of Banking and Finance* **113**, 105483.

Kaplan, G., Moll, B. and Violante, G. L. (2018), 'Monetary Policy According to HANK', *American Economic Review* **108**(3), 697–743.

Kaplan, G. and Schulhofer-Wohl, S. (2017), 'Inflation at the Household Level', *Journal of Monetary Economics* **91**, 19–38.

Kaplan, G., Violante, G. L. and Weidner, J. (2014), 'The Wealthy Hand-to-Mouth', *Brookings Papers on Economic Activity* .

Lee, S. (2022), 'Optimal Monetary Policy under Heterogeneous Consumption Baskets', *Bank of Korea Working Paper 2022-19* .

Mankiw, N. G. and Reis, R. (2003), 'What Measure of Inflation Should a Central Bank Target?', *Journal of the European Economic Association* **1**(5), 1058–1086.

Nakamura, E. and Steinsson, J. (2010), 'Monetary Non-neutrality in a Multisector Menu Cost Model', *The Quarterly journal of economics* **125**(3), 961–1013.

Neyer, U. and Stempel, D. (2022), How Should Central Banks React to Household Inflation Heterogeneity?, Technical report, DICE Discussion Paper.

Pasten, E., Schoenle, R. and Weber, M. (2020), 'The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy', *Journal of Monetary Economics* **116**, 1–22.

Taylor, J. B. (1993), Discretion versus Policy Rules in Practice, *in* 'Carnegie-Rochester Conference Series on Public Policy', Vol. 39, Elsevier, pp. 195–214.

Appendix A Model Derivation

A.1 Ricardian Households

Log-linearising the Ricardian agent's price level, we get:

$$\hat{P}_{r,t} = \gamma_r \hat{P}_{s,t} + (1-\gamma_r)\hat{P}_{f,t}$$

From this, we obtain

$$\hat{\Pi}_{r,t} = \gamma_r \hat{\Pi}_{s,t} + (1-\gamma_r)\hat{\Pi}_{f,t}$$

See that, from the definition of the Ricardian CPI,

$$P_{s,t} = \left[\frac{P_{r,t}}{P_{f,t}^{(1-\gamma_r)}}\right]^{\frac{1}{\gamma_r}}$$

Defining $Q_{r,s,t} = \frac{P_{s,t}}{P_{r,t}}$ as the relative price of the sticky good to the Ricardian CPI and equivalently $Q_{r,f,t} = \frac{P_{f,t}}{P_{r,t}}$, we can re-write the above as

$$Q_{r,s,t} = (Q_{r,f,t})^{\frac{-(1-\gamma_r)}{\gamma_r}}$$

log-linearising,

$$\hat{Q}_{r,s,t} = \frac{-(1-\gamma_r)}{\gamma_r}\hat{Q}_{r,f,t} \tag{A.1.1}$$

See that

$$Q_{r,s,t} = \frac{P_{s,t}}{P_{r,t}} \times \frac{P_{s,t-1}}{P_{s,t-1}} \times \frac{P_{r,t-1}}{P_{r,t-1}}$$

From this, we obtain

$$\hat{\Pi}_r = \hat{\Pi}_{s,t} + \hat{Q}_{r,s,t-1} - \hat{Q}_{r,s,t}$$

substituting (A.1.1), we get

$$\hat{\Pi}_r = \hat{\Pi}_{s,t} + \frac{(1-\gamma_r)}{\gamma_r}\Delta\hat{Q}_{r,f,t} \tag{A.1.2}$$

where $\Delta\hat{Q}_{r,f,t} = \hat{Q}_{r,f,t} - \hat{Q}_{r,f,t-1}$

Since prices in the flexible sector are equal to the real marginal cost in this sector, then, using the HtM labour supply condition

$$Q_{r,f,t} = \frac{N_{h,t}^{\psi} C_{h,t}^{\sigma}}{A_t A_{f,t}} \left(\frac{Q_{r,s,t}}{Q_{r,f,t}}\right)^{\gamma_h - \gamma_r}$$

Log-linearising,

$$\hat{Q}_{r,f,t} = \frac{\gamma_r}{\gamma_h}\left[\psi \hat{N}_{h,t} + \sigma \hat{C}_{h,t} - \hat{A}_t - \hat{A}_{f,t}\right] \tag{A.1.3}$$

Solution to the Ricardian household's optimisation problem yields three first order conditions (FOCs):

$$\lambda_{r,t} = \frac{C_{r,t}^{-\sigma}}{P_{r,t}}$$

$$\lambda_{r,t} = \beta \mathbb{E}_t \lambda_{r,t+1} R_t$$

$$\lambda_{r,t} = \frac{N_{r,t}^{\psi}}{P_t W_{r,t}}$$

where $\lambda_{r,t}$ is the Lagrange multiplier on the nominal budget constraint. Combining the above FOCs gives us the Euler equation and labour supply condition in the main text. Log-linearising the Euler equation, using $R = \frac{1}{\beta}$ and $\Pi_r = 1$ as useful steady state relationships, we obtain:

$$\hat{C}_{r,t} = \mathbb{E}_t \hat{C}_{r,t+1} - \frac{1}{\sigma}[\hat{R}_t - \mathbb{E}_t \hat{\Pi}_{r,t+1}] \tag{A.1.4}$$

Log-linearising the labour supply condition, noting that $\frac{P_t}{P_{r,t}} = \left(\frac{Q_{r,s,t}}{Q_{r,f,t}}\right)^{(\gamma-\gamma_r)}$, yields:

$$\psi \hat{N}_{r,t} + \sigma \hat{C}_{r,t} = (\gamma - \gamma_r)[\hat{Q}_{r,s,t} - \hat{Q}_{r,f,t}] + \hat{W}_{r,t}$$

substituting in (A.1.1), we can rewrite the above as:

$$\psi \hat{N}_{r,t} + \sigma \hat{C}_{r,t} = \frac{(\gamma_r - \gamma)}{\gamma_r} \hat{Q}_{r,f,t} + \hat{W}_{r,t} \tag{A.1.5}$$

