The substitutability of immigrants and native workers in France: use of a production function

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Abstract:

This article examines the relationships of substitutability and complementarity between native workers and immigrants in France, depending on skill level, using a translog production function. We analyze the impact of immigrant workers on employment and wages of native workers by taking into account the interrelations between all factors. In general, there is a relationship of complementarity between immigrant workers and native workers, although high and intermediate-skilled migrant workers are respectively substitutable for intermediate and low-skilled native workers.

Résumé :

Cet article examine les relations de substituabilité et de complémentarité entre les travailleurs natifs et immigrés en France, en fonction du niveau de qualification, à partir d'une fonction de production translogarithmique. On analyse l'impact des travailleurs immigrants sur l'emploi et les salaires des travailleurs natifs en prenant en considération les interrelations entre l'ensemble des facteurs. Une relation de complémentarité s'impose entre les travailleurs immigrés et les travailleurs natifs, même si les travailleurs immigrés hautement et moyennement qualifiés sont respectivement substituables aux travailleurs natifs moyennement et faiblement qualifiés.

Keywords : Immigration, Substitutability, Production Function, Employment, Wages

Mots clés : Immigration, Substituabilité, Fonction de production, Emploi, Salaire

JEL Classification : J61, C39, D24

Introduction

The theme of international migrations is a recurrent subject in the current affairs of most developed countries (Human Development Report (2009)). During a period of economic slowdown, fears of a negative effect from immigration on host countries resurface and migratory policies harden in developed countries, and in European countries in particular. Justification of restrictive policies in terms of immigration is generally reliant on the idea of negative impact from immigration for the economy of the destination country and its native workers.

Several theoretical or empirical economic studies have attempted to analyse and evaluate the effects of international migrations on the labor market of host countries. The predictions of theoretical studies generally rely on hypotheses underlying economic models and they are not truly adapted to steering migratory or structural policies (Dustmann, Fabbri and Preston (2005)). The conclusions of the theoretical models also depend on the direct or indirect effects associated with immigration, such as immigrant workers' skill levels, institutions and flexibility of the labor market, the supply and demand of employment or goods and services… The numerous empirical studies, conducted mainly in Anglo-Saxon countries, conclude that the effects of migrations on wages or on unemployment of natives are very limited (Okkerse (2008); Longhi et al. (2010)). However, the conclusions can diverge according to the context of analysis (geographical, temporal and methodological). Borjas (1994) emphasises that "the most important lesson is that the economic impact of immigration varies by time and place and can be beneficial or harmful". Moreover, an inflow of immigrants causes labor force supply shock of a given skill (Jean and Jiménez (2011)) and can be more or less beneficial to the group of native workers considered. If the aptitudes of the migrants are complementary to those of domestic workers, then the two groups should gain benefit. If these competences are similar, then the competition will be increased and native workers would probably be subjected to negative repercussions. The measure of the degree of substitutability between native and immigrant workers can be different depending on the groups, but also on the county of interest (Card (2009)). A study of complementarity-substitutability based on a production function enables the determination of the values of substitution (without conducting *arbitrage*, contrary to the approach in terms of aggregate factor share (Borjas (2003, 2008); Ottaviano and Peri (2005, 2008)) in a national context (Borjas, Freeman et Katz (1997)) to take into account the institutional characteristics (Angrist et Kugler (2003)).

In empirical literature, few studies relying on the notion of complementarity-substitutability have been conducted in Europe, and in particular in France. Constant (2003) shows that despite the importance of the migratory question for the French economy, very few economic studies have taken direct interest in the French case: Garson et al. (1987), Hunt (1992), Gross (1999), Jayet, Ragot and Rajaonarison (2001) and Ortega and Verdugo (2009). These studies show that immigration would have very limited impact on natives' income and the employment. Most empirical research employing the production theory approach is conducted almost essentially in the United States. Borjas (1983, 1986) suggests that immigrants and male native workers are complementary. Greenwood, Hunt and Kohli (1996), Borjas (1987), Grossman (1982) and Kohli (1999) find low negative effects of immigration on natives' wages. It would seem that immigration may have direct impact on immigrants already settled in the country, through a phenomenon of substitution. In Canada, Akbari and DeVoretz (1992) conclude that immigrants are neither substitutable nor complementary to natives. Islam (2009) shows that the immigration flows to Canada do not have significant negative effects on Canadian natives' wages and that recent immigrants are complementary to native workers. Few studies have taken an interest in Europe: one for Germany (Bauer, 1997) and one for Switzerland (Sheldon, 2000). To our knowledge, no study using this research methodology has been conducted in France.

This article proposes an assessment of the potential displacement of native workers and possible variation of wages, by estimating a multi-factor translog production function, taking into account the interrelations between all the factors. Hence, the originality of this article is to propose a study utilizing a methodology never employed in France, but also to accept a dual differentiation of the production factors, depending on the skill levels, for natives, and also for immigrant workers. An analysis of the interactions and of the degree of complementarity-substitutability between immigrant workers and the native labor force is conducted, taking into account the heterogeneity of the employment factor and a disaggregation of wages and employment, with rigidities on the labor market.

