

Currency devaluation with dual labor market :
Which perspectives for the Euro Zone ?
First Draft

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BETA¹

Abstract

In this paper, we assume a world of two countries in a fixed exchange rate system. The main difference between the two countries lies in the features of their labor markets. In the home country, we assume the existence of a dual labor market, with formal and informal sectors. In the foreign country, the labor market is homogeneous and characterized by a nominal wage rigidity. In this context, the situation of labor market in each country is not optimal through a misallocation of workers between sectors in domestic economy, and unemployment in foreign economy.

Our article shows that a devaluation of domestic currency implies a fall in production in each country, an increase of unemployment in foreign economy and a worse reallocation of workers by a growth of informal sector in domestic economy.

Keywords : efficiency wage, dualism, exchange rate, devaluation

JEL classification : F41, J31, J41

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1 Introduction

The sovereign debt crisis in Europe sheds light on the possibility for some countries to exit the Monetary Union in order to recover money as a tool of economic policy. For instance, some politicians have notably proposed that Greece leaves the Euro Zone. In this context, the recovery of a devaluated Drachma could sustain employment and growth in this country.

Indeed, there is a growing body of literature dealing with the departure from monetary union in various directions. Proctor [2006], Athanassiou [2009], Dor [2011] and Thieffry [2011] study this issue from an institutional and legal point of view. Another field of research focuses on exit strategies from a Monetary Union. Eichengreen [2010] argues that technical and legal difficulties of reintroducing national currency, while surmountable, should not be underestimated. In this context, he suggests several measures to prevent a break-up. Cooper [2012] suggests that countries who do not respect fiscal discipline, have to be punished through a credible exit strategy from the Monetary Union. Indeed, he argues that "Euroization", as an incomplete exit strategy, enables to reach this objective.

Nevertheless, these papers do not analyze the macroeconomic consequences of an exit from the monetary union. Our paper intends to contribute to this debate by focusing on the role of labor market. More precisely, in the European Union, some countries present an important informal sector, as shown by Schneider [2005] and Hazans [2011]. Moreover, Pouliakas and Theodossiou [2010] empirically confirm that Greece, Ireland, Italy, Portugal and Spain appear to be characterized by dual labor market. Few papers, among over Demekas [1990], Agénor and Santaella [1998], and Cook and Nosaka [2005] have already considered the case of segmented labor market in open economies. ²However, these articles only consider a small open economy framework. Taking into account the fact that countries presenting dual labor market in Europe represent a very significant weight in the GDP of euro zone, the hypothesis of a small open economy does not appear relevant.

In this paper, we assume a world of two countries in a fixed exchange rate system. The main difference between the two countries lies in the features of their labor markets. In the home country, we assume the existence of

²Christoffel, Kuester, and Linzert [2006] or Mattesini and Rossi [2009] have also considered the case of dual labor market, but focus on the efficiency of monetary policy in closed economy.

a dual labor market, with formal and informal sectors. Wage in this first sector takes into account workers effort and corresponds to efficiency wage above concurrential wage as Shapiro and Stiglitz [1984]. Wage in this second segment is concurrential. In the foreign country, the labor market is homogeneous and characterized by a nominal wage rigidity. In this context, the situation of labor market in each country is not optimal through a misallocation of workers between sectors in domestic economy, and unemployment in foreign economy.

In this framework, we analyze the effects of an exchange rate policy. Our article shows that a devaluation of domestic currency implies a fall in production in each country, an increase of unemployment in foreign economy and a worse reallocation of workers by a growth of informal sector in domestic economy. So, the devaluation is clearly counterproductive for the domestic economy since it damages the labor market and production. Furthermore, it is straightforward to demonstrate that this deterioration is greater, the bigger the domestic economy is.

We start by describing the model, notably the features of labor markets (section 2). We then analyze the equilibrium and the effects of a devaluation in the domestic country (section 3). We finally conclude in last section (section 4).

2 The model

We assume a world of two countries in a fixed exchange rate system: the country H (home country) and the country F (foreign country). Each country produces a single tradable good, noted h and f respectively for country H and F . We denote the price of the good h in the home market by p_h and the price of the good f in the foreign market by p_f . We assume that the "Law of one price" holds³.

The main difference between the two countries lies in the features of their labor markets. In the home country, we assume the existence of a dual labor market, with formal and informal sectors. In the foreign country, the labor market is homogeneous and characterized by a nominal wage rigidity.

³Since there are two goods in two countries, we should consider four prices, which can be noted p_i^j with $i = h, f$ and $j = H, F$. The "Law of one price" being verified, we have the following relations : $p_i^H = Ep_i^F$ for $i = h, f$ where E , the nominal exchange rate, represents the number of home currency units of one foreign currency unit (notice that the EMU corresponds to the case where $E = 1$). In what follows, we will only work with the two prices variables $p_h \equiv p_h^H$ and $p_f \equiv p_f^F$ defined in the text.

