Savings glut without saving: retirement saving and the interest rate decline in the United States between 1984 and 2013

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Abstract

This paper shows that domestic secular trends of demographics, inequality, and long-run productivity growth explain about half of the decline in the economy-wide interest rate on households' financial assets between 1984 and 2013. The incorporation of a decline of productivity growth and income contingent life expectancies in an overlapping generations model of retirement saving reconcile the fall in the interest rate with stylised facts on US savings, namely the absence of an increase in the aggregate saving rate and the divergence of income conditional saving rates respectively. The aggregate increase and divergence of life expectancies further offer an explanation for the inexistence of an inheritance boom.

Keywords: Interest rate decline; Overlapping generations; Retirement saving; Income inequality; Heterogeneous life expectancy; Productivity growth

JEL: D91; E21; E22; J11; O41

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1 Introduction

Since the early 80s, the U.S. economy exhibits the conundrum of coinciding falls in both the real interest and the personal saving rate. In the absence of an international dimension creating asset demand from abroad, a 'global savings glut' (Bernanke et al., 2005), this coinciding fall seems to be contradictory. This paper solves the conundrum in a domestic, closed-economy, neoclassical growth model with a prominent role for heterogeneity at retirement through income contingent life expectancies. A substantial decline of 60% of the observed decline in the interest rate between 1982 and 2013 - driven by underlying secular domestic trends of demographics, inequality, and productivity, a domestic 'savings glut' - is reconciled with a non-increasing saving rate.

The key for this result lies in the misleading character of the personal saving rate. It constitutes a current flow of active saving relative to realised disposable income, and therefore lacks the consideration of net worth, expectations about the future, and the growth rate of the economy. Nevertheless, all three are crucial elements in agents' decision making. The paper therefore addresses these elements when analysing the effect of secular domestic trends on the interest rate. It firstly accounts for increases in net worth to some degree through an incorporation of holding gains in the computation of the saving rate, which already counteracts the fall to a great extent (see Figure 2). Secondly, including the secular decline of labour productivity growth has the effect of reducing both the interest rate - it is the main driver - and the saving rate in steady state, as capital evolves along a flatter growth path. Finally, a dynamic model allows to replicate the saving rate given agents' expectations.

Recent literature has considered the role of exogenous drivers on the interest rate, particularly demographic change (Carvalho et al., 2016; Gagnon et al., 2016), but also productivity growth and inequality (Eggertsson et al., 2017). The latter is therefore the most comparable paper, extending the common drivers of slowing population growth and rising life expectancy with a rise in inequality and decline in productivity growth. Nevertheless, there are three important differences. Firstly, this paper proposes a comprehensive argument for a domestically driven decline in the interest rate by reconciling it with stylised facts of saving. The model will exhibit not only a constant aggregate saving rate, but also a strong divergence in income contingent saving rates of 35 to 44 year-olds. Secondly, it focuses on inequality and in particular heterogeneity in saving and asset-holdings for old age, drawing on the increasing amount of research on heterogeneity of net worth at retirement (e.g. De Nardi et al., 2010). In this vein, a new and important dimension of inequality is introduced with asymmetric life-expectancies, which becomes the second most important driver, before the aggregate increase in life expectancy, to explain the declining interest rate and the main channel how inequality works in this model. Finally, this paper considers the actual return on agents' financial assets as observed in the Survey of Consumer Finances, rather than safe or estimated interest rates.

The inequality dimension further links to the discussion about a potential inheritance boom (Piketty and Zucman, 2015; Wolff and Gittleman, 2014). The consideration of inequality highlights, that despite an increase of net worth, the annual inheritance flow decreases strongly due to demographics, particularly the divergence of life-expectancies across agents. These decrease the death probability of wealthy agents early at retirement and thus reduce accidental bequests and in turn wealth transmission.

The paper is structured as follows. The next section will highlight the related literature in terms of the interest rate decline, as well as inequality and saving for retirement. The following section analyses the U.S. data and presents the stylised facts of the interest rate, the saving pattern, and the underlying exogenous changes. Section 4 outlines the theoretical model including the calibration, while section 5 and 6 present the results for a static and dynamic model respectively. Finally, section 7 concludes.

2 Literature

Two strands of literature relate to this paper. Firstly, research postulates the decline of the real interest rate over the last three decades and the potential of a persistent negative interest rate - a secular stagnation equilibrium. This can be caused by excess liquidity, the so-called liquidity trap, which links to the second strand of literature, agents' saving decision. Some domestic factors which may affect it are a decline in technological growth, demographic change, and increasing income inequality.

Decline of the interest rate

Related literature has treated the interest rate decline in three distinct ways. Firstly, linked to secular stagnation, it postulates theoretical prerequisites of agents' saving behaviour for a low (negative) interest rate and its repercussions. In contrast, the other two cases analyse the actual decline of the interest rate over time, by empirically estimating it in one case, and by quantifying, using a structural model, the effect of underlying trends through changing behaviour on it in the other case.

In the literature, secular stagnation equilibria arise through excessive asset demand. The models distinguish the standard liquidity trap (Eggertsson and Krugman, 2012) and the safety trap. The former refers to an excess of liquidity in the economy - excessive demand of assets - in general, caused by exogenous drivers increasing saving propensity domestically (Eggertsson and Mehrotra, 2014) or internationally (Eggertsson et al., 2016), which affects all interest rates in the economy. Thus it is closely related to the argument of a global savings glut (Bernanke et al., 2005). The latter, on the other hand, is driven not by an increased saving propensity per se, but by an excessive demand for safe assets - a 'flight for safety' - through increased risk-aversion, also either domestically (Caballero and Farhi, 2014) or from abroad (Caballero et al., 2015), which particularly pushes down the safe interest rate and creates a divergence between safe and risky rates[1]. This presents therefore a framework of two channels to consider, how savings behaviour may affect the interest rate. The liquidity channel which raises saving propensity in general and the safety channel which affects the preferences between risky and risk-free assets[2].

As the cyclical pattern and the diversity of existing interest rates,

[1]In equilibrium it may also drive down the risky rates, albeit to a lower degree.

[2]Iachan et al. (2015) propose financial innovation as an additional channel for the decline of the interest rate. This, however, is not a channel for saving decisions to affect the interest rate and therefore less related to this paper, as the paper decouples the investment from the underlying saving decision.

which exhibit a declining trend to different degrees, complicate the analysis of whether there is a secular decline of the interest rate, empirical research has tried to estimate the natural interest rate, an economy-wide interest rate that supports full employment at a given moment in time. Laubach and Williams (2003) and Johannsen and Mertens (2016) estimate a fall of 2.94 and 0.74 percentage points between 1984 and 2013. The question of the empirical decline and the diversity of interest rates will be discussed further in Section 3.1.

Finally, other research, as in the vein of this paper, designed theoretical models which quantify the effect of exogenous underlying trends on the interest rate through adjusted savings behaviour. Barro and Mollerus (2014) and Hall (2016) postulate the importance of the safety channel, and highlight that a rise in risk-aversion drives down the safe interest rate and increases the equity premium. However, an increasing number of papers have considered the liquidity channel and particularly secular trends within the United States. The most prominent trend in this area has been demographics (Carvalho et al., 2016; Gagnon et al., 2016). In these models, a fall in population growth and an increase in life expectancy, raise the dependency ratio and the latter also increases saving propensity through consumption smoothing[3]. The relative importance of each of the two has been debated. Carvalho et al. (2016) highlight the importance of life expectancy, while Gagnon et al. (2016) argue for the significance of population growth decline due to the effect on family composition. The most similar paper to this one, however, is by Eggertsson et al. (2017) who also consider not only demographics, but also, similar to Thwaites (2015) the price of investment, and more importantly for this paper a decline in productivity growth and an increase in inequality. Analysis of data on growth supports the notion of a secular slow-down in technological growth, particularly since the early 2000s (Antolin-Diaz et al., 2016; Fernald, 2015). Fernald and Jones (2014) argue that the vanishing transitional dynamics of increases in educational attainment and research intensity during the latter half of the 20th century, which permitted higher growth rates than today, are a potential line of reasoning for this trend. The relationship between the growth rate and the interest rate is sup-

[3]This assumes a constant retirement age, which is in line with OECD data on the average effective retirement age in the United States, which has not changed significantly during the last 30 years.

ported theoretically[4] and corroborated empirically through a strong comovement between estimated falling GDP growth and the natural rate of interest (Holston et al., 2017). As growth dynamics are not only an important driver of the interest rate, but also for the saving rate (Chen et al., 2006), this paper will employ this slow-down to reconcile the decline in the interest rate with the stylised facts of saving rates in the United States. Moreover, it will focus on inequality in a novel dimension.

Inequality and (retirement) saving

The negative relationship between inequality and the interest rate has been posited through the advent of heterogeneous agent models (Aiyagari, 1994; Huggett, 1993). Indeed, most models identifying trends in inequality as a driving force in the interest rate employ this framework, in which growing income inequality raises precautionary savings and depresses in turn the interest rate (Auclert and Rognlie, 2016; Benigno and Fornaro, 2015; Eggertsson and Mehrotra, 2014; Eggertsson et al., 2017; Favilukis, 2013). This hypothesis, while internally valid, exhibits issues of external validity as an increase in income inequality produces a counterfactual decline in wealth inequality. A well-known fact, however, at least since the work by Piketty and Saez (2003) as well as Saez and Zucman (2016) is, that in the data the rise in income and wealth inequality go hand in hand.

A different channel on how inequality may affect saving in line with an increase in wealth inequality is offered by the literature on retirement saving, which recognises asymmetric saving propensities for old age. The drivers of the asymmetric saving propensities, manifested in heterogeneity of wealth at retirement and of decumulation rates, are difficult to disentangle, as all indicate higher saving propensities for high income agents (De Nardi et al., 2010). One factor of this may be precautionary saving due to heterogeneous medical expense risks, which increase exponentially with age and are considerably higher for high income households (De Nardi et al., 2010). Another factor may be a skewed bequest distribution, generated by higher bequest utility

[4] A simple neoclassical growth model with an infinitely lived representative agent suggests an elasticity of the interest rate to the growth rate equal to the intertemporal elasticity of substitution.

for wealthy agents, which also helps to match the heterogeneity of wealth at retirement (De Nardi and Yang, 2014). Finally, differences in income conditional life expectancy may also explain this heterogeneity (De Nardi et al., 2009). Moreover, this heterogeneity not only plays a role to explain wealth heterogeneity at retirement, but also for the whole cross-section (De Nardi, 2004). Thus, asymmetric saving for old age, will generate an increasing wealth inequality with income inequality in line with the data, but also create saving propensities which increase with income as observed in the data (e.g. Jappelli and Pistaferri, 2010).

This paper will therefore offer an explanation of the decline of the interest rate through the liquidity channel based on domestic factors, employing the known demographic trends, and extending them by the secular trends of income inequality and productivity growth, which add to the effect on the interest rate on one hand and reconcile it with the implications on savings on the other.

3 Secular trends in the United States

The paper shows the link between different trends in the U.S. economy and disentangles the effects of five underlying trends on the real interest rate and the saving dynamics within the economy. This section thus illustrates firstly the trends which will be explained, the interest rate and saving dynamics, and then comes to the underlying trends which are employed to explain this pattern and are supposed to be exogenous within this paper for simplicity.

3.1 Interest rate

Since the 1980s there has been a secular decline in the real interest rate, observable in a number of different metrics of the interest rate. Figure 1 shows two distinct interest rates between 1982 and 2013. As previously noted, the safe-interest rate, as measured by the 1-year constant maturity treasury return adjusted by the Survey of Professional Forecasters median inflation expectations, exhibits the strongest decline by 7 percentage points. However, also more general measures support the hypothesis of a secular decline in the interest rate, albeit the fall is not as pronounced. The literature has estimated the decline in the natural interest rate between 1984 and 2013 to be 0.74[5] (Johannsen and Mertens, 2016) and 2.94[6] (Laubach and Williams, 2003) percentage points. All these measures are problematic. The safe interest rate does not reflect the return on agents' total financial assets, which, however, is the quantity of interest in this paper. The estimates of the natural interest rate on the other hand are questionable due to the large gap between the estimates and large confidence intervals in both estimations.

This paper will therefore employ a different measure which overcomes both of these issues. Following Glover et al. (2011) a rate of return on all financial assets is derived directly from the data (Survey of Consumer Finances), allowing for the portfolio composition of the agents[7]. This rate is also illustrated in Figure 1 and exhibits a similar decline to Laubach and Williams (2003), namely 3.16 percentage points between 1982 and 2012.

3.2 Saving rates

At the same time the most prominent measure of the saving rate, the personal saving rate as reported by NIPA has also fallen continuously. This measure includes saving as a percentage of disposable income. It has more than halved from 10.6% in 1984 to 5% in 2013 with a through of 2.6% in 2005. This rate, however, may not be the most compelling rate for this paper's analysis. Alternative saving rates include consumption of consumer durables and capital holding gains. Figure 2 shows four alternative saving rates.