This paper will first present the model and the method of assessment accepted (Section I), before examining the construction of the data base (Section II) and the econometric strategy chosen (Section III). The results of the calculations of the elasticities of complementarity and of the model with rigidity of employment and wages will then be stated and discussed (Section IV), before concluding. (Section V).

1. Model and methodology

The use of a production function provides a methodological and theoretical framework of analysis to establish if immigrants and natives are substitutes or complements in production, and allows the interrelations between factors to be taken into account, maximising production for given quantities of inputs, subject to technological constraints determined by the possibilities of production. We consider a production function rather than a cost function in order to detect the underlying technologies. Employing a cross-analysis, it is reasonable to suppose that quantities, rather than prices, are fixed. Furthermore, application of the Shephard duality theorem (1953) shows that the production function contains the same information as the cost function[1]. We suppose that economic activity in France can be characterised by a translog production function[2] defined by Christensen, Jorgenson, Lau (1971), which allows the elasticities of substitution between factors to be established, following the example of Akbari and DeVoretz (1992), Grossman (1982), Grant and Hamermesh (1981) and Sheldon (2000). The Translog function presents a relatively flexible form which sets no restriction a priori on the structure of the production[3].

Contrary to previous studies, a finer distinction between the different categories of labor is implemented using the notion of skill, to take into account the heterogeneity of the employment factor (Barro and Sala-i-Martin, 1995). We assume that these factors are weakly separable from the seventh factor: capital. The weak separability signifies that the marginal rate of substitution between the employment factors will be independent from the quantity of capital used in production. Grossman (1982), Borjas (1983) and Akbari and DeVoretz (1992) concluded that capital and employment are separable for this type of production relationship.

In order to establish a relationship between the production factors, we will estimate a translog production function with seven factors (1), linearly homogeneous, of the following form:

$$\ln Y = \ln \alpha_0 + \sum_i \alpha_i \ln x_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln x_i \ln x_j$$

With $i = tq, tqi, q, qi, nq, nqi, k$ and $x_i = TQ, TQI, Q, QI, NQ, NQI, K$ (1)

1 The choice to use a production function, rather than a cost function, is discussed in the article of Grant and Hamermesh (1979).
2 There is a debate having no conclusion as to the most appropriate form of production function. Borjas (1987) shows that “there is no reason a priori to prefer one function to another, since both (in comparison with a Leontief fuction) are second order approximations to any arbitrary production function” .
3 The concept of flexible linear form and the revelation of their second order approximation property were defined by Diewert (1971).

According to Christensen, Jorgenson, Lau (1971), the Translog production function can be considered as a second order Taylor approximation. Hence, the twice differentiable production function can be written in the following way:

$$Y = f(TQ, TQI, Q, QI, NQ, NQI, K) \tag{2}$$

The weak separability hypothesis is a necessary and sufficient condition for the production functions to take the following forms:

$$Y = h\left(f(TQ, TQI, Q, QI, NQ, NQI); K \right) \tag{3}$$

where *Y* represents the value added in volume, *NAT, TQ, TQI, Q, QI, NQ, NQI* represent respectively the number of nationals employed, the number of very skilled natives employed, the number of very skilled immigrants employed, the number of intermediate-skilled natives employed, the number of intermediate-skilled immigrants employed, the number of low-skilled natives employed, the number of low-skilled immigrants employed and *K* represents capital.

Accepting conditions of perfect competition on the markets factor, companies' cost minimisation behaviour enables the calculation of the share of each factor in the value added in volume:

$$\frac{\partial \ln Y}{\partial \ln x_i} = \frac{\partial Y}{\partial x_i} . \frac{Y}{x_i} = \frac{w_i x_i}{Y} \tag{4}$$

Thus $s_i = \frac{w_i x_i}{Y}$, where w_i represents the factor price *i*, and s_i the share of each factor in the value added. Consequently, the application of (4) in (1) gives all the shares of the equations:

$$s_i = \gamma_i + \sum_j \gamma_{ij} \ln X_j \tag{5}$$

Economic theory imposes additional restrictions on the constants (α_i) and the technological parameters (γ_{ij}) of the Translog production model. The symmetry of the crossed factors, referring to Young's theorem, imposes that $\gamma_{ij} = \gamma_{ji} : \forall_{i,j}$. Knowing that the sum of the factor shares must be equal to one, this implies that the production function is linear and homogeneous:

$$\sum_i \alpha_i = 1 \text{ and } \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0 \tag{6}$$

Studies conducted on a national level consider the quantity of production inputs to be exogenous and prices endogenous. In order to assess the effect of a variation of the quantities on the prices, it is then necessary to use Hicks elasticity of complementarity to evaluate the substitutability between factors[4]. The first measurement used in our study is Hicks partial complementarity C_{ij} which is defined in the following way (Hamermesh (1986)):

$$C_{ij} = \frac{YY_{ij}}{Y_i Y_j} \text{ where } Y_i = \frac{\partial Y}{\partial x_i} \text{ and } Y_{ij} = \frac{\partial^2 Y}{\partial x_i \partial x_j} \tag{7}$$

