The Impact of Growth on Unemployment in a Low and High Inflation Environments

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Abstract

The standard search model of unemployment predicts, under realistic assumptions about household preferences, that disembodied technological progress leads to higher steady state unemployment. This prediction is at odds with the 1970s experience of many OECD economies. This paper shows that introducing nominal price rigidity helps in reconciling the model's prediction with experience. Faster growth is shown to lead to lower unemployment when inflation is relatively high, as was the case in the 1970s. In general, the effect of growth on unemployment is shown to be non-monotonic. There is a threshold level of inflation below (above) which faster growth leads to higher (lower) unemployment.

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1 Introduction

Many OECD economies experienced a combination of slow productivity growth and higher unemployment during the 1970s. In a seminal theoretical contribution, Pissarides (1990, ch. 2) argues that the observed negative relationship is consistent with the prediction of the standard search model of unemployment. Pissarides (1990) shows that, under the assumption of exogenous and constant interest rate, exogenous job destruction and

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disembodied technological progress, the model predicts a negative effect of growth on steady state unemployment. This is due to positive "capitalization" effect—by lowering the effective discount rate, higher growth raises the present discounted value of lifetime revenues and therefore job creation. However, subsequent research has shown that under alternative and plausible assumptions the standard model actually gives counterfactual predictions. For example, under the more plausible assumption of an endogenous interest rate and low degree of intertemporal substitution in consumption, Aghion and Howitt (1994) and Eriksson (1997) show that faster growth leads to higher unemployment. The reason is that the real rate of interest (and as a result the effective discount rate) rises with consumption growth implying a negative capitalization effect.[1]

The present paper is motivated by the observation that the 1970s were characterized not only by a slowdown in productivity growth but also by higher inflation rates and shows that introducing nominal price rigidity helps reconcile the prediction of labor search-type models with the experience of the 1970s. As is well known, nominal price rigidity implies a role for inflation in the determination of real variables. There is an expanding literature examining the effects of trend inflation on steady state output (e.g., Ascari (2004), Graham and Snower (2008), and Amano et. al (2009)) and the role of nominal frictions for unemployment dynamics (e.g., Trigari (2006), Christoffel and Kuester (2008) and Blanchard and Gali (2010)). The former set of papers abstract either from growth considerations or unemployment considerations while the latter set of papers abstract from growth considerations.

In the presence of nominal price rigidity faster growth is shown to lead to lower unemployment if the rate of inflation is high enough. More generally, the effect of growth on unemployment is non-monotonic—positive at low levels of inflation and negative at high levels of inflation. These results are shown using a balanced growth version of a two-sector framework with nominal price rigidity, labor market frictions and exogenous disembodied technological progress. Firms in sector 1 produce differentiated final goods using an intermediate input but adjust prices infrequently (so that price setting is forward looking).

[1]This result also appears when relaxing the assumption of exogenous job destruction in Pissarides (1990). For instance, Prat (2007) shows that, by raising a worker's outside option disembodied technological progress intensifies the rate of job separation, an effect that outweighs, for plausible parameter values, the capitalization effect so that disembodied technological progress raises unemployment. Aghion and Howitt (1994) also identify a creative destruction effect brought about by embodied technological progress: by reducing the duration of an existing job match faster growth leads to higher job destruction and therefore unemployment. Pissarides and Vallanti (2007) provide empirical evidence for a negative effect of growth on unemployment, thus supporting the view that, if unemployment is a result of search frictions, then technology must be disembodied. Nevertheless, the authors conclude that, even if one assumes technology is mainly disembodied, a significant part of the impact of growth on unemployment remains unexplained.

Firms in sector 2 produce the intermediate input under a perfectly competitive output market and face labor hiring costs (so that hiring decision is forward looking).[2]

The model has the property that, along the balanced growth path, faster growth implies (i) higher real rate of interest, (ii) higher future labor productivity relative to the present one and (iii) higher future aggregate final good demand relative to the present one. The interaction of effects (i) and (ii) determines the capitalization effect and is well known in the growth and unemployment literature cited above. The interaction of effects (i) and (iii) determines what we call the *markup effect*, an effect that is novel. Higher interest rate induces final good producing firms to lower their price markups (as it mitigates future erosion of their price markup by ongoing inflation) while faster growth of aggregate demand induces them to raise their price markups (as it exacerbates future erosion of their price markups by ongoing inflation).

Under the maintained assumption that the intertemporal substitution in consumption is low (Eriksson (1997)), growth has two opposing effects. On the one hand, the interest rate effect dominates the capitalization effect, so that, given the relative price of the intermediate good, faster growth leads to higher unemployment. On the other hand, the interest rate effect dominates the demand effect, so that faster growth lowers price markups of final good firms (i.e., raises the relative price of the intermediate good) and thus lowers unemployment. The reduction in price markup acts like a tax-cut on the intermediate input and thus induces intermediate good firms to hire more. Moreover, the tax-cut like effect of faster growth is stronger the higher is the level of inflation. It is shown that (a) there is a threshold rate of inflation below (above) which faster growth leads to higher (lower) unemployment and (b) the threshold level of inflation in turn depends on labor market parameters, such as the job destruction rate and workers' bargaining power. This is demonstrated by calibrated versions of the model (one version is calibrated to the US and the other is calibrated to continental Europe).[3]

The paper is organized as follows. In section 2 we present the model and in section 3 we discuss the model's balanced growth path and give the intuition underlying our results. In section 4 we undertake comparative static analysis (i.e., the effect of growth

[2]The two-sector framework is standard in the business cycle literature (see, e.g., Trigari (2006), Christoffel and Kuester (2008) and Blanchard and Gali (2010)). The assumption that hiring costs are the source of labor market rigidity follows closely Blanchard and Gali (2010).