The inclusion of consumer durables raises the saving rate, but represents a similar, slightly more subtle, declining trend from 22.5% (1984) to 15.0% (2013). The consideration of consumer durables is important, as in a model without explicit durable consumption, as the one in this paper, part of the durable consumption will be represented in savings. More importantly, however, is that once holding gains

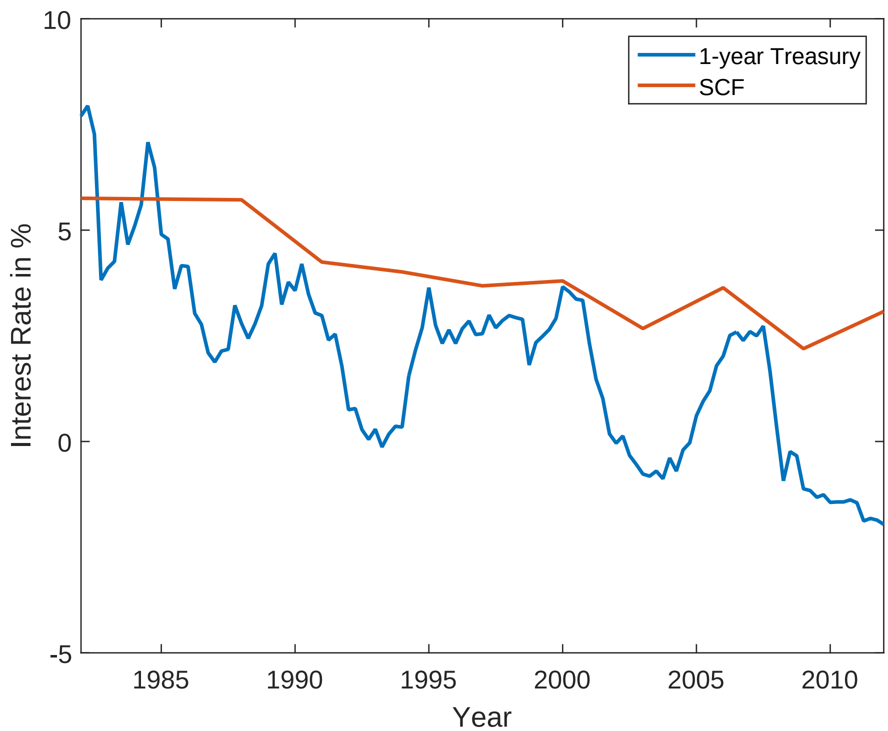
[5] Smoothed estimates in 1984Q1 was 1.71% and in 2013Q4 0.97%.

[6] Two-sided updated estimates are 3.22% in 1984Q1 and 0.28% in 2013Q4

[7] The construction of this rate follows Glover et al. (2011). It takes the sum of interest or dividend income, capital gains, and a third of business, self-employment, or farm income and divides it by total assets excluding defined benefit pension and deducting vehicles and the primary residence.

from capital are included the decline of the saving rate disappears and it becomes relatively flat, however, with significant volatility despite smoothing it through a 5-year centred rolling average. The saving rate including holding gains and consumer durables hardly changed from 18.4% in 1984 to 18.2% in 2013. Holding gains are an important factor for the consideration of saving, when agents smooth their consumption subject to their life-time wealth. In other words, an increase in holding gains in turn increases life-time income, without affecting current income, so that given consumption smoothing, it induces a fall

Figure 1: Real Interest Rate

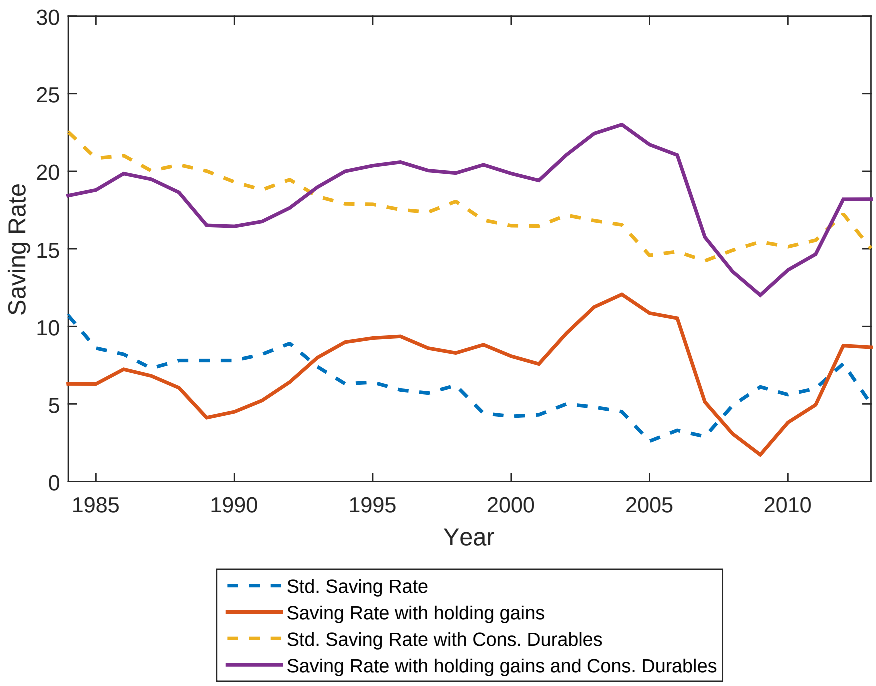


Note: The 1-year treasury refers to the 1-year fixed maturity treasury of 1-year from FRED, adjusted by the median inflation expectation of the Survey of Professional Forecasters. The SCF rate is computed as the real return on assets in the Survey of Consumer Finances as proposed by Glover et al. (2011).

in the personal saving rate which does not reflect lower propensities to save. For this reason this broader saving rate will be used to analyse the model. However, even with this saving rate there remains an oxymoronic coexistence of a domestic savings glut, driven by increasing saving propensity, with a constant saving rate in the economy, which will be shown to be possible through a decline in the growth rate of the economy and forward looking agents.

The saving rate, however, is highly heterogeneous across agents. It has been previously documented that the propensity to save increases with income (e.g. Jappelli and Pistaferri, 2010). However, a novel stylised fact presented here is, that for a certain age group, which is in

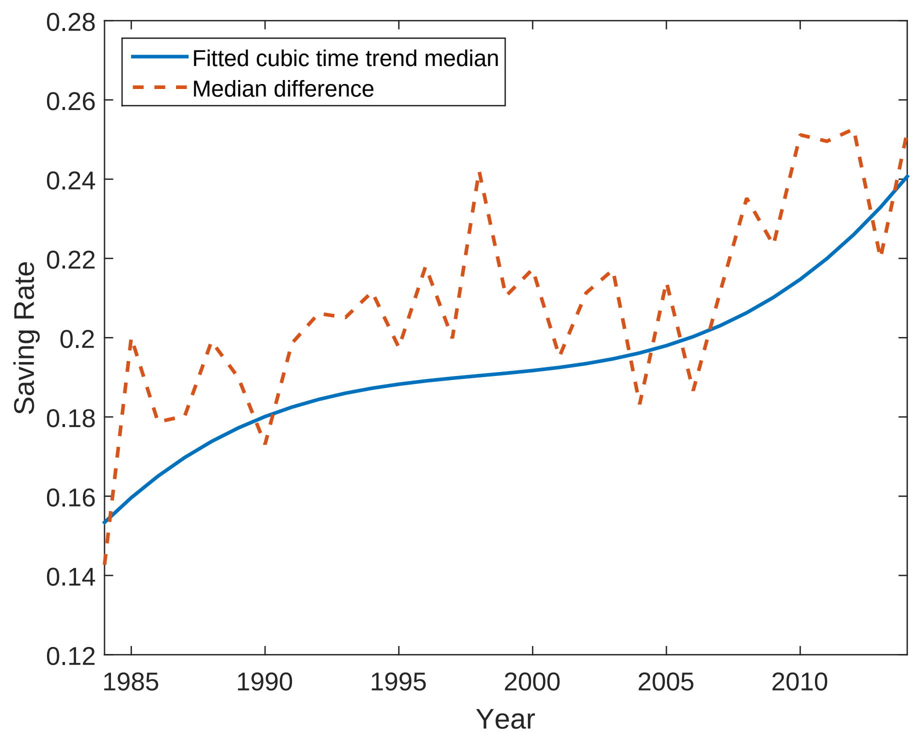
Figure 2: Aggregate Saving Rate



Note: The data is taken from NIPA. The saving rates which include holding gains have been smoothed and represent a 5-year centred rolling average of the actual saving rates.

prime age for retirement saving (35-44 year-olds), there is a diverging trend in the difference between saving rates by income. The focus on a certain age group captures changes in the saving pattern for retirement over generations, as older generations leave the sample quicker

Figure 3: Difference in income conditional median saving rates (bottom 80% vs top 20%, 35-44 year-olds)



Note: The saving rates are obtained from the interview (family) survey of the Consumer Expenditure Survey 1984-2014. It is constructed as the residual share of family income after subtracting the total (annualised) expenditure. The sample annualises each quarterly observation and treats it as a full cross-section. The fitted trends are obtained through a pooled regression on a cubic polynomial time trend interacted with a dummy variable for the top 20% income as well as the dummy variable itself, controlling for age, family structure and an overall cubic time trend. The regression is a quantile regresion on the median.

and do not bias the results. An additional measure to assess changes in saving for retirement is net worth at retirement, which seems to have increased over time (see Appendix A.1), however, is less reliable, particularly when considering differences across incomes, due to the strong effect of asset prices in this period. Figure 3 illustrates the divergence of saving rates, firstly through the raw differences in the median saving rate of the top 20% income group and the bottom 80% income group, and secondly through the fitted cubic trend of the following pooled quantile regression of the median saving rate

$$s_{i,t} = \mathbf{W}_{\mathbf{i}}'\mu + \mathbf{T}'\theta + \beta_1 I_i + \beta_2 I_i t + \beta_3 I_i t^2 + \beta_4 I_i t^3 \tag{3.1}$$

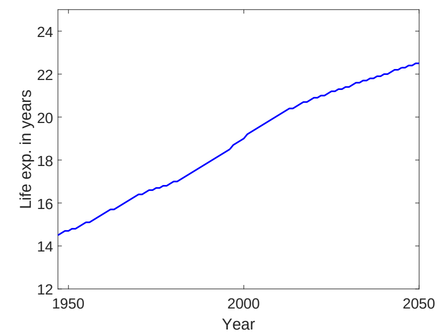
The dependent variable of the pooled regression is the saving rate $s_{i,t}$, which is obtained as the residual total household income after subtracting total household expenditure. The term W_i controls for the age of head, age of spouse, number of males (females) age 16 and over in the household, number of males (females) age 2 to 15 in the household, and number of household members under 2. The second term T' describes a cubic polynomial in time, which is normalised to zero in 1984, while the final four terms capture the actual divergent time trend between the two income groups, by interacting a cubic polynomial with the dummy variable I_i which is one for the top 20% income group and zero otherwise.

Thus the stylised facts regarding saving are twofold, firstly a relatively stable, possibly decreasing, aggregate saving rate, and secondly diverging income conditional saving rates, with a widening gap in saving propensities between low and high income families.

3.3 Underlying trends

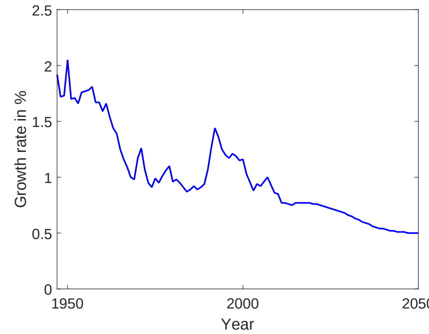
It will be shown, that a number of underlying trends, can account for these two stylised facts of saving and a contemporaneous strong decline in the interest rate. This section will exhibit the five exogenous secular trends (Figure 4), which will be fed into the model. All of these will have a negative impact on the interest rate, while an asymmetric development of income conditional life-expectancies and a decline in productivity growth are particularly important to generate the observed saving patterns. While the focus lies on the period between 1984 and 2013, the figure represents data from 1947 till 2013 - until 2050 in the case of aggregate demographics, which are forecast

Figure 4: Exogenous secular trends



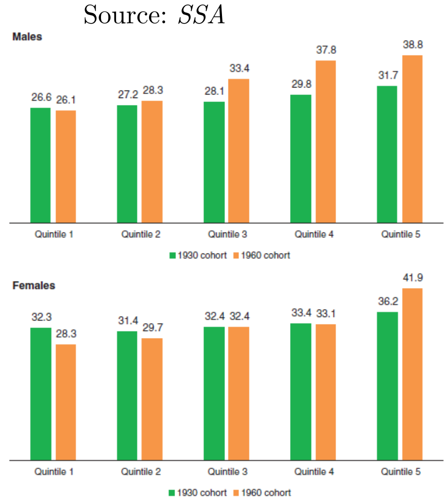
(a) Life Expectancy at the age of 65

Source: *SSA*



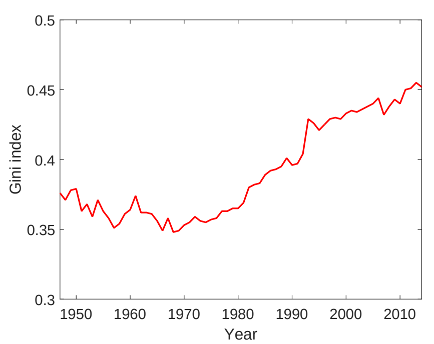
(b) Population growth

Source: *US Census*



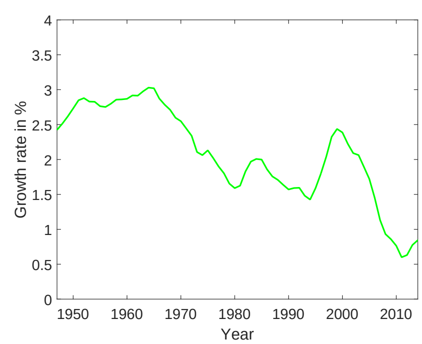
(c) Income and gender conditional life expectancies by cohort at 50

Source: *Lee et al. (2015)*



(d) Income inequality (GINIALLRF)

Source: *FRED*



(e) Labour productivity growth

Source: *Antolin-Diaz et al. (2016)*

relatively reliably - as this longer time series will be employed in the dynamic model.