The application of (7) in the Translog production fucnction leads to:

$$\begin{cases} C_{ij} = \dfrac{\gamma_{ij} + s_i s_j}{s_i s_j} : i \neq j \\ C_{ii} = \dfrac{\gamma_{ii} + s_i{}^2 - s_i}{s_i{}^2} : i = j \end{cases} \tag{8}$$

The factors *i* and *j* are substitutable if C_{ij} is negative, and complementary if C_{ij} is positive. The second measurement used is the partial elasticity ε_{ij} of the price factor w_i in comparison with the change in the quantity x_j, which is defined in the following way (referring to Sato and Koizumi (1973)):

$$\begin{cases} \dfrac{d \ln w_i}{d \ln x_j} = s_j c_{ij} : i \neq j \\ \dfrac{d \ln w_i}{d \ln x_i} = s_i c_{ii} : i = j \end{cases} \tag{9}$$

The elasticity of the factor price in comparison with quantities can also be written in the following way: $\varepsilon_{ij} = s_j C_{ij}$. If ε_{ij} is positive, the production factors x_i and x_j are classed as q-complements. If ε_{ij} is negative the production factors are said to be q-substitutes. It is to be noted that the ε_{ii} (like the C_{ii}) must be negative to be consistent with economic theory, owing to the hypothesis of quasi-concavity of the production function.

4 When the production function has more than two factors, several elasticities of substitution can be defined, in particular Hicks' and Allen's. Numerous debates (Blackorby and Russel (1989), Frondel (2003)) on this question conclude that the elasticities are calculated to respond to a particular aim and that it is not possible to define a "universal" elasticity.

2. Data

The data focusing on the quantity of workers in France in 2008, depending on their nationality and skill level, come from a request to Eurostat and to the LFS (Labour Force Survey) team. The available data base provides the quantities of natives and migrants employed in thousands, depending on the skill levels and the sector of activity. The three skill levels are determined using the ISCED (International Schedule of Education) international nomenclature[5]. Assuming that low-trained workers (NQ and NQI) belong to the group of low-skilled persons with a level of education falling between levels 0 and 2 in the ISCED nomenclature. Intermediate-skilled workers (Q and QI) present a level of education between 3 and 4. Finally, high-skilled workers (TQ and TQI) belong to the group of individuals presenting levels of education between 5 and 6 (Level 5: First stage of higher education; Level 6: Second stage of higher education).

Employed workers are distributed by sector of activity depending on the statistical nomenclature of economic activities in the European Community, Nace Rev. 2 88 divisions[6]. A cross-check had to be made to harmonise the data with the INSEE’s (French national institute of economic and statistical information) 36 positions (NES 36) nomenclature of economic activities. The grouping of the statistical data depending on different nomenclatures was made utilizing Eurostat[7] and INSEE’s[8] correspondence tables. This correspondence enables the number of native and immigrant workers to be linked with the fixed capital, keeping the divison by sector of economic activity. The capital volume corresponds to the amount of gross fixed capital at the end of the year 2008, in volume, to the chained prices of the previous year (in billions of euros 2000). Employing INSEE data, it is possible to determine the share of fixed capital in the value added by sector. The explained variable of the Translog production function is assimilated to the gross value added of France in 2008. The availability of the gross value added by sector enables a correspondence to be drawn with the factor quantities, and consequently, to calculate the relative share of each factor to the country’s value added, for the 36 sectors of activity considered.

5 International Classification Type of Education (C I T E) 1997, Organisation of the United Nations for Education, Science and Culture

6 The NACE Rév. 2 is the new revised version of the NACE Rév. 1 and is the result of a major revision of the international integrated system of economic nomenclatures which took place between 2000 and 2007. For more information: http://circa.europa.eu/irc/dsis/nacecpacon/info/data/en/

7 Source: http://epp.eurostat.ec.europa.eu/portal/page/portal/nace_rev2/correspondence_tables

8 Source: http://www.insee.fr/fr/methodes/default.asp?page=nomenclatures/

In order to take into account the specific characteristics of the immigrants, the divergences according to the level of education and the sector disparities in terms of earnings, we refined the data base[9]. Concerning wages, we retain the average annual gross salary by full-time[10] equivalent. To take into consideration the sector disparities in terms of earnings, we took account of the differences in wages according to the level of education, in the different sectors of activity[11]. Referring to the OECD's report on international migrations[12], wages of native and immigrant workers were weighted according to skill level and nationality of the worker, determining a rating by the calculation of the median[13].

In the end, the data base for France in 2008 is composed of the number of native and immigrant workers, depending on three skill levels, broken down by sector of economic activity, and of the relative share of each factor in the sector value added.