[3]We point out that, while we use nominal price rigidity as a rationale for thinking about the real effects of inflation, as in much of the business cycle literature, alternative frameworks exist that rationalize the real effects of inflation. For e.g., Vaona (2013) studies the relationship between inflation and unemployment in a flexible price model with efficiency wages. It would be interesting to know the prediction of such a model when augmented with productivity growth.

on unemployment) while in section 5 we show sensitivity to key labor market parameters. In section 6 we give concluding remarks.

2 The model

Following Blanchard and Gali (2010) we use a simple two-sector framework with price staggering as well as labor market frictions. This framework is augmented to allow for productivity growth, which is labor augmenting and disembodied (e.g., as in Pissarides (1990) and Eriksson (1997)), so that productivity growth is reflected in all existing and newly employed workers. Furthermore, growth in labor productivity A_t is assumed to be deterministic, where $\Gamma = A_t/A_{t-1}$ denoted gross productivity growth. As in Eriksson (1997), the rate of interest is endogenous and is related to consumption growth. As is standard, the economy exhibits balanced growth.[4]

2.1 Households

There is a representative household with a continuum of members over the unit interval. Similar to Eriksson (1997), household utility is of the form $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$, where $\sigma > 1$.[5] The household consumes a continuum of differentiated goods produced by an imperfectly competitive final goods sector (details of which are given below). C_t is a Dixit-Stiglitz composite of final goods: $C_t = \left(\int_0^1 C_{k,t}^{1/\mu} dk\right)^{\mu}$ where each good is indexed by k, $\mu \equiv \frac{\theta}{\theta-1}$ and $\theta > 1$ is the elasticity of substitution between the differentiated final goods. Optimal consumption allocation across goods gives the demand equation: $C_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\theta} C_t$ where

$$P_t = \left(\int_0^1 P_{k,t}^{1-\theta} dk\right)^{\frac{1}{1-\theta}} \tag{1}$$

is the price index. Optimal consumption allocation across time is derived from maximization of the lifetime utility, $E_t \sum \beta^i U(C_{t+i})$, subject to the budget constraint

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} + D_t,$$

where β is the subjective discount factor, R_t is the nominal interest rate on bond holdings B_t, W_t is the nominal wage and D_t is the aggregate nominal profit income from firm

[4] For a similar approach see, e.g., Tesfaselassie (2013).

[5] The utility function may also include disutility from work (as for e.g., in Shimer (2010)) but this is not essential for our results.

ownership. It is straightforward to derive the familiar Euler equation

$$1 = E_t\left(\frac{Q_{t,t+1}R_t}{\Pi_{t+1}}\right), \tag{2}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is gross inflation rate and $Q_{t,t+1} \equiv \beta U'(C_{t+1})/U'(C_t)$ is the familiar stochastic discount factor. It can be rewritten as

$$Q_{t,t+1} \equiv \beta\Gamma^{-\sigma}\left(c_{t+1}/c_t\right)^{-\sigma}, \tag{3}$$

where $c_t = C_t/A_t$. The steady state of equation (2) is $R/\Pi = \Gamma^{\sigma}/\beta$, which shows that higher trend growth implies higher gross real rate R/Π and in turn a stronger discounting of future payoffs.

2.2 Firms

2.2.1 Intermediate goods sector

There is a continuum of firms in the intermediate goods sector. The representative firm produces output Y_t^I with a linear technology using the input N_t of employed workers: $Y_t^I = A_tN_t$. Employment evolves according to the dynamic equation

$$N_t = (1-\delta)N_{t-1} + H_t, \tag{4}$$

where at the beginning of period t a fraction δ of previously employed workers are separated from the firm and H_t is hiring in period t.[6]

In every period, each household member can either be employed or unemployed. The size of the labor force is normalized to one so that the stock of unemployed workers in period t before hiring takes place is given by $U_t = 1-(1-\delta)N_{t-1}$. Assuming workers start working immediately after getting hired, the unemployment rate (after hiring takes place) is $u_t = 1 - N_t$.

As in Blanchard and Gali (2010), frictions in the labor market take the form of hiring costs, which take the form[7]

$$HC_t = G_tH_t, \tag{5}$$

[6] Thus δ represents an exogenous job separation rate.

[7] This section draws on Blanchard and Gali (2010). The assumption that firms can hire a worker instantaneously subject to paying hiring costs simplifies our analysis. Alternatively, one may assume vacancy posting costs as in the labor search and matching literature (see, e.g., Christoffel and Kuester (2008)). In the present paper, we do not need to track vacancies, which is necessary when one is interested, say, in the Beveridge curve (the relationship between vacancies and unemployment).

where $G_t \equiv BA_t x_t$ is the cost per hire, $B > 0$ and $x_t \equiv H_t/U_t$ is the job finding rate.[8] Hiring costs are expressed in terms of the CES bundle of final goods. Since the model features balanced growth the presence of A_t ensures that along the balanced growth path the cost per hire increases at the same rate as aggregate final output. For future reference the detrended version of equation (5) is

$$hc_t = g_t H_t, \tag{6}$$

where $g_t = Bx_t$. Intermediate good firms face perfectly competitive output market and sell output at the nominal price P_t^I. The presence of hiring costs makes the hiring decision intertemporal. To see this, a firm's lifetime discounted profit is given by

$$E_t \sum_{i=0}^{\infty} Q_{t,t+i} \left(p_{t+i}^I A_{t+i} N_{t+i} - w_{t+i} N_{t+1} - G_{t+i} H_{t+i} \right), \tag{7}$$

where $p_t^I \equiv P_t^I / P_t$ is the relative price of the intermediate good and $w_t \equiv W_t/P_t$ is the real wage. In any given period profits are equal to revenues net of the total wage bill and the total hiring cost. Maximizing the sum of discounted profits (7) subject to the employment dynamics (4) leads to the first order condition for an optimum level of hiring,

$$p_t^I A_t = w_t + G_t - (1-\delta) E_t \left\{ Q_{t,t+1} G_{t+1} \right\}. \tag{8}$$