The first two trends are well-known aggregate demographic changes (Figures 4a and 4b). The conditional life expectancy at retirement (age 65) increases steadily and relatively linearly, while population growth exhibits the opposite trend[8].

The third and fourth panel (4c and 4d) comprise the heterogeneous changes. The latter one shows the income GINI index and the former one are estimates of variation in life-expectancies for 50 year-olds by gender and permanent income quintile for two generations[9] - the cohorts born in 1930 and 1960 respectively (Lee et al., 2015) - which represent relatively well the retiring agents in 1984 and 2013 respectively. Comparing the life-expectancies, it is clear that the life expectancies increase by income group. However, most striking is the fact that for low income groups life-expectancies have not actually increased, particularly among women. As this study, considers the increase of income inequality driven by the top, it will pool the bottom 80% of men and women and the top income quintile of men and women. The respective life expectancies are then computed assuming that the differences are unchanged at the age of 65 and ensuring that the resulting average life expectancy matches the aggregate life expectancies provided by Social Security data.

The final panel (4e) shows estimates of the long-run labour productivity growth. Antolin-Diaz et al. (2016) estimate a multi-factor model of long-run growth and show that the development in long-run labour productivity has been driving a decline in the long-run growth rate particularly since the early 2000s. In this paper technological change will be labour-augmented, so that the labour productivity growth data is ideal to feed into the model.

[8] Note that there exists a spike in the early 90s, which may be due to changes in the census data collection, as the growth rate is obtained from Historical data, Intercensal Data in the 1990s and 2000s and National Population Projections 2012.

[9] The permanent income is obtained as non-zero earnings reported to Social Security between the age of 41 and 50. If the household comprised more than one individual, the total household income is divided by the square root of 2 to account for economies of scale and assigned to both people.

Table 1: Summary of underlying secular trends

Change	1984	2013
Population Growth	1.33%	1.01%
Average Life Expectancy at 65	17.3	20.4
Life Expectancy Gap	3.8	9.21
Income Inequality (GINI)	0.363	0.412
Labour Productivity Growth	2.01%	0.84%

This highlights a number of secular trends which have taken place between the early 80s and today, which will be taken as exogenous in the model. The overall changes between 1984 and 2013[10]- which will be used in the static model - are summarised in Table 1.

4 Model

4.1 Theoretical model

The model is a relatively simple perfect-foresight, closed economy, neoclassical growth model with overlapping generations, which, however, can account for the five underlying trends reported above. It comprises ex-ante heterogeneity across two types in labour productivity, life-expectancy, preferences, and inheritance. The following representation focuses on the static problem, thus abstracts from the time dimension as far as possible, for simplicity, as time variation can be incorporated relatively easily but makes the notation cumbersome.

Technology

The production is given by a standard neoclassical technology with a Cobb-Douglas production function

$$Y_t = K_t^{\theta}(z_t L_t)^{1-\theta}$$

where K_t and L_t are aggregate stock of capital and labour supply respectively, and z_t is the average labour productivity in the economy,

[10]Note that population growth and Gini index are 38- and 40-year backward looking averages, to account for changes over the working-life, considering the average as steady state values.

which grows exogenously by γ so that it evolves according to the following law of motion

$$z_{t+1} = (1+\gamma)z_t$$

The rental rates of the inputs equal their respective marginal productivities, as markets are competitive

$$\omega_t = F_L(K_t, z_t L_t)$$
$$r_t + \delta_K = F_K(K_t, z_t L_t)$$

The depreciation of capital is denominated by δ_K and considered to be constant[11].

Households and their Preferences

Each period a mass $(1+\eta)^t$ of new agents is born, where η is the exogenous population growth rate. Each cohort is born with two types of agents $i = H, L$, a measure μ of the high-type and a measure of $1-\mu$ of the low-type[12] which differ first of all in their labour productivity ϕ_i, such that the average productivity equals 1

$$\mu\phi_H + (1-\mu)\phi_L = 1 \tag{4.1}$$

The agents live with certainty for R periods, during which they supply labour inelastically, earn type-specific labour income net of tax $(1-\tau)\phi_i\omega$, and die with certainty after N periods. During retirement they face a geometrically increasing death probability with age g of $1-\lambda_i^{g-(R-1)}$, where $\lambda_i \in [0,1]$ is the survival probability of the respective type[13] and earn a state pension which is a fraction ξ of the last earned wage.

In period Ti the agents inherit a type-specific inheritance ι_i from the agents who died between the last and this period of their specific

[11] A potential secular increase in the depreciation rate, as proposed by Gomme and Rupert (2007), would cause a stronger decrease in the interest rate.

[12] The measure of each type is true at birth and remains so for each cohort during working-life. At retirement, as the low type has a higher death probability, the measure of high-type for a given cohort increases.

[13] Ideally, the model would consider the actual death probability process, which can be obtained by age, however, not by age and permanent income, so that the process of income conditional death probabilities is constructed in this way.

type. These inheritances occur not only due to accidental bequests, as the model does not offer annuity markets, but also due to a warm-glow bequest motive. The bequest motive allows for an additional channel how saving for retirement may have changed over time. De Nardi et al. (2010) highlight, how large medical expenses are an important driver in savings for old-age and how strongly these vary by income. Without modelling medical expenses explicitly, this model therefore aims to obtain the same pattern through a joy-of-giving parameter on one hand and preference heterogeneity on the other. De Nardi et al. (2010) also outline how non-homothetic preferences may capture the medical expense risks. Instead of using a non-homothetic bequest motive, this model, drawing on literature which has also used heterogeneity in the discount factor (Carroll et al., 2014; Krueger et al., 2016; Krusell and Smith, 1998), will allow for a different discount factor between the two types in order to account for differences in income-contingent saving rates to mimic the differences in expected medical expenses and bequests.

Thus, the household problem for each type is given during working-life, where they face no uncertainty by

$$\max_{c_i,a_i} \sum_{t=0}^{R-1} \beta_i^t \frac{c_{i,t}^{1-\sigma}}{1-\sigma} + \beta_i^R V_i(a_{i,R})$$

subject to the intertemporal budget constraint and a no-borrowing constraint

$$\sum_{t=0}^{R-1} \frac{c_{i,t}}{(1+r)^t} + \frac{a_{i,R}}{(1+r)^R} \leq \sum_{t=0}^{R-1} \frac{(1-\tau)\phi_i(1+\gamma)^t\omega}{(1+r)^t} + \frac{\iota_i}{(1+r)^{Ti}}$$

$$a_{i,t} \geq 0 \qquad \forall i,t$$

where $i = L, H$ denotes the type of agent. During retirement then the agent faces survival uncertainty and each type of agent solves the following value function

$$V_{i,t}(a) = \max \left\{ \frac{c_i^{1-\sigma}}{1-\sigma} + \lambda_i^{t-(R-1)} \beta_i V_{i,t+1}(a_i') + (1-\lambda_i^{t-(R-1)})\alpha\beta_i \frac{(a_i')^{1-\sigma}}{1-\sigma} \right\}$$

subject to the budget constraint

$$c_i + a_i' \leq (1+r)a_i + \xi\phi_i\omega(1+\gamma)^{R-1}$$

Fiscal Policy

The fiscal policy is straight forward. The government provides a state-pension which is a fraction ξ of the last wage during working age and finances it through a labour tax. The government cannot borrow and thus taxation has to balance the costs of the state-pension. The government budget constraint is the following

$$\tau \omega L = \xi \frac{\omega}{(1+\gamma)} \left(\mu \phi_H m_H + (1-\mu)\phi_L m_L\right)$$

where m_i is the productivity adjusted mass of retired agents of type i

$$m_i = \sum_{t=R}^{N} \frac{\prod_{j=R}^{t} \lambda_i^{j-R}}{[(1+\eta)(1+\gamma)]^{t-R}} \qquad for \quad i = H, L$$

given that the survival probabilities are different for each type of agent and they receive different pension payments.

Perfect-Foresight Equilibrium

Definition 1. *A stationary perfect foresight equilibrium are prices* $\{r, \omega\}$, *allocations* $\{c_{i,t}\}_{t=0}^{N}$ *and* $\{a_{i,t}\}_{t=0}^{N}$, *fiscal policy* $\{\xi, \tau\}$, *and the inheritance* $\{\iota_i\}$ *such that*

1. *Given prices* $\{r, \omega\}$, *fiscal policy* $\{\xi, \tau\}$ *and inheritance* $\{\iota_i\}$ *the allocations* $\{c_{i,t}\}_{t=0}^{N}$ *and* $\{a_{i,t}\}_{t=0}^{N}$ *solve the household problem for both types (H,L).*
2. *Given allocation* $\{K, L\}$ *prices* $\{r, \omega\}$ *solve the firm's problem.*
3. *Given wages* $\{\omega\}$ *and transfers* $\{\xi\}$ *the tax rate* τ *balances the government budget constraint.*
4. *Labour markets clear.*

$$L = \frac{1+\eta}{\eta} \left[1 - \left(\frac{1}{1+\eta}\right)^R\right]$$

5. *Capital markets clear.*

$$K = \sum_{i \in H,L} \sum_{t=0}^{R-1} \left(\frac{1}{1+\eta}\right)^t a_{i,t} + \sum_{i \in H,L} \sum_{t=R}^{N} \left(\frac{1}{1+\eta}\right)^t a_{i,t} \prod_{j=R}^{t} \lambda_i^{j-R}$$

6. *Inheritance equals bequests for each type of agent*

$$\iota_i = (1+r)\left(\sum_{t=R}^{N-1} \frac{1-\lambda_i^{t+1-R}}{(1+\eta)^{t-Ti}} \prod_{j=R}^{t} \lambda^{t-R} a_{i,t} + \frac{\lambda^{N-R} a_{i,N}}{(1+\eta)^{N-Ti}}\right) \forall i \in H, L$$

7. *Product markets clear by Walras Law*

4.2 Calibration

The model will be calibrated to 1984, before the five exogenous changes will be fed into it. One set of parameters is determined exogenously (see Table 2). These include the parameters determining the life-cycle. Agents' working life is assumed to be 40 years, so that R=40 at an annual horizon. An effective retirement age of 65 in the United States, implies that the agents start their working life at the age of 25. They inherit at the age of 55, so that Ti=30. The cut-off age is chosen to be 100 (N=75), as the measure of agents surviving this age is very small.

Furthermore, the trends highlighted above allow to pin-down the demographic parameters $\lambda_H, \lambda_L, \eta$ as well as the productivity parameters γ, ϕ_H, ϕ_L. Note that ϕ_H pins-down ϕ_L through equation 4.1. Finally, Pensions at a Glance of the OECD provides data on the net replacement ratio of pensions. These are only available for more recent years, however, as the model assumes a constant net replacement rate the 2005 values (Queisser and Whitehouse, 2005) are taken to determine the net replacement ratio of the last wage given the net replacement ratio of the average working-life wage.

The choice of σ, which determines the elasticity of intertemporal substitution, is crucial when considering the effect of the exogenous changes on the interest rate[14]. In the benchmark model, it is taken in standard range for the U.S. as $\sigma = 1.5$, which implies an elasticity of intertemporal substitution of 0.66. Furthermore, a sensitivity analysis shows how changing this parameter will affect the decline of

[14]This is particularly the case for the effect of the productivity growth. Note that in an infinitely lived, representative agent, neoclassical growth model the interest rate is given by $\log R = \log\left(\frac{1}{\beta}\right) + \sigma \log(1+\gamma)$, so that σ multiplies the effect of productivity growth. Even though this simple relationship breaks down and the effect will be lower in this OLG framework, it still remains crucial.

Table 2: Exogenous Parameters

Parameter		Data		Source
μ	0.2	Share of agents of High-Type		
N	75	Certain death at	100	
R	40	Effective retirement age	65	OECD
Ti	30	Age of inheritance	55	
λ_H	0.996896	Life expectancy at 65	17.3	SSA2015
λ_L	0.995038	Life expectancy gap at 50	3.8	NAP2015
η	0.013	Population growth		US Census
γ	0.0201	Labour productivity growth		ADP2016
ϕ_H	2.815	Income GINI	0.363	US Bureau of the Census
ξ	0.348	Average net replacement rate	0.51	OECD2005
σ	1.5	1/EIS		

the interest rate, however, not in a fashion which changes the overall results of this paper.

The other set of parameters is endogenously calibrated by matching the net-worth to output ratio, the aggregate saving rate including capital holding gains and durable consumption, the saving rate differential, the average net worth at retirement in 1984 and the interest rate in 1982. The parameters which are determined endogenously are illustrated in Table 3 and the fit of the model comparing the moments in Table 4. The model mimics all moments relatively well, apart from the asset holdings at retirement. The joy-of-giving parameter raises asset holdings at retirement, however, not sufficiently to match the data with a sensible parameter value. The α parameter would have to rise to 10.6 - implying a 10.6 higher utility of bequests than consumption - to match net worth at retirement. Therefore the parameter is picked as one. Nevertheless, sensitivity analysis shows that the joy-of-giving parameter has virtually no effect on the results of this paper.