3. Econometric strategy

3.1. Estimation of the elasticities

The parameters of the production function will be estimated using a system of equations composed of the production function (1), and the relative shares of the different production factors in the value added (5). This method of joint estimation is the most prevalent in economic literature. The number of degrees of freedom increases owing to the use of more data to estimate the same number of coefficients. However, by respecting parametric restrictions, the estimation of the system composed of the equations (1) and (5) is not possible as it stands, since the matrix of variance-covariance is singular. To deal with this econometric problem, we retain the solution proposed by Christensen and Greene (1976), which consists in excluding one of the factor share equations.

9 Refer to Annex 1
10 Source: INSEE; full-time occupations: wages offered by sector of activity
11 Source: Insee and DADS / Data 2008; OECD, average and median earnings by level of education
12 OECD, 2008, Perspectives of international migrations, SOPEMI, p. 89
13 The calculation is detailed in Annex 1.

We deduce the following equations in the system (10):

$$
\begin{aligned}
s_{tq} &= \alpha_{tq} + \gamma_{tq,tq}\ln(TQ/K) + \gamma_{tq,tqi}\ln(TQI/K) + \gamma_{tq,q}\ln(Q/K) + \gamma_{tq,qi}\ln(QI/K) + \gamma_{tq,nq}\ln(NQ/K) + \gamma_{tq,nqi}\ln(NQI/K)\\
s_{tqi} &= \alpha_{tqi} + \gamma_{tqi,tq}\ln(TQ/K) + \gamma_{tqi,tqi}\ln(TQI/\ln K) + \gamma_{tqi,q}\ln(Q/K) + \gamma_{tqi,qi}\ln(QI/K) + \gamma_{tqi,nq}\ln(NQ/K) + \gamma_{tqi,nqi}\ln(NQI/K)\\
s_{q} &= \alpha_{q} + \gamma_{q,tq}\ln(TQ/K) + \gamma_{q,tqi}\ln(TQI/K) + \gamma_{q,q}\ln(Q/K) + \gamma_{q,qi}\ln(QI/K) + \gamma_{q,nq}\ln(NQ/K) + \gamma_{q,nqi}\ln(NQI/K)\\
s_{qi} &= \alpha_{qi} + \gamma_{qi,tq}\ln TQ/K) + \gamma_{qi,tqi}\ln(TQI/K) + \gamma_{qi,q}\ln(Q/K) + \gamma_{qi,qi}\ln(QI/K) + \gamma_{qi,nq}\ln(NQ/K) + \gamma_{qi,nqi}\ln(NQI/K)\\
s_{nq} &= \alpha_{nq} + \gamma_{nq,tq}\ln(TQ/K) + \gamma_{nq,tqi}\ln(TQI/K) + \gamma_{nq,q}\ln(Q/K) + \gamma_{nq,qi}\ln(QI/K) + \gamma_{nq,nq}\ln(NQ/K) + \gamma_{nq,nqi}\ln(NQI/K)\\
s_{nqi} &= \alpha_{nqi} + \gamma_{nqi,tq}\ln(TQ/K) + \gamma_{nqi,tqi}\ln(TQI/K) + \gamma_{nqi,q}\ln(Q/K) + \gamma_{nqi,qi}\ln(QI/K) + \gamma_{nqi,nq}\ln(NQ/K) + \gamma_{nqi,nqi}\ln(NQI/K)
\end{aligned}
$$

The integration of capital in the other share equations enables the property of homogeneity of degree one of the production function to be satisfied. Hence, the parametric constraints $\sum_i \alpha_i = 1$ and $\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0$ are imposed thanks to the omission of one equation from the system. We assume that the production function satisfies the conditions of symmetry with a Wald test and we verify the concavity of the Translog function. Once the parameters of the production function have been estimated, we verify that the matrix of the coefficients γ_{ij} is semi-defined negative and the factor shares are non negative (property of monotony). The hypothesis of quasi-concavity of the production function will also be verified by observing the negativity of the values of ε_{ii} which implies convex isoquants.

In order to estimate the coefficients, respecting the parametric constraints, the SUR (*Seemingly Unrelated Regressions*) method introduced by Zellner (1962) will be used. This method applies to a particular case of simultaneous equation systems, within the framework of apparently unlinked equation models. The choice of the SUR method, used in several studies employing production functions (Sheldon (2000), Islam (2009), …) is preferred to other methods of econometric estimation in particular owing to the limited number of observations by factors and the possible correlation and the existence of a simultaneous equation bias.

3.2. Introduction of rigidities: short-term elasticity of employment and long-term elasticity of wages

The admittance of immigrant workers leads to an adjustment of the labor market through a modification of the combination of the employment level and price level. The modelling requires certain hypotheses to be put forward. Firstly we assume that additional immigration presents similar characteristics. In the long run, it can be considered that flexibility of real wages enables the adjustment of the labor market, but in the short run, it can be thought that wages are rather inflexible. The adjustment would operate by variations in the level of employment. We assume that the production price does not vary and that the relative variations of real wages are identical to those of nominal wages. In our demonstration, we introduce extreme hypotheses of total rigidity of real wages. Assuming a 1% increase in immigrants, we quantify the impact of immigration on low, intermediate and high-skilled native workers' employment and wages.