The left hand side of equation (8) is the marginal revenue product of labor while the right hand side is the cost of the marginal worker, which includes the real wage and the hiring cost net of discounted savings in future hiring costs. Dividing through by A_t and slightly manipulating the resulting equation gives

$$\begin{aligned} p_t^I &= w_t^d + g_t - (1-\delta) E_t \left\{ Q_{t,t+1} \Gamma g_{t+1} \right\} \\ &= w_t^d + g_t - (1-\delta) \beta \Gamma^{1-\sigma} E_t \left\{ (c_{t+1}/c_t)^{-\sigma} g_{t+1} \right\}, \end{aligned} \tag{9}$$

where $w_t^d \equiv w_t / A_t$ and $g_t \equiv G_t / A_t$ are stationary variables and the second equality follows from using equation (3) to substitute out $Q_{t,t+1}$.

[8] In this setup, a vacancy is filled instantaneously if the firm pays the hiring cost. As a matter of comparison, in the standard search and matching model the job-posting cost is constant for each posted vacancy. Assuming a matching function of the form $H_t = U_t^{\alpha_0} V_t^{1-\alpha_0}$, where V_t is the number of posted vacancies, the hiring cost is proportional to the expected vacancy duration, which is equal to the inverse of the job-filling rate H_t/V_t. It can be shown that $V/H = x^{\alpha}$, where $\alpha \equiv \alpha_0/(1-\alpha_0)$. Our specification of the cost per hire assumes implicitly that $\alpha_0 = .5$, which is close to empirical estimates (see Blanchard and Gali (2010)).

From the right hand side of equation (9) we see that there are two offsetting effects of higher productivity growth on the firm's hiring policy. On the one hand it implies larger savings in future hiring costs from current hiring (this effect is analogous to the so-called "capitalization" effect with respect to vacancy creation, as in Pissarides (2000)). On the other hand, it implies faster consumption growth, and in turn higher real interest rate (i.e., stronger discounting of future savings in hiring costs). Given the maintained assumption $\sigma > 1$ (see, e.g., Eriksson (1997)) the interest rate effect dominates the capitalization effect so that, all else equal, faster growth reduces the returns to hiring and raises unemployment.

Wage setting. The presence of hiring costs implies that existing employment relationships earn an economic surplus. The value (in terms of current consumption) to the household of an employed worker is given by

$$V_t^e = w_t + E_t\left(Q_{t,t+1}\left[(1-\delta(1-x_{t+1}))V_{t+1}^e + \delta(1-x_{t+1})V_{t+1}^u\right]\right),$$

where $\delta(1-x_{t+1})$ is the probability that an employed worker is separated from his job at the end of period t and stays unemployed in period $t+1$ while $1-\delta(1-x_{t+1})$ is the probability that an employed worker keeps his current job in period $t+1$ or he is separated from his current job at the end of period t but finds a job in period $t+1$.

The corresponding value of an unemployed worker is given by

$$V_t^u = z_t + E_t\left(Q_{t,t+1}\left[x_{t+1}V_{t+1}^e + (1-x_{t+1})V_{t+1}^u\right]\right),$$

where z_t represents the opportunity of cost of employment (which may include, among others, unemployment benefits). As is standard, z_t is assumed to be proportional to labor productivity, $z_t = bA_t$, where $b > 0$. The household's surplus from an employment relationship is then given by

$$S_t^h(\equiv V_t^e - V_t^u) = w_t - bA_t + (1-\delta)E_t\left(Q_{t,t+1}(1-x_{t+1})S_{t+1}^h\right). \tag{10}$$

Similarly, the firm's surplus from an employment relationship is

$$S_t^f = p_t^I A_t - w_t + (1-\delta)E_t\left(Q_{t,t+1}S_{t+1}^f\right), \tag{11}$$

which is the sum of current period profit and future expected surplus. Equations (9) and (11) imply that

$$S_t^f = G_t. \tag{12}$$

That is, the firm's surplus from an additional hire is equal to the cost per hire. Under the common assumption of Nash bargaining the real wage is such that it maximizes the Nash product $(S_t^h)^\eta(S_t^f)^{1-\eta}$, where $0 < \eta < 1$ is the relative bargaining power of the household. Wage setting satisfies the optimality condition $S_t^h = \nu S_t^f = \nu G_t$, where $\nu \equiv \eta/(1-\eta)$ and the second equality follows from equation (12). Then equation (10) can be rewritten as

$$w_t = bA_t + \nu\left(G_t - (1-\delta)E_t\left\{Q_{t,t+1}(1-x_{t+1})G_{t+1}\right\}\right). \tag{13}$$

Dividing equation (13) through by A_t and slightly manipulating the resulting equation gives

$$w_t^d = b + \nu\left(g_t - (1-\delta)\beta\Gamma^{1-\sigma}E_t\left\{(c_{t+1}/c_t)^{-\sigma}(1-x_{t+1})g_{t+1}\right\}\right). \tag{14}$$

The chosen wage is increasing in current hiring cost (g_t), as this raises the firm's surplus from an existing relationship, and decreasing in expected future hiring costs (g_{t+1}) and in the probability ($1 - x_{t+1}$) of not finding a job next period in the event that the worker separates from the firm, both of which raise the continuation value to currently employed worker and hence reduce the required wage today. All else equal, the higher is productivity growth the smaller is the continuation value and hence the larger is the real wage.