2.1 Production and labor markets

In the home country, we introduce a segmented labor market with two sectors. Each sector contributes to the production of the single domestic good h . In the primary sector, called formal sector, only skilled workers can be employed. Wage in this first sector takes into account workers effort and corresponds to efficiency wage above concurrential wage. In the secondary sector, called informal sector, both skilled and unskilled workers can work. Wage in this second segment is concurrential. As a consequence, no unemployment can emerge in the home country since skilled workers who do not find a job in the formal sector can always be hired in the informal one.

In the formal sector (sector 1), the aggregate production function of good h is:

$$Y_{h1}(e, L_1) = e^\beta L_1^\alpha \quad (1)$$

where Y_{h1} represents the production of good h , e is the worker's effort and L_1 the number of workers in the formal sector. We suppose decreasing returns to scale ($\alpha + \beta < 1$) and $0 < \beta < \alpha < 1$.

The effort is not observable, so that employers determine the efficiency wage developed by Shapiro and Stiglitz [1984]. Assume that consumption and effort decisions are separable, and that they depend only on the real wage earned w and the disutility of effort e . The representative worker utility function is defined by $u(w, e) = w - e$. The level of effort provided by skilled workers is strictly positive when employed and not shirking in the primary sector, or zero when shirking while employed in the primary sector or working in the informal sector. The optimal effort level of a skilled worker is deduced by the following non shirking condition :

$$w_1 - e \geq (1 - \pi)w_1 + \pi w_2 \quad (2)$$

where w_1 represents the real wage of formal workers in the primary sector and w_2 the real wage of informal workers in the secondary one. The left hand-side in expression (2) measures the expected utility derived by a formal worker who is not shirking and provides a level of effort equal to e , while the right hand-side measures the expected utility of a shirking worker as a weighted average of the wage earned if caught shirking and fired (with a probability π), and if not caught shirking (with a probability $1 - \pi$) in which case the level of effort is zero.

The level of effort required by firms is assumed to be such that formal workers are indifferent between shirking and not shirking, in which case workers choose not to shirk, so that condition (2) hold with equality. Solving for

the required level of effort yields to :

$$e(w_1, w_2) = \pi(w_1 - w_2) \quad (3)$$

Relation (3) shows that the level of effort produced by workers depends positively on the real wage difference between formal and informal sectors. Moreover, it can readily be established that an increase of the probability of being caught shirking raises the level of effort.

The representative producer of good h in the formal sector maximizes his real profit $\frac{\Pi_{h1}}{P_H}$, where P_H is the general level of prices in home country⁴, that is, using equations (1) and (3) and assuming that firm incurs no hiring or firing costs:

$$\max_{(Y_{h1}, w_1)} \frac{\Pi_{h1}}{P_H} = \left\{ \frac{p_h Y_{h1}}{P_H} - \frac{w_1 Y_{h1}^{1/\alpha}}{e(w_1, w_2)^{\beta/\alpha}} \right\}$$

The first order conditions are:

$$\frac{\partial \frac{\Pi_{h1}}{P_H}}{\partial Y_{h1}} = \frac{p_h}{P_H} - \frac{1}{\alpha} \frac{w_1 Y_{h1}^{(1-\alpha)/\alpha}}{e(w_1, w_2)^{\beta/\alpha}} = 0 \quad (4)$$

$$\frac{\partial \frac{\Pi_{h1}}{P_H}}{\partial w_1} = -Y_{h1}^{1/\alpha} \left[\frac{e(w_1, w_2)^{\beta/\alpha} - \pi w_1 \frac{\alpha}{\beta} e(w_1, w_2)^{\beta/\alpha - 1}}{e(w_1, w_2)^{2\beta/\alpha}} \right] = 0 \quad (5)$$

From expression (5), we derive a relation between the efficiency wage and competitive wage:

$$w_1 = \sigma w_2 \text{ with } \sigma = \frac{\alpha}{\alpha - \beta} \quad (6)$$

At the equilibrium, wage in the formal sector is above the competitive wage in the informal sector. The optimal level of effort is deduced from expressions (3) and (6):

$$e^*(w_1) = \delta w_1 \text{ with } \delta = \frac{\beta \pi}{\alpha} \quad (7)$$

We find that at equilibrium, the level of effort is increasing with the formal sector wage. Combining the optimality condition (4) and equilibrium effort

⁴The general level of prices P_H will be determined precisely in the next section.

(7) gives both the good h supply by firms in the formal sector and the formal worker demand:

$$Y_{h1}(w_1, z) = (\alpha z)^{\frac{\alpha}{1-\alpha}} \delta^{\frac{\beta}{1-\alpha}} w_1^{\frac{\beta-\alpha}{1-\alpha}} \text{ with } \frac{\partial Y_{h1}}{\partial w_1} < 0 \text{ and } \frac{\partial Y_{h1}}{\partial z} > 0 \quad (8)$$

and

$$L_1^d(w_1, z) = (\alpha z)^{\frac{1}{1-\alpha}} \delta^{\frac{\beta}{1-\alpha}} w_1^{\frac{\beta-1}{1-\alpha}} \text{ with } \frac{\partial L_1^d}{\partial w_1} < 0 \text{ and } \frac{\partial L_1^d}{\partial z} > 0 \quad (9)$$

where $z = \frac{p_h}{P_H}$ denotes the price of the good h relatively to the general price level.