In order to estimate the seven variable model (*TQ,Q,NQ,TQI,QI,NQI,K*), we pose constraints relative to the rigidity of wages of the unskilled and skilled, which yields $dK=0=dW_{NQ}=dW_{NQI}=dW_{Q}=dW_{QI}$. Hence the adjustment will be exerted through a modification of the level of employment. Moreover, we also assume that high-qualified immigrant or native workers have identical wage variations: $dW_{TQ}=dW_{TQI}$. Native workers have sufficient negotiating power to protect themselves from a decrease in real wages. Following Grossman's (1982) reasoning, with seven variable complex model (instead of four):

$X=f(TQ,Q,NQ,TQI,QI,NQI,K)$, we know that W_i represents the real wage of factor *i*, with *i* = $(TQ, Q, NQ, TQI, QI, NQI, K)$. Assuming that firms are represented by production functions (exhibit constant returns to scale), the conditions for profit maximization ensure that:

$$W_i=\frac{\partial f}{\partial x_i}=f_i \tag{11}$$

This yields $dW_i=f_{i,TQ}dTQ+f_{i,Q}dQ+f_{i,NQ}dNQ+f_{i,TQI}dTQI+f_{i,QI}dQI+f_{i,NQI}dNQI+f_{i,K}dK$

$$\text{with } i = (TQ, Q, NQ, TQI, QI, NQI, K)\text{, where } f_{i,j}=\frac{\partial f_i}{\partial x_j} \tag{12}$$

Returning to equation (12) avec $i=(Q, NQ, QI, NQI)$ and knowing that $dW_{TQ}=dW_{TQI}$, following a mathematical development summarized in *annex 2*, the equation (17) enables evaluation of the impact of immigration on wages owing to the flexibility of high-skilled workers' wages and equation (16) allows the impact of the admittance of immigrants in the presence of rigidity of wages to be seen, which then imposes an adjustment of the level of employment of the factor of interest. To evaluate the short-run elasticity of employment, calculations[14] were carried out for each factor using equation (16).

14 We had to resort to the Matlab digital calculation software.

Following this, equation (17) enables the evaluation of the impact of immigration on the wages of high and intermediate-skilled native workers' wages. In order to estimate $\frac{d\ln W_i}{d\ln x_j}$, we simply replace each $\frac{d\ln x_i}{d\ln x_j}$ of equation (17) by the result obtained by the calculations from equation (16).

4. Results and discussions

Table 1 presents the estimations of the technological parameters of the model, of which the production function equation presents an R^2 of 0.57 and a Durbin-Watson statistic of 1.85. Observing the elasticities of complementarity, on the whole a relationship of complementarity is imposed between immigrant and native workers. However, the intermediate-skilled immigrant workers are substitutable for low-skilled natives. As for low-skilled immigrant workers, they are complementary to intermediate and low-skilled natives, but substitutable with skilled immigrant workers. Moreover, the own elasticities are negative and are consistent with economic theory.

Table 2 presents the partial elasticities $\varepsilon_{i,j}$ and own elasticities $\varepsilon_{i,i}$. Firstly, the negativity of the own elasticities confirm the hypothesis of quasi-concavity of the production function. Next, we note that the impact of immigration on natives' wages is, generally speaking, very steady. By detailing some results, we notice that the increase of 1% in the number of high-skilled immigrants increases the very skilled natives' wages by 0.03%. A rise in the number of skilled immigrants generates a fall in low-skilled native workers' earnings in the region of 0.69%. Finally, the rise of 1% in the number of low-skilled immigrants lead to a wage drop of 0.12% for intermediate-skilled immigrant workers and wage rises of 0.05% for low and intermediate-skilled natives.

For the model introducing rigidities with short-run elasticity of employment and long-run elasticity of wages, with rigidity of wages, the adjustment operates by the employment level of the factor concerned. The results of tables 3 and 4 come from calculations of equations (17) and (16) which enable measurement of the incidence of the admittance of immigrants on high-qualified native workers' wages (*Table 4*) and the impact of the admittance of immigrants with rigidity of wages, which imposes an adjustment of the employment level of the factor concerned (*Table 3*).

Hence, the impact of immigration on wages and employment is very limited. Immigrant and native workers are generally complementary inputs on the labor market. So it would seem that the characteristics of immigrants are different to those of the national labor force and/or that immigrants hold jobs that native workers are not ready to accept. In general, the immigrant labor force is more flexible and more mobile, especially when it presents an intermediate skill level, which reinforces its adaptability. It can be considered as a "variable of adjustment", in the sense that immigrant workers tend to respond better than natives to changes imposed by the economic situation, in particular in terms of acceptance of insecure contracts, holding occupations spurned by natives and sector or geographical mobility. Moreover, the increase of the overall offer of this type of competences enables companies to hire workers with a lower level of wage expectations.