5 Comparative Statics

This section discusses the effects of the exogenous changes in demographics, inequality, and productivity growth (see Table 1) on steady state equilibrium outcomes. It focuses on the interest rate, aggregate

Table 3: Endogenous Parameters

Parameter	Name	Parameter value
β_H	Discount Factor High Type	0.98255
β_L	Discount Factor Low Type	0.9654
α	Joy-of-Giving	1
θ	Capital Share	0.3289
δ_K	Depreciation	3.40%

Table 4: Moments matched

		1984		2013	
Symbol	Moment	Data	Model	Data	Model
r	Interest rate	6.24%	6.24%	3.08%	4.36%
s	Aggregate Savings Rate	0.185	0.184	0.182	0.180
Δs_{35}	Differential Savings rate 35-44 y/o	0.153	0.153	0.241	0.193
$\frac{A}{Y}$	Net Worth to Output Ratio	3.41	3.41	4.76	4.24
A_{65}	Net Worth at 65-69 in per cap. output	8.45	7.98	12.00	8.79
$\frac{\iota}{Y}$	Annual Inheritance Flow		6.12%		3.82%
τ	Labour tax		8.37%		14.67%

saving rate, saving rate differential for 35-44 year-olds, and the annual inheritance flow.

The first subsection considers the benchmark model, calibrated as outlined previously. The exogenous changes generate a rising dependency ratio, redistribution to high income agents, a changing composition of retired agents in favour to high income agents, and an increase of retirees' productivity relative to the work-force. All these imply that the fiscal policy has to adjust through either of its two parameters (labour tax or net replacement rate) to balance the budget constraint. In the benchmark model labour taxation increases, providing more fiscal redistribution across ages.

Subsection 5.2 shows that the effect on the interest rate will be even stronger, if instead the labour tax remains constant and the net replacement rate decreases, as the private market has to take on part of the fiscal redistribution. Subsection 5.3 then considers varying intertemporal elasticities of substitution, corroborating the previous results. The following subsection 5.4 shows that different values for the joy-of-giving do not alter the results in any way. Finally, subsection 5.5 postulates the importance of the inequality dimension. A pure representative cohort model underestimates the effect on the interest rate by 20% and does not create the observed effect on the annual inheritance flow, due to the lack of composition effects during retirement.

5.1 Benchmark model

Calibrated to match the 1984 moments, the benchmark model also matches relatively well the moments in 2013 (Table 4). In particular the developments in the saving rate exhibit only slight quantitative differences. The two moments related to net worth of agents are less informative, given the strong asset price increases in stock prices in the 90s and house prices in the 2000s. In that respect, it is not surprising that the model stays far below the values in the data.

Table 5: Counterfactuals

Trend	Parameters	r	s	Δs_{35}	s_{35}^H	s_{35}^L	$\frac{\iota}{Y}$
Population Growth	$\eta\uparrow$	6.13%	0.187	0.151	0.203	0.052	5.99%
Wage Inequality	$\phi_H\uparrow, \phi_L\downarrow$	6.13%	0.187	0.155	0.210	0.056	6.10%
Life Exp. Gap	$\lambda_H\uparrow, \lambda_L\downarrow$	5.88%	0.192	0.197	0.235	0.038	4.94%
Agg. Life Expectancy	$\lambda_H\uparrow, \lambda_L\uparrow$	5.88%	0.192	0.169	0.224	0.055	5.22%
Inequality	$\phi_H\uparrow, \phi_L\downarrow,$ $\lambda_H\uparrow, \lambda_L\downarrow$	5.74%	0.195	0.199	0.229	0.030	4.87%
Agg. Demographics	$\eta\uparrow, \lambda_H\uparrow, \lambda_L\uparrow$	5.75%	0.195	0.169	0.213	0.044	5.09%
All excl. Prod. Growth	$\eta\uparrow, \phi_H\uparrow, \phi_L\downarrow,$ $\lambda_H\uparrow\uparrow, \lambda_L\uparrow$	5.28%	0.205	0.217	0.236	0.019	3.54%
Prod. Growth	$\gamma\downarrow$	5.16%	0.163	0.133	0.223	0.090	6.46%
All	$\eta\uparrow, \phi_H\uparrow, \phi_L\downarrow,$ $\lambda_H\uparrow\uparrow, \lambda_L\uparrow, \gamma\downarrow$	4.36%	0.180	0.193	0.249	0.056	3.82%
Model 1984		6.24%	0.185	0.153	0.214	0.061	6.12%

Interest rate

The model generates 60% of the 3.16 percentage point decline of the interest rate observed in the data (1.88 p.p.) when all exogenous changes are present. Table 5 summarises the model's counterfactual results, when certain exogenous changes are shut-down - i.e. only the given trend is changed to its 2013 value, while the others remain at their 1984 value - and Figure 5 illustrates these results for the interest rate. The main driver of the fall in the interest rate is the slowing productivity growth, which is stronger on its own (1.08 p.p.), than the other four drivers combined (0.96 p.p.).

Nevertheless, the model also highlights the importance of the other drivers. The aggregate demographic changes account for 0.49 p.p. decline in the interest rate, keeping the other drivers constant. Most strikingly, developments of inequality affect the interest rate even stronger than aggregate demographics (0.50 p.p.). The main channel how these two, both aggregate demographics and inequality work, is the composition of the retirees and the saving propensity linked to it. The rise in life expectancy increases the saving propensity for all agents and the share of agents with relatively high asset-holdings in the population - the retirees who saved for retirement and live longer now. The rise in the life expectancy gap, works in the same way, however, rather than across generations within each generation. It increases the life expectancy of the savings affine agents and the share of these agents - those who hold most assets - per cohort during retirement.

This is the major difference to the results by Gagnon et al. (2016) who consider household composition effects during working life. This increases the effect of population growth, while allowing for composition effects during retirement takes into account the different savings and asset-holding patterns of agents at old age and stresses the effect of life expectancy. This expresses itself in the fact that the effect of the rise in life expectancy is almost three times as strong as the slowdown in population growth. Even the rise in wage inequality has a slightly stronger effect than the slowdown in population growth.

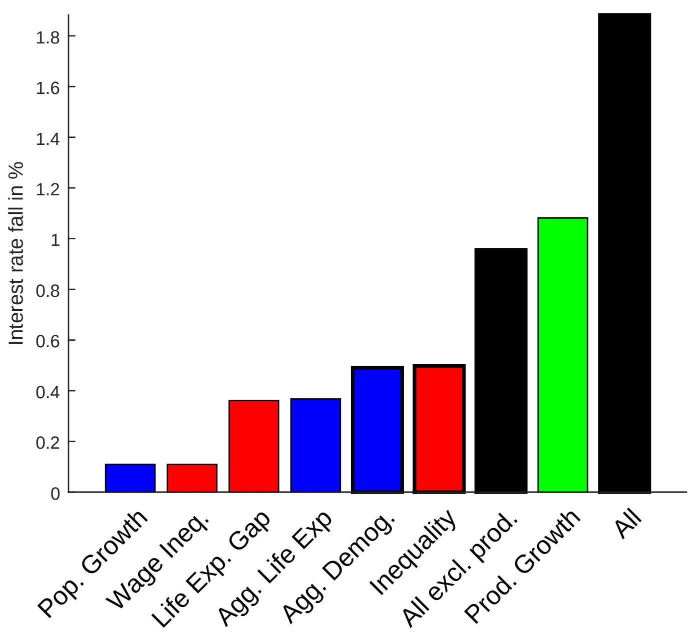
Aggregate saving rate

To analyse how these drivers affect the saving rate, it has to be reminded, that along the balanced growth path the saving rate has to be

$$s = (\delta_K + \gamma)K \tag{5.1}$$

The most important driver, again turns out to be the fall in productivity growth (γ), which generates two opposing forces. On one hand it reduces the saving rate directly, on the other it increases it through an increase in capital indirectly. The former effect, however, dominates strongly. The fall in productivity growth implies a decrease of the saving rate by 2.2 p.p. to 0.163 on its own. Nevertheless, the other four drivers increase the saving rate through their effect on capital accumulation. Therefore, overall the lack of an increasing saving rate highlights, that there has not been a sufficient drive for saving, to offset the negative effect of the productivity growth slowdown on the aggregate saving rate. The positive cumulative effect of the four other drivers - in the absence of any change in productivity growth - would have resulted in a saving rate of 0.205, which is an increase of 2 percentage points. The full model suggests a modest decline of savings of 0.5 percentage points to 0.180 replicating the relatively constant saving rate between 1984 and 2013 observed in the data.

Figure 5: Interest rate decline by secular trend



Note: The blue boxes denote aggregate demographic trends, the red boxes inequality trends, the green box denotes change in productivity growth. Black edges around the inequality and aggregate demographics denote the combined trends, while complete boxes are various trends combined.

Saving rate divergence

The saving rate for the age group 35-44 diverges in the model, albeit less than in the data. The dynamic model will be capable of matching the stronger divergence. The difference between the bottom 80% and the top 20%, however, increased even in the static model substantially to 0.193. This divergence is driven both from top and bottom. The saving rate of the top 20% has increased 16%, while the main part of the population, the bottom 80% decreased their saving rate by about 9%. This highlights, how, the endogenous effect of the interest rate offsets the exogenous effects of increased life expectancy for the bottom 80%. The main driver in the divergence of the saving rates, is naturally, the increasing gap in life expectancy. However, also the declining interest rate discourages the bottom 80% more from saving than the top 20%. Even an aggregate increase in life expectancy across types, increases the saving rate of the high type slightly (from 0.214 to 0.224) and decreases simultaneously the one of the low type (from 0.061 to 0.055). Productivity growth on the other hand decreases the difference at this age, which can be explained through the overall front-loading of saving due to a flatter wage profile. This seems to have a stronger effect for the bottom 80% than for the top 20%.

Annual inheritance flow

Piketty and Zucman (Piketty and Zucman, 2014, 2015) argue that the inheritance flow has been rising and will likely remain so, because, firstly, the net worth to output ratio, and secondly, the wealth transmission increases. This argument can be illustrated through the following accounting relationship

$$\frac{\iota_t}{Y_t} = \Phi_t m_t \frac{K_t}{Y_t}$$

where $\frac{\iota_t}{Y_t}$ is the annual inheritance flow. Φ_t is the ratio of net worth of decedents to average net worth of the population and m_t is the mortality rate, which together imply wealth-transmission. Finally, $\frac{K_t}{Y_t}$ is the net worth to output ratio. This model, however, provides an opposing argument, in line with Wolff and Gittleman (2014) who lack to observe an inheritance boom in the data. In this model, the net worth to output ratio increases considerably, nevertheless the annual inheritance flow decreases notably. The main reason is that the rise in net worth is driven by consumption smoothing and particularly due to the richer households. Nevertheless, these households live longer too, and thus use up their resources. Therefore by the time of death, the assets are mostly dissaved, which decreases the net worth of decedents to average net worth ratio (Φ_t) strongly and thus decreases the annual inheritance flow. When only considering the decline in the productivity growth, one observes a slight increase in the annual inheritance flow as argued by Piketty and Zucman (2015), however, when allowing for the other secular developments the annual inheritance flow declines by 40%. This fall is driven almost entirely by the developments in life expectancy (aggregate and gap) - shutting all other drivers out the inheritance flow is 3.73% compared to 3.82% in the full model and 3.54% when keeping only productivity growth constant. In other words, the savings in total increase which raises net worth, but the rise in life expectancy more than offsets this effect. Moreover, as net worth increases asymmetrically, just like life expectancy, this effect is intensified as the wealthy agents leave less accidental bequests. Therefore, the rise in inequality, diminishes the importance of inheritance.

5.2 Alternative fiscal adjustment

The demographic drivers, both increasing life expectancy and slowing population growth increase the dependency ratio. In the model it increases from 23% to 28%. Moreover, the falling productivity growth, also increases the transfers trough an increase of the pension payments relative to current wages. Thus, in the benchmark model the labour tax almost doubles from 8.34% to 14.67% in order to balance the government budget constraint.

An alternative fiscal policy is to fix labour taxation at the former level, and adjust the net replacement rate of pensions to wages. It is isomorphic to a pension reform to reduce pension payments in order to balance fiscal spending. This policy reduces the state redistribution towards old age and therefore increases the private saving propensity which has an additional effect on the interest rate. Table 6 shows the results for the full model in the two cases. It highlights, that the fiscal redistribution across ages through increasing labour taxation avoids an additional 0.70 p.p. decline of the interest rate, as it only falls to 4.36% compared to 3.66% under the alternative fiscal policy. The model with constant labour taxation increases private savings, which results in higher asset holdings, and a slightly higher annual inheri-

Table 6: Alternative Fiscal Adjustments

Symbol	Moment	Adjustment τ	NRR	Data
r	Interest rate	4.36%	3.66%	3.08%
s	Aggregate Savings Rate	0.180	0.198	0.182
Δs_{35}	Differential Savings rate 35-44 y/o	0.193	0.203	0.241
s_{35}^{H}	Saving rate 35-44 y/o high-type	0.249	0.263	
s_{35}^{L}	Saving rate 35-44 y/o low-type	0.056	0.060	
$\frac{A}{Y}$	Net Worth to Output Ratio	4.24	4.66	4.76
A_{65}	Net Worth at 65-69 in per cap. output	8.79	10.10	12.00
$\frac{\iota}{Y}$	Annual Inheritance Flow	3.82%	4.12%	
τ	Labour tax	14.67%	8.37%	
NRR	Avg. net replacement rate	0.51	0.271	

tance flow due to accidental bequests. The additional saving is mainly driven by the richer households which are more patient. Therefore the saving rate of these increases strongly while for the poor only slightly, increasing the saving rate differential.

Overall, the alternative fiscal adjustment model seem to match the data better in general, as it captures more of the decline in the interest rate, the divergence in the saving rates, and of the increase in asset holdings. However, as outlined before, due to asset price developments, the asset holding moments cannot be taken as a clear target. Moreover, while it captures the divergence better, it also implies a relatively strong increase in the saving rate, which is counterfactual.