An increase in the efficiency wage implies a reduction of skilled labor demand and a decrease of supply. Even if this last negative effect seems obvious at first glance, it results from two opposite effects. On the one hand, we have a negative quantitative effect on production since a higher wage yields to a lower skilled labor demand. On the other hand, we find a positive qualitative effect on output because a higher wage rises the optimal level of effort. From expression (8), the negative qualitative effect is larger than the positive qualitative one, leading to an inverse relation between efficiency wage and production.

Moreover, when the relative price z increases, the real wage in the primary sector goes down involving simultaneously a raise in formal labor demand and in good h supply.

In the secondary or informal sector (sector 2), we assume a perfect observation of the effort by employer. For simplicity's sake, we admit that the disutility of informal worker effort is supposed to be zero. The informal wage is fully flexible and determined by market forces. The aggregate production technology in the informal sector is given by:

$$Y_{h2}(L_2) = L_2^\alpha \text{ with } \alpha < 1 \quad (10)$$

where Y_{h2} denotes the total quantity of good h produced in the informal sector and L_2 is the number of informal workers. The profit maximization program is:

$$\max_{Y_{h2}} \frac{\Pi_{h2}}{P_H} = \left\{ \frac{p_h Y_{h2}}{P_H} - w_2 Y_{h2}^{1/\alpha} \right\}$$

From the first order condition, the production of good h and the informal labor demand are:

$$Y_{h2}(w_2, z) = \left(\frac{\alpha z}{w_2} \right)^{\frac{\alpha}{1-\alpha}} \text{ with } \frac{\partial Y_{h2}}{\partial w_2} < 0 \text{ and } \frac{\partial Y_{h2}}{\partial z} > 0 \quad (11)$$

$$L_2^d(w_2, z) = \left(\frac{\alpha z}{w_2}\right)^{\frac{1}{1-\alpha}} \quad \text{with} \quad \frac{\partial L_2^d}{\partial w_2} < 0 \quad \text{and} \quad \frac{\partial L_2^d}{\partial z} > 0 \quad (12)$$

where production and level of informal workers demand are obviously increasing with relative price z and decreasing with real wage w_2 .

Let \bar{L}_H denote the total supply of labor in the domestic economy H , supposed to be constant. We note \bar{L}_{H1} the exogenous total number of skilled workers and $\bar{L}_{H2} = \bar{L}_H - \bar{L}_{H1}$ the exogenous total number of unskilled workers. Firms in the primary sector set both wage and level of formal employment. Employers then hire formal workers among the total skilled labor force in order to satisfy their labor demand. Skilled workers who do not succeed in finding a job in the formal sector enter the informal sector where wage is the adjustment variable. Formally, labor market equilibrium can be written as follows:

$$\bar{L}_{H1} - L_1^d(w_1, z) + \bar{L}_{H2} = L_2^d(w_2, z) \quad (13)$$

Introducing (6), (7), (9) and (12) in the expression of labor market equilibrium (13), we obtain the relative price z of good h as a function of the competitive wage w_2 :

$$z(w_2) = \frac{1}{K} \left(\Phi w_2^{\frac{\beta-1}{1-\alpha}} + w_2^{\frac{-1}{1-\alpha}} \right)^{\alpha-1} \quad \text{with} \quad \frac{dz}{dw_2} > 0 \quad (14)$$

where $K = \alpha \bar{L}_H^{\alpha-1}$ and $\Phi = \sigma^{\frac{\beta-1}{1-\alpha}} \delta^{\frac{\beta}{1-\alpha}}$. Substituting z given by expression (14) in labor demands (9) and (12), the skilled and unskilled labor demands are given by:

$$L_1^d(w_2) = \frac{\Phi \alpha^{\frac{1}{1-\alpha}}}{K^{\frac{1}{1-\alpha}} \left(\Phi + w_2^{\frac{-\beta}{1-\alpha}} \right)} \quad \text{with} \quad \frac{dL_1^d}{dw_2} > 0 \quad (15)$$

$$L_2^d(w_2) = \frac{\alpha^{\frac{1}{1-\alpha}}}{K^{\frac{1}{1-\alpha}} \left(1 + \Phi w_2^{\frac{\beta}{1-\alpha}} \right)} \quad \text{with} \quad \frac{dL_2^d}{dw_2} < 0 \quad (16)$$

An increase of the relative price z tends to motivate the firms of each sector to raise the level of output, implying higher formal and informal labor demands. However, the two sectors can not satisfy simultaneously their new

labor demand because of full employment condition. Consequently, some skilled workers from informal sector enter the primary one, and the decrease of labor in the secondary sector leads to raise the level of competitive wage.