A.2 HtM Households

We can use the same steps as in (A.1) to obtain the following:

$$\hat{Q}_{h,f,t} = \psi \hat{N}_{h,t} + \sigma \hat{C}_{h,t} - \hat{A}_t - \hat{A}_{f,t} \tag{A.2.1}$$

$$\hat{Q}_{h,s,t} = \frac{-(1-\gamma_h)}{\gamma_h} \hat{Q}_{h,f,t} \tag{A.2.2}$$

$$\hat{\Pi}_h = \hat{\Pi}_{s,t} + \frac{(1-\gamma_h)}{\gamma_h} \Delta \hat{Q}_{h,f,t} \tag{A.2.3}$$

Solution to the HtM household's optimisation problem yields two FOCs:

$$\lambda_{h,t} = \frac{C_{h,t}^{-\sigma}}{P_{h,t}}$$

$$\lambda_{h,t} = \frac{N_{h,t}^{\psi}}{P_t W_{h,t}}$$

Combining the above FOCs gives us the labour supply condition in the main text. Log-linearising, we obtain:

$$\psi\hat{N}_{h,t} + \sigma\hat{C}_{h,t} = \frac{(\gamma_h - \gamma)}{\gamma_h}\hat{Q}_{h,f,t} + \hat{W}_{h,t} \tag{A.2.4}$$

We can write the HtM agent's budget constraint as:

$$\frac{P_{h,t}}{P_t}C_{h,t} = W_{h,t}N_{h,t} + \frac{\tau}{\mu}D_t$$

Log-linearising, noting that dividends are zero in steady state and approximating $\hat{d}_t \approx \frac{D_t}{Y_s}$, we obtain

$$\frac{(\gamma - \gamma_h)}{\gamma_h}\hat{Q}_{h,f,t} + \hat{C}_{h,t} = \hat{W}_{h,t} + \hat{N}_{h,t} - \frac{\tau}{\mu}\frac{Y_s}{Y}\Big(\frac{W_h N_h}{Y}\Big)^{-1}\hat{mc}_{s,t} \tag{A.2.5}$$

where the last term follows from the definition of dividends, $D_t = Y_{s,t} - Y_{s,t}mc_{s,t}$, where $mc_{s,t}$ is the real marginal cost in the sticky price sector.

A.3 Flexible Price Sector

The log-linearised production function is given by

$$\hat{Y}_{f,t} = \hat{A}_t + \hat{A}_{f,t} + \hat{N}_{h,t} \tag{A.3.1}$$

We can use the HtM agent's labour supply condition to rewrite (19) as:

$$Q_{f,t} = \frac{N_{h,t}^{\psi}C_{h,t}^{\sigma}}{A_t A_{f,t}}\Big(\frac{Q_{h,s,t}}{Q_{h,f,t}}\Big)^{\gamma_h - \gamma}$$

Log-linearising, and using (A.3.1), we obtain

$$\hat{Q}_{f,t} = \psi[\hat{Y}_{f,t} - Y_{f,t}^*] \tag{A.3.2}$$

where

$$Y_{f,t}^* = \frac{(1+\psi)}{\psi}(\hat{A}_t + \hat{A}_{f,t}) - \frac{\sigma}{\psi}\hat{C}_{h,t} + \frac{(\gamma_h - \gamma)}{\psi\gamma_h}\hat{Q}_{h,f,t} \tag{A.3.3}$$

can be interpreted both as the natural rate of output in the flexible price sector and a supply shock in this sector.

A.4 Sticky Price Sector

The log-linearised production function is given by

$$\hat{Y}_{s,t} = \hat{A}_t + \hat{A}_{s,t} + \hat{N}_{r,t} \tag{A.4.1}$$

Manipulating the sticky price firms' symmetric FOC, we obtain:

$$\frac{P^*_{s,t}}{P_{s,t}} = \left(\frac{\eta}{\eta - 1}\right)\frac{\mathbb{E}_0 \sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} MC_{s,t+k} Y_{s,t+k}}{\mathbb{E}_0 \ \sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} \Pi^{-1}_{s,t+k,t} Y_{s,t+k}}$$

which tells us that monopolistic firms in the sticky price sector optimally set their relative price according a function which multiplies their discounted future costs and revenues multiplied by the mark-up, $\mathcal{M} = \frac{\eta}{\eta-1}$. Using (23) and log-linearising, we can derive a Phillips Curve for the sticky price sector:

$$\hat{\Pi}_{s,t} = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\hat{MC}_{s,t}) + \beta \mathbb{E}_t \hat{\Pi}_{s,t+1} \tag{A.4.2}$$

where

$$\hat{MC}_{s,t} = \hat{W}_{r,t} - \hat{A}_t - \hat{A}_{s,t} \tag{A.4.3}$$

A.5 IS Curve

Combining the Ricardian household's Euler equation and the resource constraint, we obtain

$$\hat{Y}_t = \mathbb{E}_t\Big[\hat{Y}_{t+1} - \mu\Delta\hat{C}_{h,t+1}\Big] - \frac{(1-\mu)}{\sigma}\Big[\hat{R}_t - \mathbb{E}_t\hat{\Pi}_{r,t+1}\Big] \tag{A.5.1}$$

where

$$\mu\Delta\hat{C}_{h,t+1} = \mu\mathbb{E}_t\hat{C}_{h,t+1} - \mu\hat{C}_{h,t}$$