5.3 Intertemporal elasticity of substitution

An important parameter for the effect on the interest rate is σ, as it captures the intertemporal elasticity of substitution. A value of 1.5 (IES=0.66) as in the benchmark model is a standard value for the U.S., however, Table 7 outlines two different calibrations of the model for log utility ($\sigma = 1$) and $\sigma = 2$ - matching the 1984 moments of the benchmark model - to illustrate the sensitivity of the results to this parameter.

Firstly, it is to note that the interest rate effect between the two

Table 7: Alternative Intertemporal Elasticities of Substitution

Symbol	Moment	Models			Data
σ	1/IES	1	1.5	2	
β_H	Discount Factor High Type	0.96889	0.98255	0.995	
β_L	Discount Factor Low Type	0.95756	0.9654	0.972	
α	Joy-of-Giving	2.31	1	0	
θ	Capital Share	0.3289	0.3289	0.3289	
δ_K	Depreciation	3.40%	3.40%	3.40%	
r	Interest rate	4.88%	4.36%	3.41%	3.08%
s	Aggregate Savings Rate	0.169	0.180	0.190	0.182
Δs_{35}	Differential Savings rate 35-44 y/o	0.185	0.193	0.202	0.241
$\frac{A}{Y}$	Net Worth to Output Ratio	3.98	4.24	4.47	4.76
A_{65}	Net Worth at 65-69 in per cap. output	8.40	8.79	9.18	12.00
$\frac{\iota}{Y}$	Annual Inheritance Flow	4.16%	3.82%	3.84%	
τ	Labour tax	14.67%	14.67%	14.67%	

extreme values for σ does not vary extremely. Considering $\sigma = 1$ a lower bound, the minimum effect on the interest rate is 1.36 p.p., while the choice of parameter in σ may add up to 0.95 percentage points in the case of $\sigma = 2$. While as expected the results are sensitive to the choice of σ, the majority share of the effect is independent of this choice, as even the minimum effect captures 40% of the total decline in the interest rate.

Secondly, the saving rates suggest that the correct parameterisation for σ is close to the benchmark model and if any, more towards $\sigma = 2$, indicating a stronger effect on the interest rate. As the aggregate saving rate and the differential between the saving rates for 35-44 year-olds increase with σ and would be too low otherwise. The relationhsip between the moments and the choice of the intertemporal elasticity of substitution are very straight forward. The lower the intertemporal elasticity of substitution, the more the agents save and so the asset holdings increase and the interest rate decreases. The only moment, which is non-linear in σ is the annual inheritance flow, which is highest for the logarithmic utility, decreases at $\sigma = 1.5$ and then increases again for $\sigma = 2$. The reason for this is, that despite lower asset holdings with logarithmic utility, agents dissave much slower in old age in this calibration due to a higher joy-of-giving parameter,

Table 8: Alternative Joy-of-Giving

Symbol	Moment	Models			Data
α	Joy-of-Giving	0	1	10.6	
σ	1/IES	1.5	1.5	1.5	
β_H	Discount Factor High Type	0.9834	0.98255	0.97776	
β_L	Discount Factor Low Type	0.9661	0.9654	0.9603	
θ	Capital Share	0.3289	0.3289	0.3289	
δ_K	Depreciation	3.40%	3.40%	3.40%	
r	Interest rate	4.37%	4.36%	4.35%	3.08%
s	Aggregate Savings Rate	0.180	0.180	0.180	0.182
Δs_{35}	Differential Savings rate 35-44 y/o	0.194	0.193	0.200	0.241
$\frac{A}{Y}$	Net Worth to Output Ratio	4.24	4.24	4.25	4.76
A_{65}	Net Worth at 65-69 in per cap. output	8.74	8.79	8.92	12.00
$\frac{\iota}{Y}$	Annual Inheritance Flow	3.50%	3.82%	5.36%	
τ	Labour tax	14.67%	14.67%	14.67%	

which is needed to match the asset holdings at retirement from the benchmark calibration for 1984 with a high intertemporal elasticity of substitution.

5.4 Joy-of-Giving

The previous subsection noted the effect of the joy-of-giving parameter on the inheritance level. Nevertheless, this subsection shows that apart from the levels of inheritance flow and net worth at retirement, the choice of this parameter does not affect any of the results. In fact, Table 8 illustrates that the results are almost identical with different calibrations of α to the benchmark model's moments. Solely net worth at retirement and inheritance increase with α, unsurprisingly. In terms of the trends, the increase in net worth becomes smaller with α (0.48 for $\alpha = 10.6$ and 0.9 for $\alpha = 0$) while the decline in the annual inheritance flow increases with it (3 p.p. for $\alpha = 10.6$ and 2.2 p.p for $\alpha = 0$). The general results on these trends nevertheless, still hold.

5.5 Representative cohort model

The model draws on ex-ante heterogeneity across agents in every cohort and shows that the rise in inequality can explain 0.5 p.p. in the

Table 9: Representative Cohort Model

Symbol	Moment	Models		Data
σ	1/IES	1.5	1.5	
β_H	Discount Factor High Type	0.98255	0.97659	
β_L	Discount Factor Low Type	0.9654		
α	Joy-of-Giving	1	3.25	
θ	Capital Share	0.3289	0.3289	
δ_K	Depreciation	3.40%	3.40%	
r	Interest rate	4.36%	4.73%	3.08%
s	Aggregate Savings Rate	0.180	0.172	0.182
$\frac{A}{Y}$	Net Worth to Output Ratio	4.24	4.05	4.76
A_{65}	Net Worth at 65-69 in per cap. output	8.79	8.67	12.00
$\frac{\iota}{Y}$	Annual Inheritance Flow	3.82%	6.83%	
τ	Labour tax	14.67%	13.07%	
	Dependency ratio	27.69%	27.72%	

decline of the interest rate. This subsection analyses the decline of the interest rate in a model which abstracts from any cohort heterogeneity and only considers the aggregate trends.

Table 9 shows that a calibrated representative cohort model also understates the effect on the interest rate by 0.37 p.p. or 20%. Moreover, it predicts a relatively pronounced decline in the saving rate and only slightly exhibits the fall of the annual inheritance flow (6.83% from 6.27%). This is due to a higher degree of accidental bequests, as everybody has an equal amount of assets and faces the same death probability. In the heterogeneity model, however, the people who hold the majority of assets do not face a high probability of dying early in retirement, thus the accidental bequests are lower. The lacking complexity of the composition of agents during retirement is also exhibited in the relatively small labour tax (13.07% compared to 14.67%) despite a slightly higher dependency ratio. As all agents receive the same pension, whereas in the heterogeneous agent model the agents with high pensions outlive the agents with low pensions, the total pension costs are higher in the heterogeneity model.

Therefore, the inclusion of inequality does not only help to consider the different trajectories of saving for rich and poor agents, but also to firstly, not understate the decline in the interest rate, and secondly, to explore the path for inheritance in the future.

6 Dynamic Model

This section extends the previous analysis dynamically, establishing the evolution of savings and interest rate over time. It thus introduces perfect foresight expectations on the future paths of the variables. The first part of the section outlines the calibration of the dynamic model, which differs from the static one. The latter part then discusses the results in relation to data and through counterfactual experiments.

6.1 Calibration

The calibration of the dynamic model departs completely from the static model. It therefore is not just a simple transition between the two previously computed steady states. Instead it will be a transition between steady states in 1947 and 2100, with a focus on the period between 1984 and 2013. This way the model allows for a 37-year phase-in period before 1984 and a phase-out period after 2013. The phase-in period ensures that the 1984 moments are obtained from the time series of the exogenous parameters, while the initial steady state loses its importance[15]. The phase-out period in turn ensures that agents optimise with rational expectations about the future. This relates particularly to the aggregate demographic variables - average life expectancy and population growth - which are relatively reliably forecast.

The exogenous time paths fed into the model for 1947 till 2013 are the actual processes. For the GINI index and labour productivity growth the values between 2013 and 2100 are fixed at the 2013 value, while average life expectancy and population growth are allowed to evolve along their forecast values until 2050 and then the final value is carried forward until 2100 for the model to converge smoothly to a

[15] The phase-in and phase-out periods are helpful, as it is highly unlikely that any choice of steady state is an actual steady state. The choice of 1947 is due to data availability. It is the first year for which data for all exogenous processes is available. The year 2100 is chosen to be sufficiently far away from 2013 to allow a normal evolution.

Table 10: Dynamic Model Endogenous Parameters

Parameter	Name	Parameter value
β_H	Discount Factor High Type	0.9826
β_L	Discount Factor Low Type	0.959
α	Joy-of-Giving	0.19
θ	Capital Share	0.29
r_{1947}	Interest rate (1947)	7.239%

new steady state. Figure 4 depicts the time paths of these four trends and the differences in income conditional life expectancy. In terms of this life expectancy gap, as data is only available for two birth cohorts (1930 and 1960), its path will be fixed at the life expectancy gap for the 1930 birth cohort until these retire in 1995. From then on the gap will widen linearly, reaching in 2025 the 1960 cohort life expectancy gap - at their retirement age - which then stays constant until 2100.

The actual calibration is then achieved with the same data moments as for the static model. Thus the preferences and initial interest rate in 1947, which together pin-down depreciation, are fixed so that, given the exogenous parameter paths, the model matches the data moments in 1984, in 1982 for the interest rate. Table 10 illustrates the calibration and Table 11 the moments of model and data. The counterfactual analysis then employs the benchmark calibration while adjusting the time paths of the exogenous processes. During the phase-in period the paths will be identical for all experiments, however, from 1984 four of the five exogenous processes are kept at the 1984 level, with only one driver allowing its normal evolution.

6.2 Results

The dynamic model corroborates the results of the static model, both in terms of the fit to the data and the role of the different drivers, determined through the counterfactual analysis.

Model vs Data

The model fits the data relatively well. Table 11 and Figure 6 show the moments in 2013 and over time respectively. They show that the steady state values are not reached as fast as 2013. The interest rate

Table 11: Dynamic Model Moments

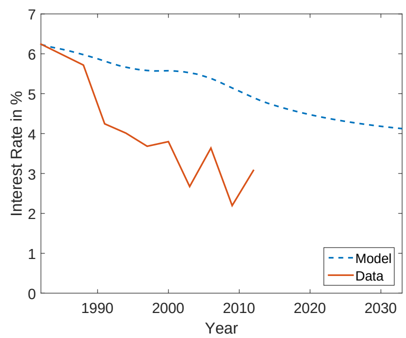
		1984		2013	
Symbol	Moment	Data	Model	Data	Model
r	Interest rate	6.24%	6.24%	3.08%	4.83%
s	Aggregate Savings Rate	0.185	0.185	0.182	0.190
Δs_{35}	Differential Savings rate 35-44 y/o	0.153	0.153	0.241	0.257
$\frac{A}{Y}$	Net Worth to Output Ratio	3.41	3.41	4.76	4.04
A_{65}	Net Worth at 65-69 in per cap. output	8.45	8.45	12.00	8.46
$\frac{\iota}{Y}$	Annual Inheritance Flow		7.98%		5.41%
τ	Labour tax		4.70%		9.91%

has not fallen as far as in the static model (1.4 percentage points compared to 1.88), so that the capital accumulation resulting in an increasing capital-to-output ratio is still ongoing (compare also Figure 13 in Appendix B.1). Moreover, just as in the static model the size of the fall in the interest rate can only be replicated partly. However, the time path in the model exhibits a slow-down of the fall in the 1990s, which is also existent in the data.

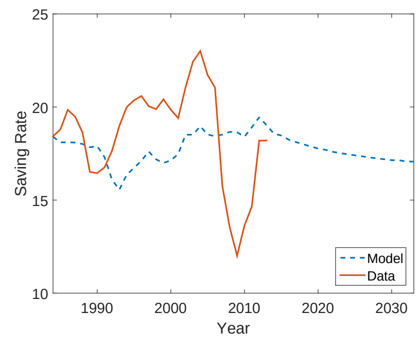
Despite the ongoing capital accumulation, the saving rate is very close to the data at 0.19. It declines then further, when capital converges to its steady state value. Moreover, note that most of the time the model's saving rate is below the data, despite the increasing capital-to-output ratio implied by the falling interest rate over the entire interval. This shows that a low saving rate is in line with a domestic savings glut, even when an accumulation of capital occurs. Moreover, as the saving rate in the data is very volatile due to the holding gains, with troughs during recessions and spikes during asset price booms, while the model's saving rate is based on a smooth rate of return, they are difficult to compare. Averaging the saving rate between 1984 and 2013, shows that the model does actually generate a saving rate (0.1788) more than 3 percentage points lower than in the data (0.2129), however also here one has to bear in mind the asset price developments in the 90s and 2000s.

The rise in the difference between the saving rates for top 20% and bottom 80% of incomes at the age of 35-44 is matched in size in the dynamic model, thus improving the static model which only obtained

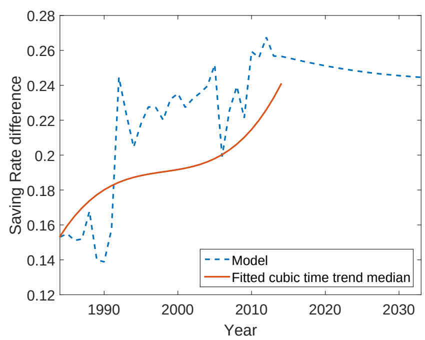
Figure 6: Model vs Data over time



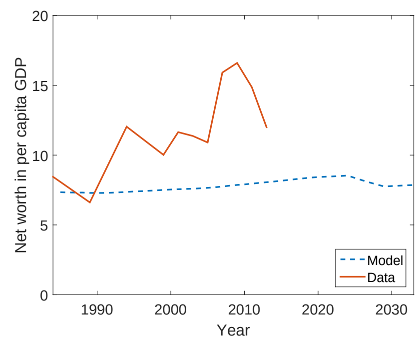
(a) Interest rate



(b) Agg. saving rate



(c) Saving rate difference 35-44 year-olds



(d) Net worth 65-69 year-olds

about half of the increase. The timing of the increases, however, do not match perfectly the timing of the estimated cubic time trend. Appendix B.1 shows the raw median differences, which are much more volatile, just as the model, and are also close to the model, particularly in the 2000s.