However, immigrant workers, with a different skill level to natives, are substitutable for native workers in the light of the elasticities of complementarity $\gamma_{q,tqi}$ and $\gamma_{nq,qi}$. This "inter-skill" substitutability can be explained by exogenous factors.

Employers may underestimate the skill level of immigrants on the French labor market owing to absence of recognition of their diplomas, language barriers or a possible inability to adapt rapidly to the host country's labor market. High-skilled immigrants could be ready to accept low-paid jobs, despite a high skill level, on a labor market presenting a relatively high unemployment rate, limited capacity of absorption and frequent occurrence of job insecurity. Hence, it could be thought that this type of worker is confronted with skills mismatch unemployment, in the sense that workers' skills are not adapted to recruiters work demand. The worker who does not find a job matching his or her skill level, could accept a less attractive and less well-paid job. Furthermore, the rarity of available jobs enables employers to be more selective, hiring labour which holds a higher qualification than the one necessary for the occupation on offer. Hence, the most skilled workers are chosen first and foremost, even for jobs which do not require a high level of skill. Consequently, the mobility of workers, and especially of immigrant workers, should grow across different skill segments. Increased competition between job seekers may generate a spate of declassification, during which each skill level would be in competition with the level below it. Immigrant workers could be confronted with a "downgrading" effect, since the employed individual holds a level of education a priori higher than the one required for the occupation filled.

Table 1. Estimation of the coefficients γ_{ij}, of the Hicks elasticities of complementarity $c_{i,j}$

$\gamma_{tq,tqi}$	**0.0048** (-1.702)*	$c_{tq,tqi}$	**2.374** (-1.702)*
$\gamma_{tq,qi}$	**0.0112** (-1.953)*	$c_{tq,qi}$	**6.094** (-1.953)*
$\gamma_{tq,nqi}$	**0.0078** (-2.303)**	$c_{tq,nqi}$	**4.868** (-2.303)**
$\gamma_{q,tqi}$	**0.0054** (-1.022)	$c_{q,tqi}$	**-1.097** (-1.022)
$\gamma_{q,qi}$	**0.0052** (0.764)	$c_{q,qi}$	**4.215** (0.764)
$\gamma_{q,nqi}$	**0.0088** (1.992)**	$c_{q,nqi}$	**6.940** (1.992)**
$\gamma_{nq,tqi}$	**0.0022** (1.046)	$c_{nq,tqi}$	**3.472** (1.046)
$\gamma_{nq,qi}$	**-0.0062** (-2.175)**	$c_{nq,qi}$	**-9.828** (-2.175)**
$\gamma_{nq,nqi}$	**0.0034** (1.698)*	$c_{nq,nqi}$	**7.573** (1.698)*
$\gamma_{q,nq}$	**0.0933** (4.630)***	$c_{q,nq}$	**7.553** (4.630)***
$\gamma_{tq,q}$	**0.0838** (3.677)***	$c_{tq,q}$	**2.524** (3.677)***
$\gamma_{qi,nqi}$	**-0.0010** (1.821)*	$c_{qi,nqi}$	**-16.417** (1.821)*
$\gamma_{tq,tq}$	**0.119** (4.098)***	$c_{tq,tq}$	**-1.064** (4.098)***
$\gamma_{tqi,tqi}$	**0.0056** (2.694)***	$c_{tqi,tqi}$	**-42.837** (2.694)***
$\gamma_{q,q}$	**-0.102** (-3.035)***	$c_{q,q}$	**-6.508** (-3.035)***
$\gamma_{qi,qi}$	**-0.0037** (1.848)*	$c_{qi,qi}$	**-65.09** (1.848)*
$\gamma_{nq,nq}$	**-0.015** (-0.983)	$c_{nq,nq}$	**-16.13** (-0.983)
$\gamma_{nqi,nqi}$	**0.0008** (1.04)	$c_{nqi,nqi}$	**-118.261** (1.04)

Table 2. The partial elasticities of the price factor compared to quantities $\varepsilon_{i,j}$

$\varepsilon_{tq,tqi}$	**0.030** (-1.702)*
$\varepsilon_{tq,qi}$	**0.078** (-1.953)*
$\varepsilon_{tq,nqi}$	**0.036** (-2.303)**
$\varepsilon_{q,tqi}$	**-0.22** (-1.022)
$\varepsilon_{q,qi}$	**0.034** (0.764)
$\varepsilon_{q,nqi}$	**0.051** (1.992)**
$\varepsilon_{nq,tqi}$	**0.24** (1.046)
$\varepsilon_{nq,qi}$	**-0.695** (-2.175)**
$\varepsilon_{nq,nqi}$	**0.056** (1.698)*
$\varepsilon_{q,nq}$	**0.534** (4.630)***
$\varepsilon_{tq,q}$	**0.507** (3.677)***
$\varepsilon_{qi,nqi}$	**-0.122** (1.821)*
$\gamma_{tq,tq}$	**0.119** (4.098)***
$\gamma_{tqi,tqi}$	**0.0056** (2.694)***
$\gamma_{q,q}$	**-0.102** (-3.035)***
$\gamma_{qi,qi}$	**-0.0037** (1.848)*
$\gamma_{nq,nq}$	**-0.015** (-0.983)
$\gamma_{nqi,nqi}$	**0.0008** (1.04)