2.2.2 Final goods sector

There is a continuum of firms producing differentiated final goods and face Calvo-type price staggering, where only a fraction $1-\omega$ of firms can reset prices in any given period. Each firm k produces a differentiated final good using the intermediate good as an input. As in Blanchard and Gali (2010) we assume a simple linear technology $Y_{k,t} = Y_{k,t}^I$, which implies that the firm's real marginal cost ($mc_{k,t}$) is given by p_t^I. Let $P_{k,t}$ denote firm k's output price. Maximizing lifetime profit $E_t\sum_{i=0}^{\infty}\omega^i Q_{t,t+i}\left(P_{k,t}/P_{t+i} - p_{t+i}^I\right)Y_{k,t+i}$ subject to the demand for good k, $Y_{k,t+i} = \left(P_{k,t}/P_{t+i}\right)^{-\theta}Y_{t+i}$, where $Y_{t+i} = C_{t+i} + HC_{t+i}$, leads to the optimality condition

$$p_t^* = \mu\frac{E_t\sum_{i=0}^{\infty}\omega^i Q_{t,t+i}p_{t+i}^I(Y_{t+i}/Y_t)\left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega^i Q_{t,t+i}(Y_{t+i}/Y_t)\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}, \tag{15}$$

where $p_t^* \equiv P_t^*/P_t$ is the relative price of optimizing firms, all of which face identical price setting problem, and μ is the price markup in the absence of price staggering. Equation

(15) can be rewritten in stationary variables

$$p_t^* = \mu \frac{E_t \sum_{i=0}^{\infty} (\beta\omega\Gamma^{1-\sigma})^i (c_{t+i}/c_t)^{-\sigma} p_{t+i}^I y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} (\beta\omega\Gamma^{1-\sigma})^i (c_{t+i}/c_t)^{-\sigma} y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}. \tag{16}$$

This is our key equation capturing the influence of steady state inflation in the presence of price staggering. We thus discuss its relevance in more detail by looking at its steady state version

$$p^* = \mu \frac{\sum_{i=0}^{\infty} (\beta\omega\Gamma^{1-\sigma}\Pi^{\theta})^i}{\sum_{i=0}^{\infty} (\beta\omega\Gamma^{1-\sigma}\Pi^{\theta-1})^i} p^I = \mu \frac{1 - \beta\omega\Gamma^{1-\sigma}\Pi^{\theta-1}}{1 - \beta\omega\Gamma^{1-\sigma}\Pi^{\theta}} p^I, \tag{17}$$

where for the sums to be convergent, we impose the restriction $\Pi < \Pi^{max} = (\Gamma^{\sigma-1}/(\beta\omega))^{1/\theta}$.[9] When the inflation rate is zero ($\Pi = 1$), the optimal relative price is a fixed markup over real marginal cost ($p^* = \mu p^I$, as is the case under flexible prices) and is independent of productivity growth. When the inflation rate is positive, firms choose a markup higher than that implied by zero inflation so as to mitigate the future erosion of their markup by an ongoing inflation (until they get the chance to reset their price). The underlying reason behind this markup distortion is the asymmetry in the profit function: profit declines more strongly with a markup that is below the optimum (under flexible prices) than with a markup above the optimum.[10] The markup distortion is smaller the higher is the rate of productivity growth owing to stronger discounting effect from higher real interest rate. As will be shown below, this negative markup effect of productivity growth mitigates the negative capitalization effect.

Under Calvo-type price staggering the price index (1) can be rewritten as

$$1 = (1-\omega) p_t^{*(1-\theta)} + \omega \Pi_{t-1}^{\theta-1}, \tag{18}$$

which shows that in steady state p^* is positively related to Π and together with equation (17) implies that, given $\Pi > 1$, the higher is productivity growth the lower is the price markup and hence the higher is the relative intermediate good price p^I. Moreover, the negative markup effect of faster growth is stronger the higher is the rate of inflation. The reduction in price markup acts like a tax-cut on the intermediate input supply and thus induces intermediate good firms to supply more output and hire more workers.

[9] For instance, assuming plausible parameter values—$\beta = 0.99, \sigma = 3, \omega = 0.75, \theta = 11$ and $\Gamma = 1.005$ (i.e., an annualized growth rate of 2 percent)—$\Pi^{max} = 1.028$ (i.e., an annualized inflation rate of about 11.35 percent).

[10] See, e.g, Amano et. al (2009) for a detailed discussion.

Next, aggregating both sides of the market clearing condition for the intermediate good and using the demand equation for the final good k leads to a relationship between aggregate final output y_t and intermediate good output y_t^I,

$$y_t^I = \Delta_t y_t, \tag{19}$$

where $\Delta_t \equiv \int_0^1 (p_{k,t})^{-\theta} dk$ is a measure of price dispersion, which can be rewritten as

$$\Delta_t = (1-\omega) p_t^{*-\theta} + \omega \Pi_t^{\theta} \Delta_{t-1}. \tag{20}$$

Finally, using intermediate good production function ($y_t^I = N_t$) in equation (19) leads to a relationship between aggregate employment and aggregate final good output,

$$N_t = \Delta_t y_t. \tag{21}$$

Thus, higher price dispersion increases the wedge between aggregate final output and aggregate employment. Since final goods are imperfectly substitutable a rise in the relative price (and correspondingly output) dispersion acts like a downward shift in labor productivity.

To summarize, the equilibrium of the model is determined by equations (4), (6), (9), (14), (16), (18), (20), (21), the equations determining the cost per hire ($g_t = Bx_t$), the job finding rate ($x_t \equiv H_t/U_t$), unemployment before hiring ($U_t = 1-(1-\delta)N_{t-1}$), unemployment after hiring ($u_t = 1 - N_t$), and the aggregate resource constraint ($y_t = c_t + hc_t$).