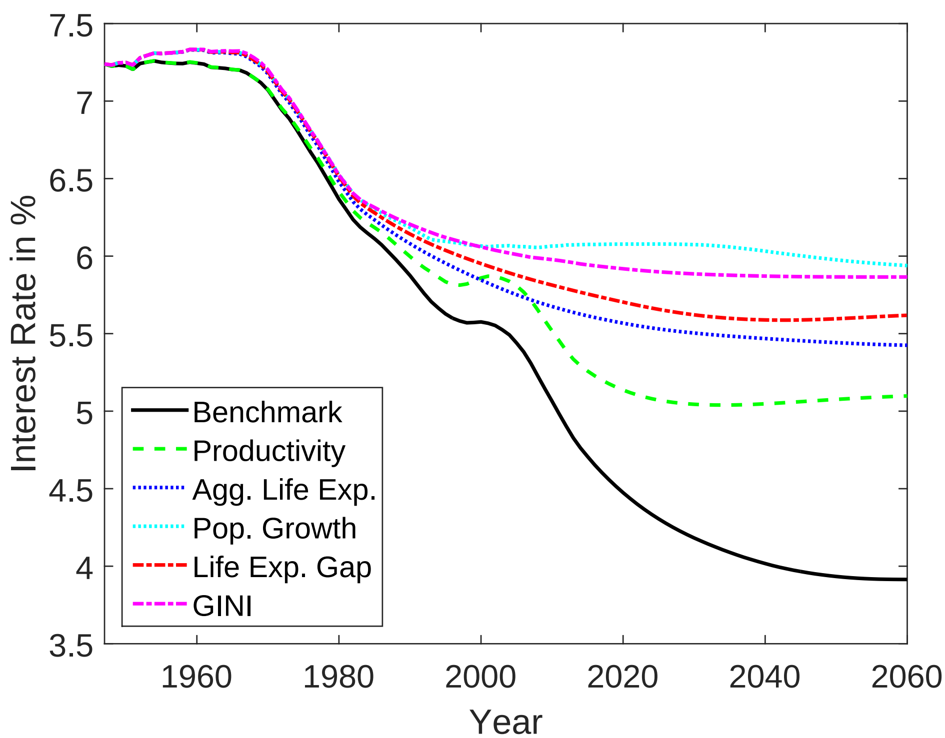
Finally, the model does not replicate a rise in the net worth at retirement. On the contrary, the net worth stays almost completely flat. This occurs due to lower net worth of the bottom 80% which drives down the average, despite the strong increase in net worth of the top 20%. This seems to be a weak point of the model. Even though it is unclear how much of the increase in the data is due to asset price hikes rather than saving, there seems to be an increase in the data too strong to explain just with asset prices. Considering the data on net worth at retirement in Appendix A.1, the net worth seems to increase particularly for the top incomes, but also for the 70th percentile. The reason for the model failing to reproduce an increase of average net worth may be due to a lack of heterogeneity, both at the extreme top, but also through the consideration of the bottom 80% as identical.

Interest rate

The evolution of the interest rate (Figure 7) is very smooth both in the benchmark model and the counterfactuals. It highlights the result of the static model of the importance of the slowdown in productivity growth. The slowdown in the 90s which matches the data seems to be generated by the increasing labour productivity growth at that time. The other drivers seem to generate an even smoother path for the interest rate, but overall the effects are comparable to the static model. Note that in contrast to the static model the average life expectancy has a stronger effect than the life expectancy gap, which is due to the average life expectancy continuing to increase until 2050. Similarly the population growth exhibits more movement after 2013, as it still decreases at that time.

Another interesting feature of the importance of the labour productivity slowdown can be observed in the 60s and 70s, while all models are subject to the same exogenous paths until 1984. Already 20 years prior to the slowdown in labour productivity growth, unique to this

Figure 7: Counterfactual evolution of the interest rate



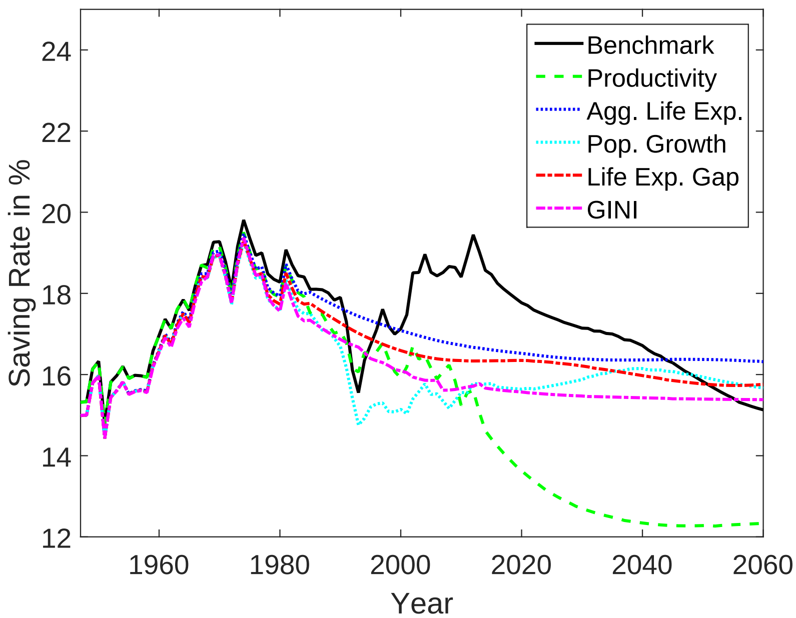
specification, it generates higher capital accumulation resulting in a lower interest rate.

Saving

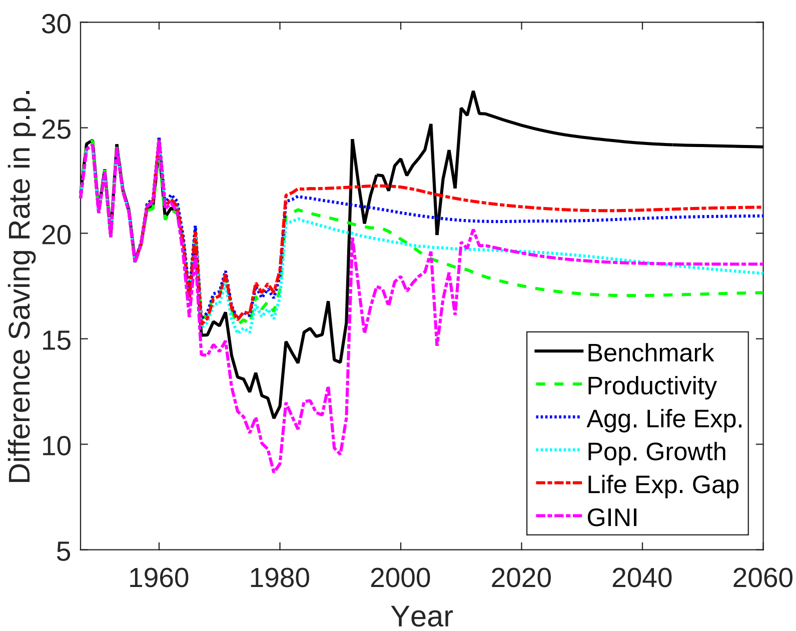
The aggregate saving rate (Figure 8a) by construction is strongly driven by the growth rate of the economy and capital accumulation (or decumulation). Therefore unsurprisingly, variation in labour productivity growth determines the temporary variation and also the general trend again supporting the results of the static model. Population growth also adds to temporary declines, in years with high population growth as a high mass of young agents who do not save much enter the model. For the other three drivers the saving rate declines smoothly from 1984 onwards. Indeed, this decline and the comparable low levels of all individual drivers relative to the benchmark model seem surprising. However, the reason for high levels of the saving rate until 1984 is capital accumulation, when five forces are at work. Once only one driver remains, the model converges relatively quickly to the steady state (see Figure 13 in Appendix B.1), capital accumulation slows down and with it the saving rate falls. The same applies for the benchmark model compared to the individual counterfactual models. The benchmark model exhibits for a long time a relatively high saving rate due to persistent capital accumulation unlike the counterfactual models.

The difference in the saving rate of the 35-44 year-olds (Figure 8b) shows an increasing trend with a lot of high frequency movement, driven by the wage inequality process of the GINI index. The difference is driven both from the top 20% and the bottom 80%, which is illustrated in Figure 14 in Appendix B.1. The decreasing GINI index in the 60s and 70s drove up the savings of the low type, which stayed high during the 80s to front-load savings given the falling wage increases in the future as a result of a strongly rising GINI index. Moreover, once the inequality jumps close to its 2013 level, the savings of the low type drop heavily, which marks the spike in the difference in the 90s. The savings of the high type behave until 1990 in the opposite way, the savings decrease in the periods when inequality is relatively low, as increasing inequality implies stronger growing wages in the future. From the 90s onwards the rise in the saving of the high type is driven, stronger than for the low type, by the spikes in the productivity pro-

Figure 8: Counterfactual evolution of saving



(a) Saving rate



(b) Saving rate difference 35-44 year-olds cesses in the late 90s and mid 2000s, to exploit the periods of high wages. For this reason, the difference rises slightly afterwards. The increase in the life expectancy gap affects the agents retiring from 1996 onwards - increasing until 2025 - thus it affects agents slowly from 1966 onwards, however until 1984, these gap seems to be negligible to have an effect, partly through the existence of the other drivers. Indeed the strong influence of wage inequality can be viewed in the uniform jump of all the specifications in 1984, once that wage inequality remains stable. From 1984 onwards the life expectancy gap has the strongest effect and around 2015 it declines towards its steady state value, which is the highest from all individual drivers. The other drivers behave similarly by increasing once the influence of the wage inequality disappears and then slowly converging downwards to the steady state values. Thus the dynamic model underpins the result that the highest effect on the difference in saving is due to the life-expectancy gap, but also adds to the analysis by showing the temporary impact of wage inequality, which seems to be in the short-run more important.

7 Conclusion

This paper shows that a domestic 'savings glut' driven by secular trends in demographics, inequality and labour productivity growth may be at the core of the decline in the U.S. interest rate between the early 1980s and 2013. The role of domestic factors, which can explain a considerable 60% of the decline of the interest rate, is not contradicted by a decline in the personal saving rate. While affecting the interest rate strongly, they do not manifest themselves in an increasing personal saving rate, due to forward looking behaviour of the agents and a decline in labour productivity growth.

Labour productivity growth plays a prominent role not only in counteracting rising saving rates through capital accumulation in the economy, but is also the main driver of the decline in the interest rate. Nevertheless, inequality possesses also an important part, particularly through asymmetric life expectancies. Firstly, it affects the interest rate strongly through on one hand increasing savings propensity of the savings affine high income households and on the other hand through a new composition of retirees in favour of this type. Secondly, it is an important driver to explain the diverging saving rates for the 35 to 44 year-olds, that for the majority savings decline, while for the top income households savings increase strongly. This occurs directly through the different life expectancies and their effect on consumption smoothing, but also through the decline of the interest rate which decreases savings stronger for the low income households. While the rise in wage inequality overall contributes less to the divergence of saving rates than asymmetric life expectancies, it is the main driver at high frequencies, causing a lot of volatility.

Moreover, the life expectancy pattern provides a new insight into the discussion, whether inheritances become more important, suggesting the opposite due to a decrease in accidental bequests.

References

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):pp. 659–684.

Antolin-Diaz, J., Drechsel, T., Petrella, I., et al. (2016). Tracking the slowdown in long-run gdp growth. Technical report.

Attanasio, O. P., Battistin, E., and Leicester, A. (2006). From micro to macro, from poor to rich: Consumption and income in the uk and the us. Technical report, working paper, University College London.

Auclert, A. and Rognlie, M. (2016). Inequality and aggregate demand.

Barro, R. J. and Mollerus, A. (2014). Safe assets. Working Paper 20652, National Bureau of Economic Research.

Benigno, G. and Fornaro, L. (2015). Stagnation traps.

Bernanke, B. S. et al. (2005). The global saving glut and the us current account deficit. Technical report.

Caballero, R. J. and Farhi, E. (2014). The safety trap. Working Paper 19927, National Bureau of Economic Research.

Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2015). Global imbalances and currency wars at the zlb. Technical report, National Bureau of Economic Research.

Carroll, C. D., Slacalek, J., and Tokuoka, K. (2014). The distribution of wealth and the marginal propensity to consume. *ECB Working Paper No. 1655*.

Carvalho, C., Ferrero, A., and Nechio, F. (2016). Demographics and real interest rates: Inspecting the mechanism. *Available at SSRN 2713443*.

Chen, K., İmrohoroğlu, A., and İmrohoroğlu, S. (2006). The japanese saving rate. *The American economic review*, 96(5):1850–1858.

De Nardi, M. (2004). Wealth inequality and intergenerational links. *The Review of Economic Studies*, 71(3):743–768.

De Nardi, M., Frenc, E., and Jones, J. B. (2009). Life expectancy and old age savings. *The American economic review*, 99(2):110–115.