*Notes: *,** and *** represent significance at the 10%, 5% and 1% levels. The t-statistics are represented between parentheses. All estimations are not represented owing to the lack of significance of certain coefficients.*

Table 3. Impact of immigration on the employment of TQ, Q and NQ

	Increase of 1% of employed immigrants		
Modification of employment in %	TQI	QI	NQI
Employment of high-skilled workers	0.0949	0.0246	1.5457
Employment of intermediate-skilled workers	-0.0784	0.0203	1.277
Employment of low-skilled workers	0.047	-0.012	0.7656

Table 4. Impact of immigration on the earnings of TQ

	Increase of 1% of employed immigrants		
Modification of wage in %	TQI	QI	NQI
Wage of high-skilled workers	0.290	0.075	4.727

It is true that the reports of the OECD (2006-2007) reveal a marked downgrading of immigrants compared to natives, even if this phenomenon becomes less marked with length of stay (Chiswick (1978), Dolton and Vignoles (2000), Dorn and Sousa-Poza (2005),…). In these conditions, the entry of new skilled immigrant workers restricts and defers the hiring of the unskilled workers already present on the labor market. It could also be thought that there is an effect of inter-skill substitution since the boundary and balance between the skill levels and the occupations on offer diminishes progressively owing in particular to the contraction of the labor market. Newcomers may also take some time to find work, which is not necessarily in keeping with their expectations, which reinforces 'wait unemployment' (Burda, 1988). Moreover, the complementary nature of high-skilled immigrant workers with very skilled natives could be explained by the fact that numerous immigrants are entrepreneurs and so hold relatively independent jobs.

Unskilled immigrant workers are complementary to all native workers, whatever their level of education. Unskilled immigrant workers bring about relatively large rises in employment and wages, perhaps owing to the need for management and supervision of this low-skilled labor force by intermediate or high-skilled native workers. The complementary nature of immigrant workers and low-skilled natives could be explained by the segmentation of the labor market (Piore, (1979); Greenwood and McDowell (1986); Gilles-Saint Paul, (2009)). It is true that low-skilled immigrant workers tend to fill trades or occupations which are often neglected by native workers, in particular in the sectors of catering, construction, maintenance… In line with this thinking, a subdivision of the market segment into two sectors on which low-skilled workers[15] develop could be observed. Moreover, following the example of several studies of complementarity-substitutability (Borjas (1987); Greenwood, Hunt and Kohli (1997)), it can be noticed that immigrant workers are substitutable for one other. So, the arrival of new immigrant workers would tend to be detrimental for skilled immigrant workers by increasing competition in work demand, which leads to a wage reduction for this group in the region of 0.12%.

On the whole, admittance of immigrant workers increases respectively the level of employment and the wages of high-skilled native workers (*+1.66%; +5.06%*), the level of employment of intermediate-skilled natives (*+1.21%)* and of low-skilled natives (*+0.8%*). We note, however, that the admittance of very skilled

15 Greenwood and McDowell (1986) mention the hypothesis of segmentation of the labor market. If the labor market or "the labor markets" are sufficiently segmented, immigration will not have direct impact on native workers.

immigrants causes a decrease in the employment of intermediate-skilled native workers of *0.078%* and the increase in the number of intermediate-skilled immigrant workers leads to a decrease in the employment of low-skilled natives (*-0.012%*), which could be translated by a downgrading or inter-skill substitution effect.

Conclusion

This paper studies the economic consequences of immigrant workers on the native labor force, employing stock data and taking into account the heterogeneity of the employment factor, in a framework of quasi-microeconomic analysis. This analysis of complementarity-substitutability, based on the estimation of a seven-factor Translog function with the Zellner iterative method, applied to the case of France, concludes that on the whole immigrant workers are complementary to native workers. This conclusion is verified by the results of Hicks elasticity of complementarity and the partial elasticities of the price factor compared to quantities, but also by the model integrating rigidities of wages and employment in order to estimate the impact of immigrant workers (differentiated by skill level) on the native labor force of the host country. In the light of the results, immigration has positive impact (low) on the earnings and employment of native workers.

Unlike other studies using the same research methodology, it is possible to interpret the results with a finer level of details. High and intermediate-skilled immigrant workers are respectively substitutable with intermediate and low-skilled native workers, which could be the expression of an underestimation of their competences on the French market, unadapted competences (OECD, 2007) or an inter-skill substitution effect. The results also seem to acknowledge the existence of segmentations on the labour “markets”, notably because of the complementarity between native workers and low-skilled immigrants. The immigration of low-skilled workers does not seem to have negative influence on French native workers. Generally, native and immigrant workers do not seem to be in competition on the French labor market and so immigrant workers could fill different occupations.