Notice that the total supply of good h , associated with labor market equilibrium, is given by:

$$Y_h(w_2) = Y_{h1}(w_2) + Y_{h2}(w_2) \quad (17)$$

Substituting w_1 and z , respectively given by equations (6) and (14), in expressions (8) and (11), we obtain:

$$Y_h(w_2) = \left(\frac{\alpha}{K}\right)^{\frac{\alpha}{1-\alpha}} \frac{1 + \Lambda w_2^{\frac{\beta}{1-\alpha}}}{\left(1 + \Phi w_2^{\frac{\beta}{1-\alpha}}\right)^\alpha} \text{ with } \frac{dY_h}{dw_2} > 0 \quad (18)$$

where $\Lambda = \sigma^{\frac{\beta-\alpha}{1-\alpha}} \delta^{\frac{\beta}{1-\alpha}}$. This result shows that the total production in home country is not constant although the full employment is always satisfied. Indeed, it means that even if each worker is employed, the total level of production can evolve thanks to workers reallocation between the two sectors. An increase in competitive wage w_2 leads to a flow of skilled workers from the informal to the formal sector. As a consequence, the supply in the primary sector grows up, whereas it declines in the secondary sector, as shown in Appendix (A). Finally, the overall effect is positive.

In the foreign economy, we assume a one sector labor market, with homogeneous workers, and a legal minimum nominal wage \bar{W} above the equilibrium wage. In other words, the labor market is characterized by unemployment. Firms produces a single good f traded on a competitive market. The production technology is given by

$$Y_f = L_f^\alpha \text{ with } 0 < \alpha < 1 \quad (19)$$

where Y_f represents the production of good f and L_f the number of workers.

The representative producer of good f maximizes his real profit $\frac{\Pi_f}{P_F}$, where P_F is the general level of prices in foreign country⁵, that is :

$$\max_{(Y_f)} \frac{\Pi_f}{P_F} = \left\{ \frac{p_f}{P_F} Y_f - \frac{\bar{W}}{P_F} Y_f^{1/\alpha} \right\}$$

⁵The general level of prices P_F will be determined precisely in the next section.

The first order conditions is:

$$\frac{\partial \frac{\Pi_f}{P_F}}{Y_f} = \frac{p_f}{P_F} - \frac{1}{\alpha} \frac{\bar{W}}{P_F} Y_f^{(1-\alpha)/\alpha} = 0 \quad (20)$$

From this condition, we can derive the supply of good f and the corresponding labor demand as:

$$Y_f(p_f) = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{\bar{W}}{p_f}\right)^{\frac{\alpha}{\alpha-1}} \text{ with } \frac{dY_f}{dp_f} > 0 \quad (21)$$

and

$$L_f^d(p_f) = \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{\bar{W}}{p_f}\right)^{\frac{1}{\alpha-1}} \text{ with } \frac{dL_f^d}{d(\bar{W}/p_f)} < 0 \quad (22)$$

These two last equations simply state that the production is increasing with the price of good f , whereas the labor demand decreases with the real cost of labor.

As supplies of goods and labor demands are determined, in the next subsection, we focus on the demand side.

2.2 Demands for goods and money

In each country, consumers face three goods : the two tradable goods h and f and the money of their country. The utility function of the representative consumer, for $j = H, F$, is given by:

$$U_j = \left(\frac{M_j}{P_j}\right)^\theta C_j^{1-\theta} - k_j \theta^\theta (1-\theta)^{1-\theta} e_j \text{ with } k_H = 1, k_F = 0, 0 < \theta < 1 \quad (23)$$

with

$$C_j = (c_{hj}^\rho + c_{fj}^\rho)^{1/\rho} \text{ with } 0 < \rho < 1 \quad (24)$$

where c_{ij} represents the consumption of the good $i = h, f$ by the consumer of the country $j = H, F$. The preferences are represented by a Cobb-Douglas function concerning the aggregate consumption and money, and separable regarding work disutility. Preferences on goods are represented by a CES function as described by (24). Since $\rho < 1$, the two goods are imperfect substitutes and the elasticity of substitution is $1/(1-\rho)$. Because of the separability of the utility function, we can derive the consumption demand for

goods and money holdings independently of the second term of U_j . Concerning the leisure decisions, the problem was treated in the previous section⁶.

Considering the utility function, the general level of prices, in each country, is defined as follows :

$$P_H = \left(p_h^{\frac{\rho}{\rho-1}} + (Ep_f)^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \quad (25)$$

$$P_F = \left(\left(\frac{p_h}{E} \right)^{\frac{\rho}{\rho-1}} + p_f^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \quad (26)$$

The representative consumer maximizes his utility function under a budget constraint where Ω_j denotes the total income. This one is composed of nominal wages W_j , the profits Π_j distributed by the firms of country j and a fixed initial quantity of money \bar{M}_j . In other words, his maximization program can be written as :

$$\left\{ \begin{array}{l} \underset{(C_j, M_j)}{Max} \left(\frac{M_j}{P_j} \right)^\theta C_j^{1-\theta} \\ \text{s.t. } P_j C_j + M_j = \Omega_j, C_j > 0 \text{ and } M_j > 0 \end{array} \right.$$

From the first order conditions, we derive the optimal demands for goods and for money.

$$C_j = (1 - \theta) \frac{\Omega_j}{P_j} \quad (27)$$

$$M_j = \theta \Omega_j \quad (28)$$

Expressions (27) and (28) states that the money demand equals a share θ of the nominal income, whereas the optimal consumption corresponds to a share $1 - \theta$ of the real income.