De Nardi, M., French, E., and Jones, J. B. (2010). Why do the elderly save? the role of medical expenses. *Journal of political economy*, 118(1):39–75.

De Nardi, M. and Yang, F. (2014). Bequests and heterogeneity in retirement wealth. *European Economic Review*, 72:182–196.

Eggertsson, G. B. and Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach*. *The Quarterly Journal of Economics*.

Eggertsson, G. B. and Mehrotra, N. R. (2014). A model of secular stagnation. Working Paper 20574, National Bureau of Economic Research.

Eggertsson, G. B., Mehrotra, N. R., and Robbins, J. A. (2017). A model of secular stagnation: Theory and quantitative evaluation. Working Paper 23093, National Bureau of Economic Research.

Eggertsson, G. B., Mehrotra, N. R., and Summers, L. H. (2016). Secular stagnation in the open economy. Working Paper 22172, National Bureau of Economic Research.

Favilukis, J. (2013). Inequality, stock market participation, and the equity premium. *Journal of Financial Economics*, 107(3):740 – 759.

Fernald, J. G. (2015). Productivity and potential output before, during, and after the great recession. *NBER Macroeconomics Annual*, 29(1):1–51.

Fernald, J. G. and Jones, C. I. (2014). The future of us economic growth. *American Economic Review*, 104(5):44–49.

Gagnon, E., Johannsen, B. K., and López-Salido, D. (2016). Family composition, life expectancy, and the equilibrium real interest rate.

Garner, T. I., Janini, G., Paszkiewicz, L., and Vendemia, M. (2006). The ce and the pce: a comparison. *Monthly Lab. Rev.*, 129:20.

Glover, A., Heathcote, J., Krueger, D., and Ríos-Rull, J.-V. (2011). Intergenerational redistribution in the great recession. Technical report, National Bureau of Economic Research.

Gomme, P. and Rupert, P. (2007). Theory, measurement and calibration of macroeconomic models. *Journal of Monetary Economics*, 54(2):460–497.

Hall, R. E. (2016). Understanding the decline in the safe real interest rate. Working Paper 22196, National Bureau of Economic Research.

Holston, K., Laubach, T., and Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of economic Dynamics and Control*, 17(5):953–969.

Iachan, F. S., Nenov, P. T., and Simsek, A. (2015). The choice channel of financial innovation. Technical report, National Bureau of Economic Research.

Jappelli, T. and Pistaferri, L. (2010). The consumption response to income changes. *Annu. Rev. Econ.*, 2(1):479–506.

Johannsen, B. K. and Mertens, E. (2016). A time series model of interest rates with the effective lower bound. *Finance and Economics Discussion Series 2016-033*, Washington: Board of Governors of the Federal Reserve System.

Krueger, D., Mitman, K., and Perri, F. (2016). Macroeconomics and household heterogeneity. *Handbook of Macroeconomics*, 2:843–921.

Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):pp. 867–896.

Laubach, T. and Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economics and Statistics*, 85(4):1063–1070.

Lee, R., Orszag, P., Auerbach, A., Charles, K., Coile, C., Gale, W., and Kinsella, K. (2015). The growing gap in life expectancy by income: Implications for federal programs and policy responses.

Li, G., Schoeni, R. F., Danziger, S., and Charles, K. K. (2010). New expenditure data in the psid: Comparisons with the ce. *Monthly Labor Review*, 133(2):29–39.

Meyer, B. D. and Sullivan, J. X. (2009). Five decades of consumption and income poverty. Technical report, National Bureau of Economic Research.

Piketty, T. and Saez, E. (2003). Income inequality in the united states, 1913-1998. *The Quarterly Journal of Economics*, 118(1):pp. 1–39.

Piketty, T. and Zucman, G. (2014). Capital is back: Wealth-income ratios in rich countries 1700-2010*. *Quarterly Journal of Economics*, 129(3):1255 – 1310.

Piketty, T. and Zucman, G. (2015). Wealth and inheritance in the long run. In Atkinson, A. and Bourguignon, F., editors, *Handbook of Income Distribution*, volume 2. Elsevier.

Queisser, M. and Whitehouse, E. (2005). Pensions at a glance: public policies across oecd countries. Technical report, OECD.

Saez, E. and Zucman, G. (2016). Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. *The Quarterly Journal of Economics*, 131(2):519–578.

Thwaites, G. (2015). Why are real interest rates so low? secular stagnation and the relative price of investment goods.

Wolff, E. N. and Gittleman, M. (2014). Inheritances and the distribution of wealth or whatever happened to the great inheritance boom? *The Journal of Economic Inequality*, 12(4):439–468.

A Data Appendix

A.1 Net worth at retirement

This section summarises the evolution of asset holdings of retirees by analysing the net worth of the agents at retirement age in the PSID. Table 12 and 13 show the respective data for a 5-year (65-69 year-olds) and 10-year (65-74 year-olds) cohort respectively between the years 1984 and 2013. Until the year 1999 the PSID had only every five years supplementary data on wealth. From 1999 onwards this data became biannual. The tables show the means, number of observations and five deciles. The values represent nominal net worth divided by nominal per capita GDP. It is therefore inflation adjusted, however, the data exhibits to a strong degree the asset price developments in the late 90s and the mid 2000s. There seems to be an increase in the net worth of households at retirement, even when abstracting from the asset price volatility.

Table 12: Net worth of households with heads at the age of 65-69 in per capita GDP (PSID)

Year	Mean	10th pct	30th pct	50th pct	70th pct	90th pct	Observations
1984	8.449	0.063	2.159	4.675	8.638	16.845	322
1989	6.612	0.065	1.658	3.512	7.292	15.313	341
1994	12.037	0.396	2.213	4.968	10.080	26.569	405
1999	10.018	0.064	2.397	4.535	8.781	26.689	262
2001	11.645	0.322	2.345	5.183	9.122	35.119	266
2003	11.369	0.171	1.651	4.637	10.490	27.850	253
2005	10.904	0.075	1.569	5.349	10.472	26.406	267
2007	15.913	0.105	1.623	5.368	10.179	28.151	282
2009	16.599	0.040	1.821	5.361	10.649	26.871	306
2011	14.867	0.078	1.637	4.470	9.964	26.872	339
2013	11.998	0.042	1.425	3.775	10.098	26.236	452

Table 13: Net worth of households with heads at the age of 65-74 in per capita GDP (PSID)

Year	Mean	10th pct	30th pct	50th pct	70th pct	90th pct	Observations
1984	6.930	0.048	1.558	3.977	7.380	15.524	601
1989	7.319	0.087	1.843	4.101	7.504	16.272	597
1994	9.545	0.144	1.800	4.176	8.549	20.701	739
1999	12.715	0.087	2.397	5.228	10.003	28.798	545
2001	11.839	0.327	2.573	5.500	10.530	31.738	536
2003	11.024	0.252	1.991	5.041	10.908	24.699	499
2005	10.778	0.0745	1.772	4.931	10.111	26.970	509
2007	14.156	0.044	1.908	5.659	10.976	29.670	518
2009	13.851	0.043	1.830	5.361	10.468	25.646	537
2011	12.482	0.040	1.637	4.731	9.542	22.699	578
2013	11.547	0.047	1.557	4.171	10.015	24.641	699

A.2 Divergent saving rates

This section discusses in detail the saving data for the cohort of 35-44 year-olds, which serves as a proxy for the retirement saving. In particular it addresses the evolution of the difference in saving rates between high income (top 20%) and normal income (bottom 80%) households between 1984 and 2014.

Data

The saving rates are based on interview survey data of the Consumer Expenditure Survey (CEX) from 1984q1 till 2014q4. The quarterly data is pooled and annualised to generate the sample for each year. From this data the saving rate is constructed as the residual share of family income, subtracting total family expenditure (totexpq+totexcq or totex4pq+totex4cq depending on the year) from total family after tax income (fincatax or finctxm depending on the year) divided by total family after tax income. The sample is made of income reporters until 2004 and once the CEX imputed income, includes also those households. Construct the saving rate in this way, has certain drawbacks due to an issue of expenditure underreporting in the CEX compared to aggregate data such as the Personal Consumption Expenditure (PCE) in the National Income and Product Accounts, an

Table 14: Saving rates 35-44 year-olds by year and income (CEX)

	Bottom 80% income				Top 20% income			
Year	25th	50th	75th	Obs	25th	50th	75th	Obs
1984	-0.253	0.081	0.272	2565	0.062	0.224	0.362	640
1985	-0.212	0.147	0.344	2614	0.138	0.347	0.470	681
1986	-0.283	0.0676	0.276	2714	0.059	0.246	0.397	724
1987	-0.182	0.097	0.282	2942	0.068	0.277	0.413	813
1988	-0.261	0.0625	0.259	2652	0.0525	0.262	0.416	728
1989	-0.258	0.066	0.262	2710	0.078	0.256	0.393	748
1990	-0.269	0.069	0.258	2806	0.037	0.242	0.375	785
1991	-0.252	0.080	0.270	2812	0.074	0.279	0.399	743
1992	-0.274	0.063	0.259	2843	0.073	0.269	0.400	772
1993	-0.227	0.077	0.266	2905	0.117	0.282	0.303	846
1994	-0.279	0.066	0.268	2953	0.086	0.278	0.431	795
1995	-0.27	0.066	0.269	2643	0.032	0.263	0.394	741
1996	-0.291	0.054	0.274	2638	0.089	0.272	0.417	678
1997	-0.278	0.059	0.277	2917	0.059	0.259	0.406	775
1998	-0.284	0.068	0.270	2751	0.081	0.311	0.426	704
1999	-0.203	0.094	0.308	3556	0.112	0.305	0.446	967
2000	-0.216	0.112	0.317	3605	0.146	0.330	0.450	972
2001	-0.184	0.130	0.320	3738	0.119	0.325	0.464	977
2002	-0.163	0.151	0.335	3965	0.185	0.362	0.478	1027
2003	-0.131	0.155	0.350	4016	0.200	0.372	0.497	1064
2004	-0.026	0.220	0.387	4475	0.214	0.403	0.523	1199
2005	-0.081	0.184	0.372	4038	0.223	0.399	0.523	1067
2006	-0.085	0.180	0.365	4251	0.186	0.367	0.499	1139
2007	-0.098	0.168	0.361	3763	0.205	0.380	0.504	1033
2008	-0.143	0.154	0.337	3709	0.230	0.389	0.507	999
2009	-0.076	0.195	0.368	3666	0.273	0.418	0.528	983
2010	-0.104	0.174	0.357	3584	0.256	0.426	0.525	949
2011	-0.112	0.173	0.354	3420	0.289	0.423	0.525	868
2012	-0.107	0.154	0.356	3385	0.262	0.407	0.511	885
2013	-0.088	0.191	0.359	3051	0.277	0.411	0.519	814
2014	-0.127	0.177	0.361	3146	0.280	0.429	0.525	846

issue which seems to have increased over time (Attanasio et al., 2006; Garner et al., 2006; Meyer and Sullivan, 2009). An alternative data source to obtain the required differences at the household level is the PSID. Nevertheless, the PSID only considers a comprehensive measure of expenditures since 1999 and was mainly limited to food expenditure previously. The expenditure data in the PSID since 1999 and the expenditure data in the CEX are nevertheless relatively similar (Li et al., 2010).

Divergence

The annualised CEX data is used to identify income conditional saving patterns over time. However, the data exhibits considerable heterogeneity and extreme values of the saving rate. This is particularly striking at the low end, with a high number of negative values. To account for these extreme values, as the objective are median households, the observations with the highest and lowest 5% of saving rates are dropped. There is a strong correlation between saving rate and household income, so that the top 5% of saving rates are mainly found among high earners. This does not pose a problem, as it even helps to adjust for the fact that the CEX is particularly prone to underreporting of expenditures for high incomes (Meyer and Sullivan, 2009).

Once the sample adjustment has been made, the observations are grouped by income. After tax family income is divided by the square root of the adults per household to obtain income per person - considering economies of scale. Deciles are constructed each year on this per person income, to group the households into top 20% and bottom 80% of incomes.

Table 14 shows the descriptive data by income group and year and Figure 9 illustrates the evolution of median saving rates for these two groups over time. Figure 3 illustrates the median difference and the fitted cubic time trend of the median difference obtained through a pooled quantile regression of the median saving rate on control variables, a cubic time trend, a dummy variable for the top 20% income group and a cubic time trend interacted with this dummy variable. The fitted time trend is computed through the coefficients of all terms which include the dummy variable. The regression results are presented in Table 15.