3 Steady state equilibrium

In a steady state equilibrium the flow into unemployment is equal to the flow out of unemployment. Starting with the intermediate good sector, in steady state the optimal hiring condition (9) becomes

$$p^I = w^d + \left(1 - \beta\Gamma^{1-\sigma}(1-\delta)\right) g, \tag{22}$$

which shows that faster growth decreases steady state hiring by decreasing the discounted savings in future hiring costs.[11] Similarly, in steady state the wage setting equation (14)

[11] Note that the effect of faster growth on optimal hiring is similar to a reduction in the subjective discount rate or an increase in the job separation rate.

becomes

$$w^d = b + \nu \left(1 - \beta\Gamma^{1-\sigma}(1-\delta)(1-x)\right) g, \tag{23}$$

which shows that faster growth increases the steady state real wage by decreasing the discounted continuation value to an employed worker. Substituting equation (23) into equation (22)

$$p^I = b + Bxh(\Gamma, x), \tag{24}$$

where $h(\Gamma, x) \equiv (1 - \beta\Gamma^{1-\sigma}(1-\delta)) + \nu (1 - \beta\Gamma^{1-\sigma}(1-\delta)(1-x))$ and we substitute out the cost per hire g using $g = Bx$. The steady state job finding rate x is given by

$$x = \frac{\delta N}{1-(1-\delta)N} \equiv x(N). \tag{25}$$

The term $h(\Gamma, x)g$ in equation (24) represents a labor market wedge (LMW) between the marginal revenue product p^I and the opportunity cost of work b. The LMW is the sum of two wedges: the first is the wedge, in the presence of hiring cost, between the marginal revenue product and the real wage (see equation (22)) and the second is the wedge between the real wage and the opportunity cost of work (see equation (23)). $LMW_\Gamma = (h_\Gamma)g > 0$, implying that, all else equal, the higher is productivity growth the larger is the LMW.

Next, from the final goods sector, in steady state the aggregate price index (18) becomes

$$p^* = p^*(\Pi) \equiv \left(\frac{1-\omega\Pi^{\theta-1}}{1-\omega}\right)^{1/(1-\theta)}, \tag{26}$$

so that the optimal relative price of a final good is pinned by trend inflation alone. It is easily seen that for $\Pi \geq 1$, $\partial p^*/\partial\Pi > 0$—the higher is trend inflation the larger is the gap between the optimally set prices and the price level. Substituting equation (26) in the steady state optimal relative price (17) and rearranging we get

$$p^I = \frac{p^*(\Pi)(1-\beta\omega\Gamma^{1-\sigma}\Pi^{\theta})}{\mu(1-\beta\omega\Gamma^{1-\sigma}\Pi^{\theta-1})} \equiv p^I(\Gamma, \Pi). \tag{27}$$

Under the special case of zero trend rate of inflation (i.e., $\Pi = 1$) $p^I(\Gamma, \Pi) = 1$ and $p^I = 1/\mu$ so that the relative intermediate good price is independent of productivity growth. By contrast, when trend inflation rate is positive ($\Pi > 1$), $p^I_\Gamma \equiv \partial p^I(\Gamma, \Pi)/\partial\Gamma > 0$.[12]

[12]The derivation is straightforward, as $\partial p^I(\Gamma, \Pi)/\partial\Gamma = (\sigma-1)\mu^{-1}\beta\omega\Gamma^{-\sigma}(\Pi-1)\Pi^{\theta-1}p^*(\Pi)/(1-\beta\omega\Gamma^{1-\sigma}\Pi^{\theta-1})^2$.

Finally, substitution of equation (27) in equation (24) leads to

$$p^I(\Gamma, \Pi) = b + Bx(N)h(\Gamma, x(N)). \tag{28}$$

The solution to the nonlinear equation (28) is an implicit function $N^* = N(\Gamma, \Pi)$, which relates the employment rate (and the unemployment rate, $u^* = 1 - N^*$) to productivity growth Γ and steady state inflation Π. In what follows we analyze the effect of productivity growth on equilibrium unemployment and how that effect depends on the level of trend inflation. For this purpose, we work with the total derivatives $dN^*/d\Gamma$ and $du^*/d\Gamma = -dN^*/d\Gamma$.

4 Comparative statics

We discuss the underlying channels whereby the level of steady state inflation affects $dN^*/d\Gamma$ so as to get a sense of the numerical analysis shown in the next section. To this end, we evaluate equation (28) at the implicit solution N^* so that

$$F \equiv p^I(\Gamma, \Pi) - Bx(N^*)h(\Gamma, x(N^*)) - b \equiv 0. \tag{29}$$

By applying the implicit function theorem (see, e.g., Chiang (1984)) on the identity (29) we get an expression for the effect of trend growth on equilibrium employment,

$$\frac{dN^*}{d\Gamma} = -\frac{F_\Gamma}{F_{N^*}} = \frac{p^I_\Gamma - Bx^*h_\Gamma}{T}, \tag{30}$$

where $x^* = x(N^*)$, $h_\Gamma = \beta\Gamma^{-\sigma}(\sigma - 1)(1 - \delta)(1 + \nu(1 - x^*) > 0$, $h_x = \nu\beta\Gamma^{1-\sigma}(1 - \delta) > 0$, $0 < x^* < N^* < 1$, $\delta < x_N < 1/\delta$, $T \equiv Bx_N(x^*h_x + h(\Gamma, x^*)) > 0$ and as is shown above $p^I_\Gamma > 0$.[13] We see that the sign of $dN^*/d\Gamma$ depends on the sign of $p^I_\Gamma - Bx^*h_\Gamma$. The first term represents the partial effect of trend growth on the intermediate good price and the second multiplicative term represents the partial effect of trend growth on the LMW. These effects arise in the presence, respectively, of nominal rigidity (via the interplay of output demand growth and the rise in real interest rate accompanying productivity growth) and labor market rigidity (via the interplay of productivity growth and the rise in real interest rate).

[13] All partial derivatives are evaluated at the equilibrium steady state employment.