As aggregate demand in each country is determined, we focus now on the optimal demand for goods in each country. In the home country, maximization program of the consumption function is:

$$\left\{ \begin{array}{l} \underset{(c_{hH}, c_{fH})}{Max} (c_{hH}^\rho + c_{fH}^\rho)^{1/\rho} \\ \text{s.t. } p_h c_{hH} + (Ep_f) c_{fH} = (1 - \theta) \Omega_H, c_{hH} > 0 \text{ and } c_{fH} > 0 \end{array} \right.$$

⁶Indeed, this particular form of the second term of expression (23) leads to the indirect utility function $u = w - e$ used in the previous section.

Optimal demands for each good can be expressed as:

$$c_{hH} = (1 - \theta) \frac{\Omega_H}{P_H} \left(\frac{p_h}{P_H} \right)^{\frac{1}{\rho-1}} \quad (29)$$

$$c_{fH} = (1 - \theta) \frac{\Omega_H}{P_H} \left(\frac{Ep_f}{P_H} \right)^{\frac{1}{\rho-1}} \quad (30)$$

Similarly, for the representative consumer in the foreign country, optimal demands for each good are:

$$c_{hF} = (1 - \theta) \frac{\Omega_F}{P_F} \left(\frac{p_h}{EP_F} \right)^{\frac{1}{\rho-1}} \quad (31)$$

$$c_{fF} = (1 - \theta) \frac{\Omega_F}{P_F} \left(\frac{p_f}{P_F} \right)^{\frac{1}{\rho-1}} \quad (32)$$

From individual demands for each good, we can deduce the aggregate demand functions by summing domestic and foreign demands for each good i such that $D_i = c_{iH} + c_{iF}$ for $i = h, f$. Using expressions (29) to (32), total demand for good h and good f are:

$$D_h(p_h, p_f) = (1 - \theta) \frac{\Omega_H}{P_H} \left(\frac{p_h}{P_H} \right)^{\frac{1}{\rho-1}} + (1 - \theta) \frac{\Omega_F}{P_F} \left(\frac{p_h}{EP_F} \right)^{\frac{1}{\rho-1}} \quad (33)$$

$$D_f(p_h, p_f) = (1 - \theta) \frac{\Omega_H}{P_H} \left(\frac{Ep_f}{P_H} \right)^{\frac{1}{\rho-1}} + (1 - \theta) \frac{\Omega_F}{P_F} \left(\frac{p_f}{P_F} \right)^{\frac{1}{\rho-1}} \quad (34)$$

As aggregate demand for each good is determined, we describe now the money market. As we assume a fixed nominal exchange rate, the equilibrium is defined by the equality between the world money supply and the world money demand. The money demand is given by equation (28). The money supply is \bar{M}_j in each country, and with a fixed exchange rate, it rises with a trade balance surplus. The definition of the money market equilibrium represents the external equilibrium, expressed as follows:

$$M_H + EM_F = \bar{M}_H + E\bar{M}_F \quad (35)$$

3 Equilibrium and exchange rate policy

3.1 Equilibrium

This world economy is characterized by five markets : the good h market, the good f market, two national labor markets and the money market. We can reduce this model to two equilibrium conditions on good markets. Since the Law of one price holds, the Purchasing Power Parity condition is always verified : $P_H = EP_F$.

The equilibrium condition on the good h market is derived from equalization of world demand, given by expression (33), and the domestic supply provided by (18). This last equation takes into account the domestic labor market equilibrium. Furthermore, using expression (14) defining the relation between w_2 and z and knowing that $z = \frac{p_h}{P_H}$, we can derive the supply of good h as a function of p_h and p_f :

$$D_h(p_h, p_f) = (1 - \theta) \frac{\Omega_H + E\Omega_F}{P_H} \left(\frac{p_h}{P_H} \right)^{\frac{1}{\rho-1}} = Y_h(p_h, p_f) \quad (36)$$

Similarly, the equilibrium condition on the good f market is obtained from equalization of expressions (34) and (21), where supply is associated to underemployment situation:

$$D_f(p_h, p_f) = (1 - \theta) \frac{\Omega_H + E\Omega_F}{P_H} \left(\frac{Ep_f}{P_H} \right)^{\frac{1}{\rho-1}} = Y_f(p_f) \quad (37)$$