Figure 9: Median saving rates bottom 80% vs top 20%, 35-44 year-olds

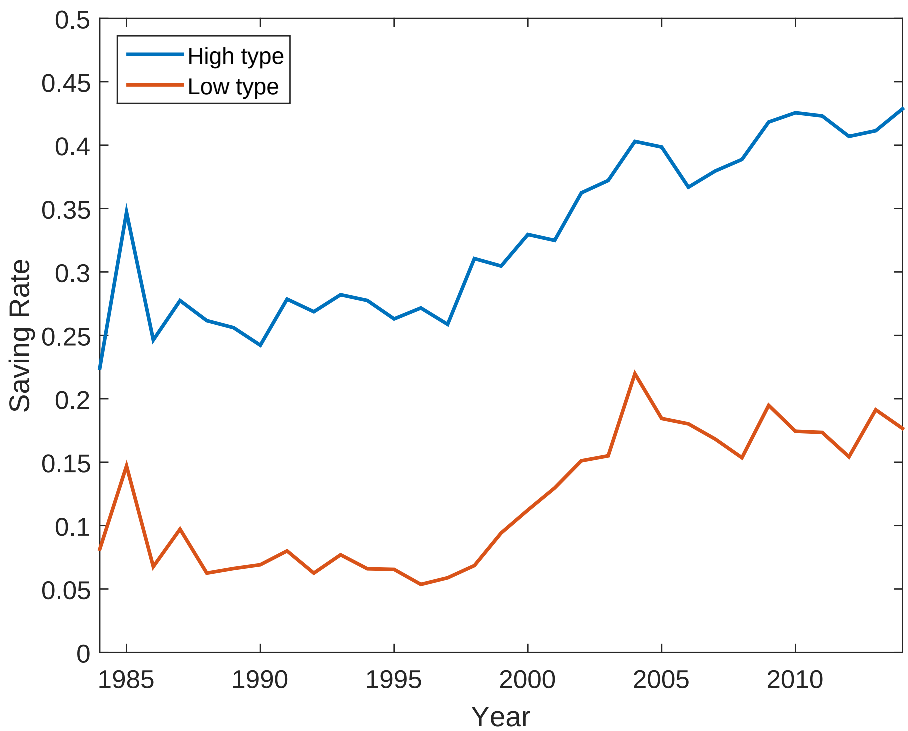


Table 15: Pooled quantile regression of the median saving rate in percent (1984-2014)

	Saving rate in %	
Age Head	-0.324***	(-7.61)
Age Spouse	0.254***	(34.12)
No. Males > 15	0.425**	(2.06)
No. Females > 15	-2.144***	(-10.83)
No. Males 2-14	-2.760***	(-18.31)
No. Females 2-14	-2.904***	(-18.62)
No. Children < 2	-2.314***	(-5.75)
Time	-1.959***	(-12.75)
Time^2	0.181***	(15.72)
Time^3	-0.00369***	(-15.05)
Type Dummy	15.34***	(11.93)
Time × Type	0.665*	(1.91)
Time^2 × Type	-0.0428*	(-1.66)
Time^3 × Type	0.00101*	(1.86)
Constant	24.18***	(13.87)
Observations	127795	

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

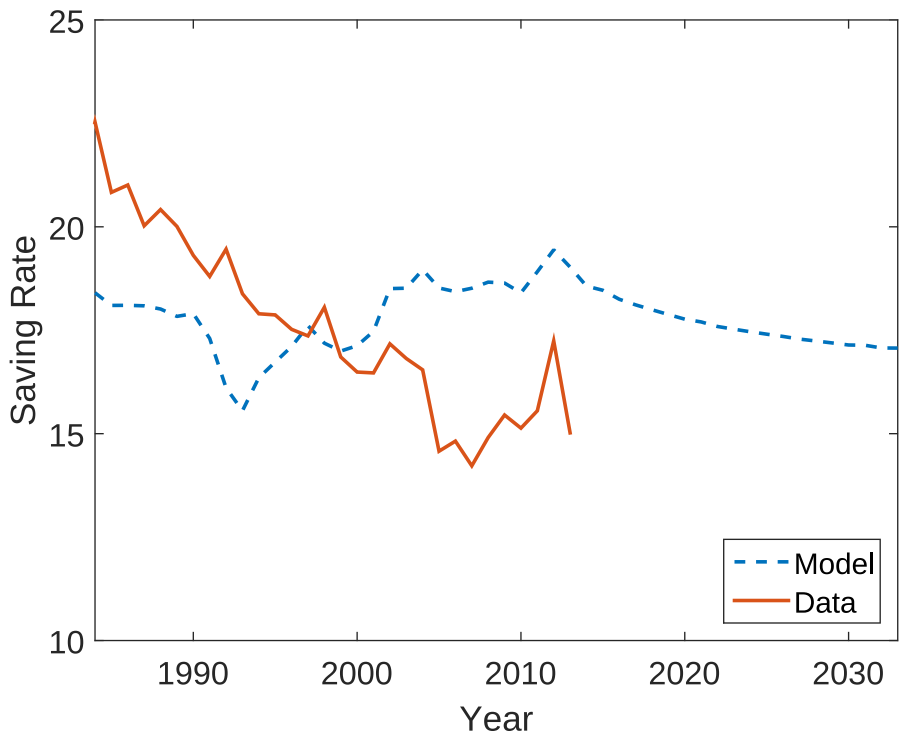
B Additional Figures

B.1 Dynamic Model

This section shows additional figures for the dynamic model to save space in the main text. The first two figures relate to the model output in comparison to the data, however, using alternative data series. Figure 10 compares the aggregates saving rate to the saving rate without taking into account holding gains on wealth. Figure 11 compares the difference of saving rates of 35-44 year-olds to the raw median differences observed in the Consumer Expenditure Survey, instead of the estimated time trend.

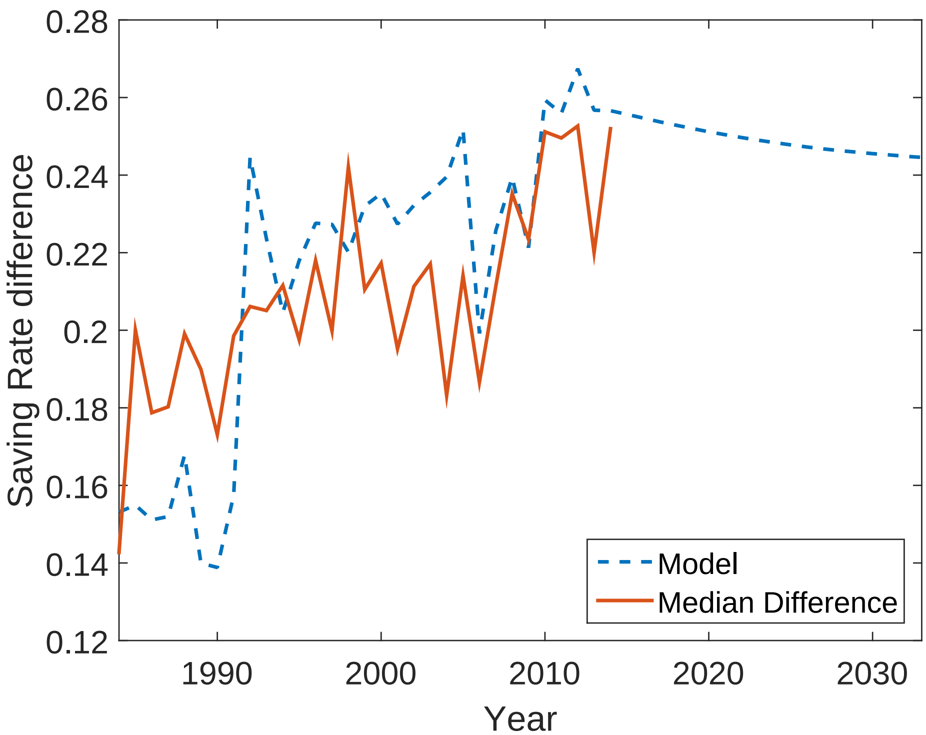
The remaining figures relate to the counterfactual analysis. Figure 12 shows the evolution of the model in terms of net worth at retirement and annual inheritance flow. It exhibits the same results as already presented in the text. On the one hand, net worth at retirement remains pretty flat over the entire horizon, driven by lower savings of the bottom 80%. On the other hand, the annual inheritance flow declines strongly due to the composition effects at retirement. Figure 13 shows the capital accumulation in the model, measured by the evolution of the capital-to-output ratio - the counterpart of the interest rate due to a constant depreciation. Finally, figure 14 highlights the saving rates of the 35-44 year-olds for the two types of agents in the benchmark model, which clarifies the evolution of the differences in the saving rates.

Figure 10: Aggregate saving rate over time: model vs data



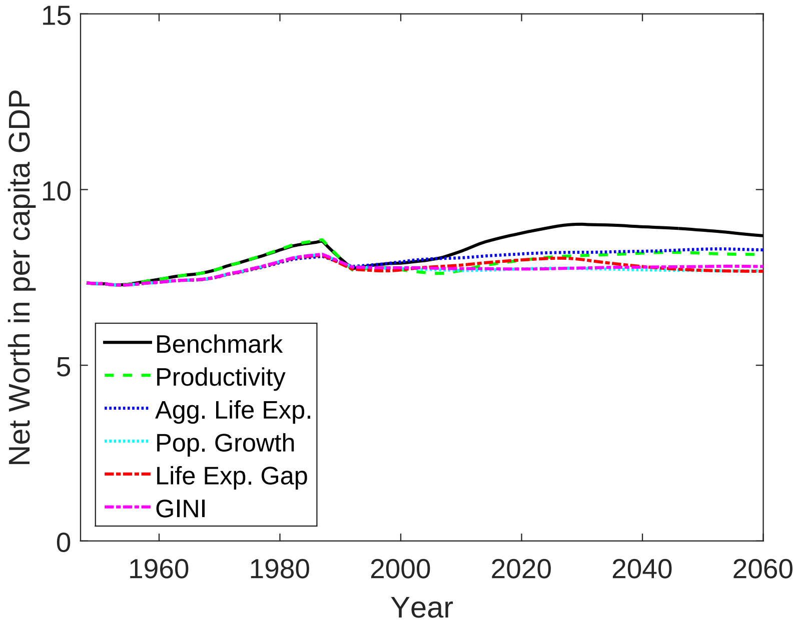
Note: The saving rate is the aggregate saving rate including consumer durables as saving but excluding holding gains from income.

Figure 11: Saving rate difference 35-44 year-olds over time: model vs data

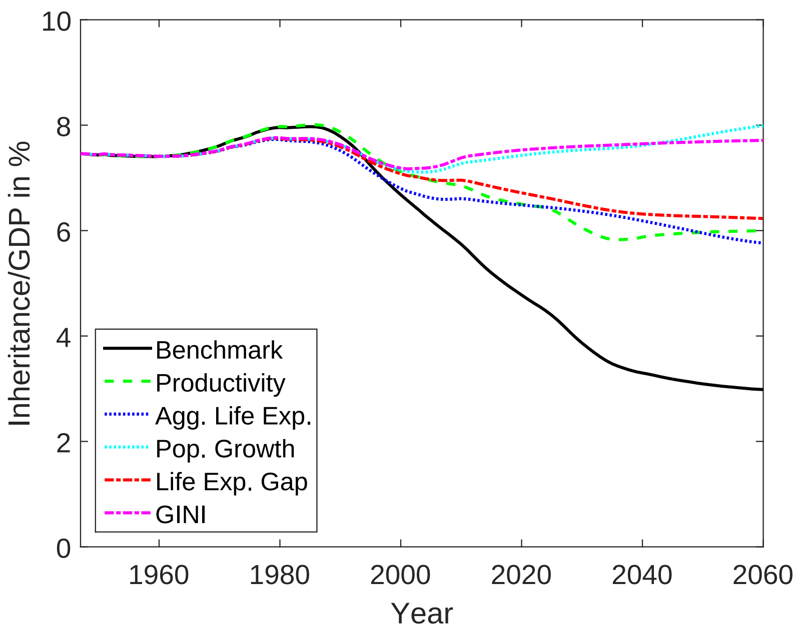


Note: The data is the actual difference between the median saving rates of top 20% of income and bottom 80% of income in each year.

Figure 12: Counterfactual evolution of net worth at retirement and inheritance flow



(a) Net worth 65-69 year-olds



(b) Annual inheritance flow

Figure 13: Counterfactual evolution of capital-to-output ratio

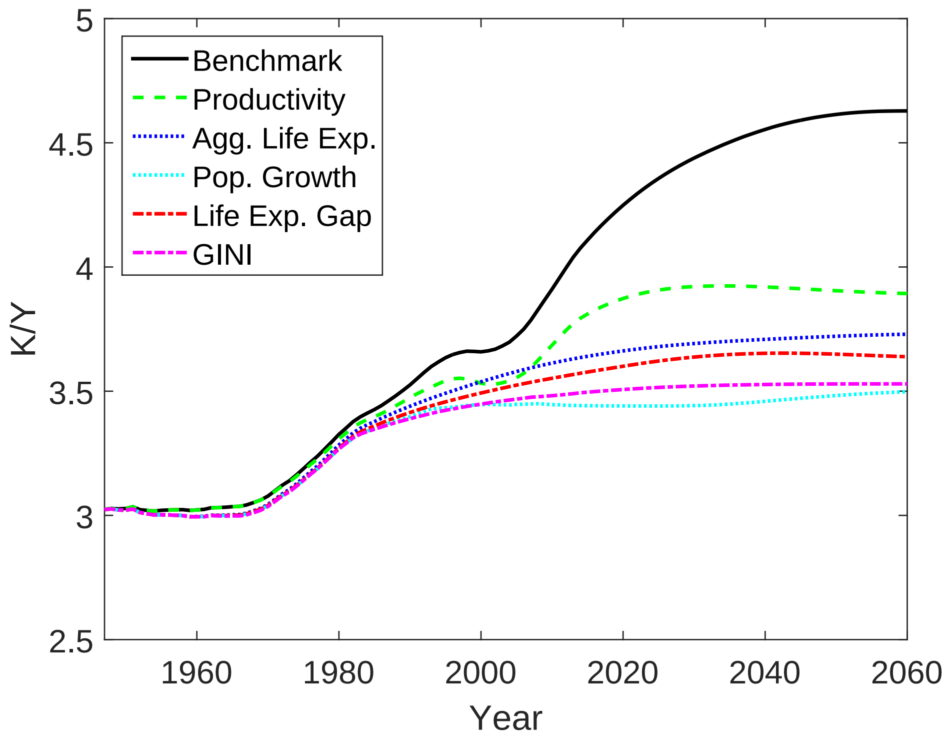


Figure 14: Saving rates 35-44 year-olds over time: benchmark model