In order to examine how trend inflation affects the sign of $dN^*/d\Gamma$, we rewrite the numerator of equation (30) as

$$p_\Gamma^I - Bx^* h_\Gamma = (\sigma - 1)\beta\Gamma^{-\sigma}\left\{\frac{\mu^{-1}\omega(\Pi - 1)\Pi^{\theta-1}p^*(\Pi)}{(1 - \beta\omega\Gamma^{1-\sigma}\Pi^{\theta-1})^2} - Bx^*(1-\delta)(1+\nu(1-x^*))\right\}.$$

We see that $p_\Gamma^I - Bx^* h_\Gamma < 0$ at $\Pi = 1$ (zero inflation rate) implying that $du^*/d\Gamma = -dN^*/d\Gamma > 0$. Moreover, looking at the terms inside the curly bracket, it can easily be checked that the first term increases monotonically with Π while the second term may increase or decrease with Π, since

$$\frac{d[Bx^*(1-\delta)(1+\nu(1-x^*))]}{d\Pi} = B(1-\delta)\frac{dx^*}{d\Pi}(1+\nu(1-2x^*)),$$

whose sign is not clear cut. For instance, under the standard assumption of symmetric wage bargaining (i.e., $\nu = 1$) the sign of the derivative depends only on the sign of $dx^*/d\Pi = p_\Pi^I x_N/T$, which is positive when Π is small enough (as then $p_\Pi^I > 0$) but negative when Π is large enough (as then $p_\Pi^I < 0$).[14] In this case, $du^*/d\Gamma < 0$ (faster growth leads to lower unemployment) if the level of inflation is sufficiently high.

In general determining the magnitude of the threshold level of inflation Π^*, above which $du^*/d\Gamma < 0$, requires knowledge of the equilibrium job finding rate $x^* = x(N^*)$. However, there is no explicit reduced-form solution to the nonlinear equilibrium condition (28). We thus resort to numerical analysis so as to illustrate our main result that the effect of growth on unemployment is non-monotonic—it is positive (negative) when the level of inflation is sufficiently low (high). We consider two alternative calibrations. In the first calibration the model's equilibrium unemployment rate is relatively low and the job finding rate is relatively high (for e.g., as in the US) while in the second calibration the equilibrium unemployment rate is relatively high and the job finding rate is relatively low (for e.g., as in continental Europe). Consistent with these, the implied job separation rate is relatively low in continental Europe and relatively high in the US. Such a distinction reflects the notion that the continental European labor market is more sclerotic than that of the US labor market (Blanchard and Gali (2010)), and as we show below, the structure of the labor market affects the threshold level of inflation.

[14] To see this, first note that by definition $p^I = (P_t^I/P_t^*)(P_t^*/P_t)$. On the one hand P_t^*/P_t $(= p^*)$ increases with inflation (see equation (26)), so that the markups of those firms whose prices are fixed in the past are eroded by higher inflation. On the other hand, P_t^I/P_t^* $(= p^I/p^*)$ decreases with inflation (see equation (17)), as optimizing firms raise their prices so as to mitigate the erosion of their future markups by higher inflation. The net effect of inflation on p^I depends on the level of inflation. When Π is small enough, $\partial p^I/\partial\Pi > 0$ due to time discounting (as $\partial(P_t^I/P_t^*)/\partial\Pi$ depends on β while $\partial(P_t^*/P_t)/\partial\Pi$ does not). The non-monotonic nature of p_Π^I is a standard property of New-Keynesian model with Calvo price staggering (see, e.g., King and Wolman (1996)).

We evaluate the derivative $du^*/d\Gamma$ at alternative equilibrium unemployment rates that are close to observed average unemployment rates—around 5 percent in the U.S. and around 10 percent in the continental Europe. As in Blanchard and Gali (2010), the exogenous job separation rate δ is set equal to 0.12 in the US calibration and 0.04 in the European calibration. Then our steady state equation relating the job finding rate to employment implies that the model's steady state job finding rate is about 0.7 in the US calibration and about 0.25 in the European calibration, values that are considered plausible by empirical standards.

We set $\Gamma = 1.0075$ for the US and $\Gamma = 1.005$ for Europe, implying an annualized productivity growth rate of 3% and 2%, respectively. These numbers are in line with long-term average growth rates (see, e.g., OECD (2003)). The value of the scale parameter B is set such that in the steady state equilibrium with a zero inflation rate the share of aggregate hiring costs in aggregate output is one percent.[15] The implied value of B is 0.12 in the US calibration and one in the European calibration. Furthermore, assuming symmetry in wage bargaining ($\nu = 1$)—a standard assumption in the labor search literature—the model's implied value of the replacement ratio b is 0.84 (0.82) in the US (European) calibration. Finally, somewhat in line with Shimer (2010) inverse of the elasticity of intertemporal substitution σ is set equal to 3 while the rest of the model parameters take values very similar to the New-Keynesian literature: $\beta = 0.99$, $\theta = 11$ (implying that firms choose a 10 percent price markup under flexible prices or when the inflation rate is zero) and $\omega = 0.75$ (prices are fixed on average for four quarters).

Results under US calibration. In the left panel of Figure 1 we plot p_Γ^I and $LMW_\Gamma = Bx^*h_\Gamma$ against the (annualized) steady state rate of inflation. It can be seen that p_Γ^I increases monotonically with inflation while LMW_Γ is nearly flat.[16] There is a threshold rate of inflation of about 2.7% below which $p_\Gamma^I < LMW_\Gamma$ and above which $p_\Gamma^I > LMW_\Gamma$.

The right panel of Figure 1 shows $du^*/d\Gamma(= -dN^*/d\Gamma)$ as a function of the rate of inflation and illustrates our main result that the relationship between unemployment and growth depends on the level of inflation. For inflation rates below (above) 2.7 percent unemployment and growth are positively (negatively) related.