From relations (28) and (35), we can express the world income as a function of the world money holdings:

$$\Omega_H + E\Omega_F = \frac{\bar{M}_H + E\bar{M}_F}{\theta} \quad (38)$$

Using relations (38) into expressions (36) and (37), the reduced model is given by the two following equations :

$$D_h(p_h, p_f) = \frac{1 - \theta}{\theta} \frac{\bar{M}_H + E\bar{M}_F}{P_H} \left(\frac{p_h}{P_H} \right)^{\frac{1}{\rho-1}} = Y_h(p_h, p_f) \quad (39)$$

$$D_f(p_h, p_f) = \frac{1 - \theta}{\theta} \frac{\bar{M}_H + E\bar{M}_F}{P_H} \left(\frac{Ep_f}{P_H} \right)^{\frac{1}{\rho-1}} = Y_f(p_f) \quad (40)$$

Since we suppose that goods are substitutes ($\rho < 1$), the sign of the partial derivatives of the goods demands with respect to prices can be established without ambiguity:

$$\frac{\partial D_h(p_h, p_f)}{\partial p_h} < 0 \quad \text{and} \quad \frac{\partial D_h(p_h, p_f)}{\partial p_f} > 0$$

$$\frac{\partial D_f(p_h, p_f)}{\partial p_h} > 0 \quad \text{and} \quad \frac{\partial D_f(p_h, p_f)}{\partial p_f} < 0$$

These derivatives confirm traditional results : the demand for each good decreases when its price increases, and due to substitutability, increases with the price of the other good.

It is straightforward to note that $\frac{\partial z(p_h, p_f)}{\partial p_h} > 0$ and $\frac{\partial z(p_h, p_f)}{\partial p_f} < 0$, and using (14) and (18), partial derivatives of the domestic good supplies with respect to prices reveals that:

$$\frac{\partial Y_h(p_h, p_f)}{\partial p_h} > 0 \quad \text{and} \quad \frac{\partial Y_h(p_h, p_f)}{\partial p_f} < 0$$

The global supply of good h is increasing (respectively decreasing) with the price p_h (respectively p_f). Even if these results seem obvious, it is important to recall that they are the consequences of more complex mechanisms, through dual labor market. Indeed, changes in prices of good affect wages and implies reallocation of workers between sectors. As explained in the previous section, an increase in p_h leads to more hirings in the formal sector at the expense of the informal one. As a consequence, wages in both sectors are higher notably because of efficiency considerations. Finally, the total supply of good h rises, indicating that the reduction of production in the informal sector is more than offset by the expansion of production in the formal sector.

Since the equilibrium is analyzed, we can now shed light on the effects of an exchange rate policy.

3.2 Exchange rate policy effects

At the equilibrium, the situation of employment is not satisfying. Indeed, in the home country, since jobs are rationed in the formal sector because of the presence of an efficiency wage, some workers have to accept informal jobs. In the foreign country, as the labor market is characterized by a minimum legal wage, unemployment emerges.

In this case, it can be interesting to analyze the effects of an exchange rate policy. More precisely, the questions are : can a devaluation of the domestic currency improve the allocation of workers by increasing formal jobs ? And what are consequences in the foreign country ? To answer these questions, we examine the effects of an increase in the exchange rate E on macroeconomic outcomes, which are appreciated by studying the elasticities of prices p_h and p_f , and of relative price z with respect to nominal exchange rate E , as shown in Appendix (B). Results are :

$$\xi_{p_h/E} = \frac{\frac{dp_h}{p_h}}{\frac{dE}{E}} > 0, \quad \xi_{p_f/E} = \frac{\frac{dp_f}{p_f}}{\frac{dE}{E}} < 0, \quad \xi_{z/E} = \frac{\frac{dz}{z}}{\frac{dE}{E}} < 0 \quad (41)$$

Thus, a devaluation implies an increase in price of good h (p_h) and a decrease in price of good f (p_f). The effect on the relative price z seems at first glance ambiguous because of the opposite effects of p_h , p_f and E on z as shown in expression (48). However, we demonstrate that the overall effect of a devaluation on the relative price is negative. Recalling that $z = \frac{p_h}{P_H}$, we can deduce from this result that the effect of devaluation on P_H is positive and higher than the positive effect on p_h . In other words, despite the decrease of foreign price, a raise in domestic price combined with a devaluation of domestic currency generates inflation in the home country.

In the home country, as the relative price is affected by the devaluation, the equilibrium of dual labor market evolves. More precisely, the structure of wages changes and a reallocation of workers between the two sectors occurs. Indeed, the inflation, generated by devaluation, leads to a contraction of the domestic production at the equilibrium. To adjust the production of good h to the lower level of demand, employers have to fire formal workers, who enter in the secondary sector. This flow of employees, increasing informal labor supply, exercises a downward pressure on informal wage, w_2 (see expression (14)). Due to efficiency considerations, a lower real wage in the informal sector allows firms of formal sector to reduce wage offered, without being exposed to shirking workers (expressions (6) and (7)).

So, the devaluation is clearly counterproductive for the domestic economy since (i) it generates inflation, (ii) it reduces the level of activity and (iii) it damages the situation of employment (less formal workers and lower real wages in both sectors).

Furthermore, if we retain $1 - I = \frac{E\bar{M}_H}{\bar{M}_H + E\bar{M}_F}$, as a proxy of the relative

size of the domestic economy, it is straightforward to demonstrate, from expressions (55) that absolute value of the elasticity of the relative price z with respect to the nominal exchange rate E is more important, the higher $1 - I$ is. In other words, the reduction of production and the deterioration of dual labor market are more pronounced, the bigger the domestic economy is.

In the foreign economy, the devaluation of the domestic currency leads to a decrease in the price of good f , notably through a contraction of the domestic demand for the good f . Thus, the real cost of labor becomes higher and unemployment increases.

As a consequence, the level of production decreases. So, in the foreign economy, as in the domestic country, a devaluation of the domestic currency is not relevant to reduce unemployment. Moreover, it is clear from expression (54) that the bigger the domestic economy is, the more amplified these negative effects are.

4 Conclusion

In this paper, we have considered a world of two countries in a fixed exchange rate system where countries differ through their labor markets. We have assumed a dual labor market in home country and the presence of a nominal wage rigidity in the foreign country. In this case, the equilibrium situation is suboptimal : in home country, setting a efficiency wage in dual labor market leads to a misallocation of workers between formal and informal sectors ; in foreign country, unemployment emerges through rigidity of real cost of labor. We analyze then the effects of domestic currency devaluation. This last one can be view as an analyze of the exit of EU zone consequence of the Greece, or others south countries of Europe. We show that currency home devaluation has important negative effects : a fall in production in each country, and a deterioration of labor markets.

Of course, this paper can be extended in several directions. Notably, it could be interesting to introduce in this framework public deficits to take into account more precisely the case of sovereign debt crisis in the analyze of the leaving of monetary union. Furthermore, a dynamic model could also be considered in order to better understanding the dynamic transition of the economies after a devaluation of the money.

5 Appendix

A Level of production in home country in formal and informal sectors

Introducing w_1 and z , respectively given by equations (6) and (14), in expressions (8) and (11), we obtain :

$$Y_{h1}(w_2) = \Lambda \left(\frac{\alpha}{K} \right)^{\frac{\alpha}{1-\alpha}} \frac{w_2^{\frac{\beta}{1-\alpha}}}{\left(1 + \Phi w_2^{\frac{\beta}{1-\alpha}} \right)^\alpha} \text{ with } \frac{dY_{h1}}{dw_2} > 0 \quad (42)$$

$$Y_{h2}(w_2) = \left(\frac{\alpha}{K} \right)^{\frac{\alpha}{1-\alpha}} \frac{1}{\left(1 + \Phi w_2^{\frac{\beta}{1-\alpha}} \right)^\alpha} \text{ with } \frac{dY_{h2}}{dw_2} < 0 \quad (43)$$

where $K = \alpha \bar{L}_H^{\alpha-1}$, $\Phi = \sigma^{\frac{\beta-1}{1-\alpha}} \delta^{\frac{\beta}{1-\alpha}}$ and $\Lambda = \sigma^{\frac{\beta-\alpha}{1-\alpha}} \delta^{\frac{\beta}{1-\alpha}}$.

B Elasticities of prices with respect to nominal exchange rate

To extract the elasticities of prices with respect to nominal exchange rate, we first express the good market equilibrium conditions (36) and (37) in logarithmic terms. We then differentiate these two expressions with respect to prices of goods and nominal exchange rate. More formally, we will express $\frac{dY_h}{Y_h}$, $\frac{dY_f}{Y_f}$, $\frac{dD_h}{D_h}$ and $\frac{dD_f}{D_f}$ as a function of $\frac{dp_h}{p_h}$, $\frac{dp_f}{p_f}$ and $\frac{dE}{E}$.

Concerning supply of good h given by expression (18), we obtain :

$$\frac{dY_h}{Y_h} = \Psi_1 \frac{dw_2}{w_2} \text{ with } \Psi_1 = \frac{\beta}{1-\alpha} w_2^{\frac{\beta}{1-\alpha}} \left(\frac{\Lambda}{1 + \Lambda w_2^{\frac{\beta}{1-\alpha}}} - \frac{\alpha \Phi}{1 + \Phi w_2^{\frac{\beta}{1-\alpha}}} \right) > 0 \quad (44)$$

We then express $\frac{dY_h}{Y_h}$ with respect to $\frac{dz}{z}$. Thanks to equation (14), we have:

$$\frac{dz}{z} = \Psi_2 \frac{dw_2}{w_2} \text{ with } \Psi_2 = \frac{1 + \Phi(1-\beta)w_2}{1 + \Phi w_2^{\frac{\beta}{1-\alpha}}} > 0 \quad (45)$$

Combining expressions (44) and (55), it is straightforward that:

$$\frac{dY_h}{Y_h} = \Psi \frac{dz}{z} \text{ with } \Psi = \frac{\Psi_1}{\Psi_2} > 0 \quad (46)$$