Results under European calibration. The model with the European calibration, illustrated in Figure 2, has similar qualitative properties to that with the US calibration. The difference is quantitative: the threshold rate of inflation under the European calibration,

[15] As Blanchard and Gali (2010) point out, one percent is a plausible upper bound given the lack of direct empirical evidence.

[16] Although not visible to the naked eye, LMW_Γ actually rises (falls) with inflation for inflation rates below (above) one percent.

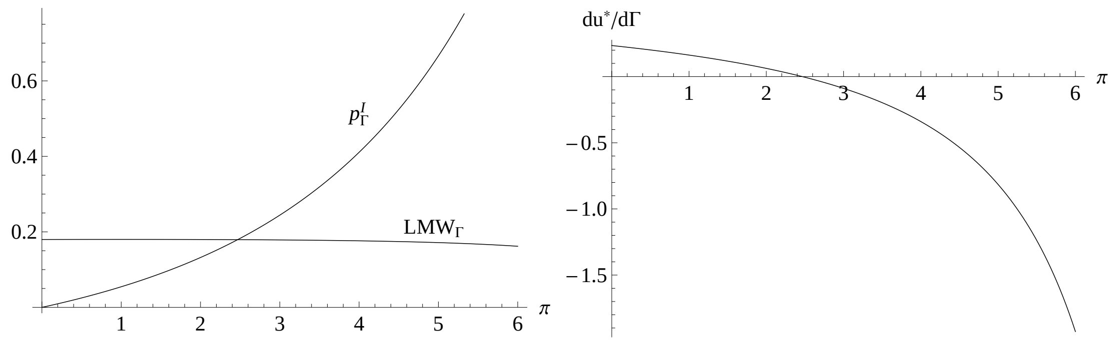

Figure 1: The threshold rate of inflation under US calibration.

which is about 5.3 percent, is higher than that under the US calibration (2.7 percent). As can be seen from the left panel of Figure 2, the difference is mainly due to LMW_Γ being larger under the European calibration.[17]

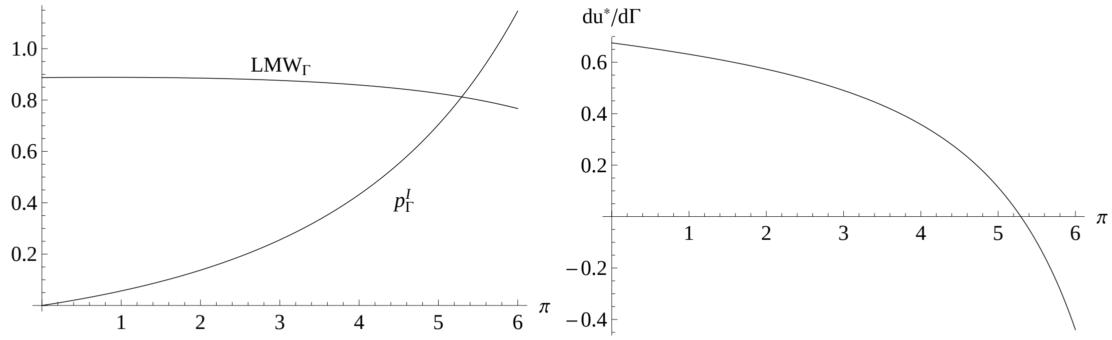

Figure 2: The threshold rate of inflation under European calibration.

5 Sensitivity to labor market parameters

In this section, we examine how the effect of growth on unemployment as well as the threshold level of inflation are influenced by changes in the exogenous labor market parameters—the scale parameter B in the cost per hire G_t (see equation (5)), the job separation rate δ, workers' relative bargaining power ν and the opportunity cost of work b. From equation (30) these labor market parameters affects $dN^*/d\Gamma$ by influencing the

[17]Note that, relative to the US calibration, the European calibration has larger values of B and $1-\delta$, both of which imply a larger value of LMW_Γ, but a smaller value of x^*, which implies a smaller value of LMW_Γ).

magnitude of LMW_Γ, which we rewrite as follows

$$LMW_\Gamma = \beta\Gamma^{-\sigma}(\sigma-1)(1-\delta)Bx^* + \beta\Gamma^{-\sigma}(\sigma-1)(1-\delta)\nu(1-x^*)Bx^*. \quad (31)$$

The first right hand side term in equation (31) captures the effect of growth on the wedge between the marginal revenue product and the real wage (see equation (9)). The second term captures the effect of growth on the wedge between the real wage and the opportunity cost of work (see equation (14)). From (9) equation, all else equal, the expected future savings in hiring costs declines with trend growth. The decline in future savings in hiring costs (and in turn the rise in the wedge between the marginal revenue product and the real wage) is more pronounced the higher is the cost per hire, $g^* = Bx^*$ and the higher is the job retention rate $1-\delta$. Likewise, from equation (14), all else equal, the continuation value to an employed worker declines with trend growth. The decline in the continuation value (and in turn the rise in the wedge between the real wage and the opportunity cost of work) is more pronounced the higher is the cost per hire, the higher is the probability of not finding a job in case of job separation, $1-x^*$, and the higher is the job retention rate $1-\delta$. The total derivative of equation (31) with respect to parameter $z \in \{B, \nu, \delta, b\}$ is given by

$$\frac{dLMW_\Gamma}{dz} = LMW_{\Gamma,z} + LMW_{\Gamma,x}\frac{dx^*}{dz}.$$

The term $LMW_{\Gamma,z}$ captures the direct effect of z. The second multiplicative term captures the indirect effect, where $LMW_{\Gamma,x} = B\beta\Gamma^{-\sigma}(\sigma-1)(1-\delta)(1+\nu(1-2x^*)) > 0$ if $x^* < 0.5$ (satisfied under the European benchmark calibration) or $\nu = 1$ (satisfied under the US and European benchmark calibrations). The overall effect depends on the signs and magnitudes of $LMW_{\Gamma,z}$ and dx^*/dz. The table below shows the sign of the effects on the unemployment rate u^*, the job finding rate x^* and the threshold rate of inflation π^T of an increase in the value of one parameter (B, ν, δ or b) while keeping the rest of model parameters at their respective baseline values, as discussed in the previous section. We only show the sign of the effect of each parameter since the comparative statics are similar under the US and European calibrations.

Table

Parameter	u^*	x^*	π^T
B	$+$	$-$	$+$
ν	$+$	$-$	$+$
δ	$+$	$-$	$-$
b	$+$	$-$	$-$

Note first the comparative static effects of the four labor market parameters on the unemployment rate and the job finding rate. For instance, the larger is the value of B (i.e, the higher is the cost per hire) the higher is the unemployment rate and the lower is the job finding rate. A higher rate of job destruction, an increase in the bargaining power of workers or an increase in the opportunity cost of work have similar effects. These are standard properties of search models of unemployment. Below we discuss the comparative static effects on the threshold level of inflation.

The effect of B: From equation (31) it is easy to check that the direct effect of B is positive ($LMW_{\Gamma,B} > 0$) while the indirect effect is negative, as $dx^*/dB = x_N(dN^*/dB) = -x_N x^* h(\Gamma, x^*)/T < 0$.[18] As shown in the table above, we find that the larger is B the larger is π^T. This result implies that under our calibrations the direct positive effect dominates the indirect negative effect so that the larger is B the larger is LMW_Γ.[19]

The effect of ν: The direct effect of ν is positive ($LMW_{\Gamma,\nu} > 0$) while the indirect effect is negative, as $dx^*/d\nu = x_N(dN^*/d\nu) = -x_N Bx^*[1 - \beta\Gamma^{1-\sigma}(1-\delta)(1-x^*)]/T < 0$. The table shows that the larger is ν the larger is π^T, implying that the direct positive effect dominates the indirect negative effect. This shows that a rise in workers' relative bargaining power has similar effects as does a rise in the cost per hire.

The effect of δ: The direct effect of δ is negative ($LMW_{\Gamma,\delta} < 0$) while the indirect effect is ambiguous a priori, as $dx^*/d\delta = x_\delta + x_N(dN^*/d\delta)$, where $x_\delta, x_N > 0$ (see equation (25)) and $dN^*/d\delta = -B\beta\Gamma^{1-\sigma}x^*(1 + \nu(1 - x^*))/T < 0$. We find that under our calibrations the larger is δ the smaller are N^* and x^*. This result implies that the indirect negative effect reinforces the direct negative effect so that the larger is δ the smaller is LMW_Γ and therefore the smaller is π^T.[20]

[18] Thus an increase in the value of B decreases the job finding rate, which in turn decreases the cost per hire and increases the probability of not finding a job in case of job separation. As discussed above these have countervailing effects on LMW_Γ.

[19] In terms of the left panel of Figures 1 and 2 the curve pertaining to LMW_Γ shifts upward, implying a higher threshold rate of inflation.

[20] In terms of the left panel of Figures 1 and 2 the curve pertaining to LMW_Γ shifts downward, implying a lower threshold rate of inflation.

The effect of b: Finally, note that $LMW_{\Gamma,b} = 0$ while the indirect effect is negative, as $dx^*/db = x_N(dN^*/db) = -x_N/T < 0$. The lower value of x^* associated with a larger value of b decreases the cost per hire and increases the probability of not finding a job in case of job separation $1 - x^*$. The table shows that the larger is b the smaller is π^T.

Together, these results show that, at any given rate of inflation, shocks, policies or institutions that contribute to higher costs of hiring (making the labor marker more rigid) or a higher bargaining power of workers relative to firms in wage negotiations also make it *less* likely that faster growth leads to lower unemployment. In this case it takes a higher rate of inflation for faster growth to lead to lower unemployment. By contrast, shocks, policies or institutions that contribute to a higher rate of job separation or raise the opportunity cost of employment also make it *more* likely that faster growth leads to lower unemployment. In this case it takes a lower rate of inflation for faster growth to lead to lower unemployment.

6 Summary and concluding remarks

Following the simultaneous slowdown in productivity growth and rising unemployment in many OECD economies during the 1970s academic research has sought to understand the effect of growth on steady state unemployment using the standard the search model of unemployment. Past research has shown that when the intertemporal substitution in consumption is weak (a plausible assumption) search-type models of the labor market imply that disembodied technological progress leads to higher unemployment, a result at odds with the experience of the 1970s and recent empirical evidence (Pissarides and Vallanti (2007)).

Motivated by the observation that the 1970s were also characterized by high and rising inflation the present paper reexamines the effect of growth on unemployment in the presence of nominal price rigidity (implying a role for inflation). The analysis leads to a novel result: faster growth leads to lower unemployment if the rate of inflation is high enough. More generally, the paper shows that the effect of growth on unemployment is non-monotonic—there is a threshold level of inflation below (above) which faster growth leads to higher (lower) unemployment. The threshold level in turn depends on labor market characteristics—hiring efficiency, the job destruction rate, workers' relative bargaining power and the opportunity cost of work—as is demonstrated by a model calibrated to the US and continental European economies.

The model is kept as simple as possible (for instance, assuming an exogenous and disembodied technological progress) so as to focus on the role of nominal rigidities and present the results in a more transparent way. A straightforward extension of the model is to allow for endogenous growth via learning-by-doing. For example, one could allow for a feedback from unemployment to growth (as in Aghion and Howitt (1994)) or introduce capital and assume positive externality from aggregate capital accumulation (as in Eriksson (1997)). However, the resulting model is no longer amenable to the comparative static analysis undertaken in the present paper, as then growth becomes an endogenous variable. One can nevertheless study how growth and unemployment respond to structural parameters.

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