We finally express $\frac{dY_h}{Y_h}$ with respect to $\frac{dp_h}{p_h}$, $\frac{dp_f}{p_f}$ and $\frac{dE}{E}$. Recalling that $z = p_h/P_H$ and using expression (25), we have:

$$\frac{dP_H}{P_H} = t \frac{dp_h}{p_h} + (1-t) \left(\frac{dp_f}{p_f} + \frac{dE}{E} \right) \text{ where } t = \frac{p_h^{\frac{\rho}{\rho-1}}}{p_h^{\frac{\rho}{\rho-1}} + (Ep_f)^{\frac{\rho}{\rho-1}}} \quad (47)$$

with $0 < t < 1$. Thus:

$$\frac{dz}{z} = (1-t) \left(\frac{dp_h}{p_h} - \frac{dp_f}{p_f} - \frac{dE}{E} \right) \quad (48)$$

Introducing expression (48) into (46), we obtain:

$$\frac{dY_h}{Y_h} = \Psi(1-t) \left(\frac{dp_h}{p_h} - \frac{dp_f}{p_f} - \frac{dE}{E} \right) \quad (49)$$

Similarly, we determine the good f supply elasticity with respect to p_f . Using expression (21), we obtain:

$$\frac{dY_f}{Y_f} = \frac{\alpha}{1-\alpha} \frac{dp_f}{p_f} \quad (50)$$

Concerning, the demand side, we derive the two elasticities of goods h and f from expressions (39) and (40)

$$\frac{dD_h}{D_h} = \frac{1-\rho t}{\rho-1} \frac{dp_h}{p_h} - \frac{\rho(1-t)}{\rho-1} \frac{dp_f}{p_f} + \left(I - \frac{\rho(1-t)}{\rho-1} \right) \frac{dE}{E} \quad (51)$$

$$\frac{dD_f}{D_f} = -\frac{\rho t}{\rho-1} \frac{dp_h}{p_h} + \frac{1-\rho(1-t)}{\rho-1} \frac{dp_f}{p_f} + \left(I + \frac{1-\rho(1-t)}{\rho-1} \right) \frac{dE}{E} \quad (52)$$

with $I = \frac{EM_F}{\bar{M}_H + EM_F}$ and $0 < I < 1$.

Finally, equilibrium total differentiation is given by equalization of expressions (49) and (51), and expressions (50) and (52) :

$$\left\{ \begin{array}{l} \frac{1-\rho t}{\rho-1} \frac{dp_h}{p_h} - \frac{\rho(1-t)}{\rho-1} \frac{dp_f}{p_f} + \left(I - \frac{\rho(1-t)}{\rho-1} \right) \frac{dE}{E} = \Psi(1-t) \left(\frac{dp_h}{p_h} - \frac{dp_f}{p_f} - \frac{dE}{E} \right) \\ -\frac{\rho t}{\rho-1} \frac{dp_h}{p_h} + \frac{1-\rho(1-t)}{\rho-1} \frac{dp_f}{p_f} + \left(I + \frac{1-\rho(1-t)}{\rho-1} \right) \frac{dE}{E} = \frac{\alpha}{1-\alpha} \frac{dp_f}{p_f} \end{array} \right.$$

In matricial form, we obtain :

$$\begin{pmatrix} \frac{1-\rho t}{\rho-1} - \Psi(1-t) & -\frac{\rho(1-t)}{\rho-1} + \Psi(1-t) \\ -\frac{\rho t}{\rho-1} & \frac{1-\rho(1-t)}{\rho-1} - \frac{\alpha}{1-\alpha} \end{pmatrix} \begin{pmatrix} \frac{dp_h}{p_h} \\ \frac{dp_f}{p_f} \end{pmatrix} = \begin{pmatrix} -I + \frac{\rho(1-t)}{\rho-1} - \Psi(1-t) \\ -I - \frac{1-\rho(1-t)}{\rho-1} \end{pmatrix} \frac{dE}{E}$$

where the determinant of the (2, 2) matrix is :

$$\Delta = \frac{\Psi(1-t)(\rho-1) + \alpha\rho t - 1}{(\rho-1)(1-\alpha)} > 0$$

We can now extract the elasticities of prices respect to nominal exchange rate from matricial form. Thus, we obtain:

$$\xi_{p_h/E} = \frac{\frac{dp_h}{p_h}}{\frac{dE}{E}} = \frac{\alpha(1-t)[\Psi(\rho-1) - \rho] + I[(\alpha\rho-1) + \Psi(1-t)(\rho-1)(1-\alpha)]}{\Psi(1-t)(\rho-1) + \alpha\rho t - 1} > 0 \quad (53)$$

$$\xi_{p_f/E} = \frac{\frac{dp_f}{p_f}}{\frac{dE}{E}} = \frac{(1-I)(1-\alpha)[1 - \Psi(1-t)(\rho-1)]}{\Psi(1-t)(\rho-1) + \alpha\rho t - 1} < 0 \quad (54)$$

The elasticity of the relative price z with respect to nominal exchange rate can be computed by introducing expressions (53) and (54) in equation (48) :

$$\xi_{z/E} = \frac{\frac{dz}{z}}{\frac{dE}{E}} = \frac{\alpha(1-t)(\rho-1)(I-1)}{\Psi(1-t)(\rho-1) + \alpha\rho t - 1} < 0 \quad (55)$$

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