Even if the geographical context of the study has real importance, from the viewpoint of economic literature, these results seem to reinforce the conclusions that the impact of immigration on natives’ wages and employment is limited. However, our results are relatively divergent to other studies such as Grossman’s (1982), which shows that immigration causes a decrease of 0.2% in natives’ wages, Borjas (1986), with a decrease of 1% in wages, Greenwood et al. (1996) which shows that a 10% increase in the number of recent immigrants leads to a reduction of 0.96% of the wage rate of American natives and of 5.6% for immigrants. In the short term, when the number of immigrants rises by 1%, they conclude that there is a 0.2% fall in natives' employment and a 0.4% growth in past immigrants' employment. On the other hand, our conclusions match the results of the studies of Akbari and DeVoretz (1992) and of Islam (2009) in Canada and the works of Garson et al. (1987), Jayet, Ragot and Rajaonarison (2001) and Ortega and Verdugo (2009) conducted in France. It could be thought that despite the institutional differences with the North-American labor market (Mouhoud and Oudinet, 2010), immigration also has very limited impact on the French labor market.

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Annex 1: Data relating to the econometric study of complementarity-substitutability

Different types of data are used in this study: the statistics come from Eurostat, from survey data on the Labor Forces Labour Force Survey (LFS), from the organisation for economic cooperation and development (OECD) and the French national institute of economic and statistical information (INSEE). Eurostat's surveys and statistics allow the immigrant population to be apprehended according to the criterion of nationality at the time of the survey and the region of residence of the previous year. Referring to the criterion of nationality, there are three categories: those who have the nationality of the host country (nationals); those who do not have the nationality of the host country and who have the nationality of a member country of the European Union (community); those who do not have the nationality of the host country and do not have the nationality of a member country of the European Union (non community). The two latter categories represent the total population of foreign-born.

In order to conduct a study of complementarity-substitutability for France, in 2008, we use the following data:

- *Quantity of workers*: The data, from Eurostat, on specific demand, provide the number of employed workers depending on nationality (nationals; community and non community), the skill level (referring to the ISCED international nomenclature), by sector of activity (NACE Rev 1. 88 divisions).
- *Quantity of capital*: The INSEE provides the amount of the fixed gross capital in volume by sector of activity (NES 36).
- *Gross value added*: The gross value added by sector of activity is provided by Eurostat (Nace Rev. 2) and by INSEE (NES 36). The sharing of value added between capital and employment (taking into account taxes on production and operating grants) by sector of activity is also available on the INSEE's website.
- *Price of employment*: The INSEE provides the average gross annual wage by full-time equivalent by sector of activity. To take into consideration the sector disparities in terms of earnings, we took into account the differences in wages depending on socio-professional category, linked to the level of education, in the different sectors of activity. Referring to the OECD's report on international migrations, which provides the median wage depending on the level of education for persons born in the country and born abroad, the wages of native and immigrant workers were weighted according to skill level and nationality of the worker, determining a rating by the calculation of the median.

  Knowing the median wage depending on the level of education for persons born in the country (me_A) and born abroad (me_I), and the average wage irrespective of skill levels by sector of activity with weighting by sector (*sal*), in addition to the proportion of immigrants (p_I) by sector, it is possible to deduce the wage of the skilled and the wage of the immigrants depending on skill level:

$$sal_{tq,\lambda} = \frac{sal_{\lambda}}{(1-p_{tqI,\lambda}) + (me_I / me_A) * P_{tqI,\lambda}}$$

$$sal_{tqI,\lambda} = \left(\frac{me_I}{me_A}\right) * sal_{tq,\lambda}$$

$$sal_{q,\lambda} = \frac{sal_{\lambda}}{(1-p_{qI,\lambda}) + (me_I / me_A) * P_{qI,\lambda}}$$

$$sal_{qI,\lambda} = \left(\frac{me_I}{me_A}\right) * sal_{q,\lambda}$$

$$sal_{nq,\lambda} = \frac{sal_{\lambda}}{(1-p_{nqI,\lambda}) + (me_I / me_A) * P_{nqI,\lambda}}$$

$$sal_{nqI,\lambda} = \left(\frac{me_I}{me_A}\right) * sal_{nq,\lambda}$$

where λ represents the sector of activity.

- *Price of capital*: The INSEE provides the distribution of the value added depending on production factors. Therefore, it is not necessary to know the capital price to determine the share of the capital factor in the sector gross value added. Nevertheless, in order to control the robustness of the data base, the calculation of the share was done by verification, considering the price of capital as the difference between fixed capital at standard price and fixed capital at constant price, for each sector of activity.
- *The distribution of the number of workers, of the quantity of capital and of the share of each factor in the total value added*: in the following table, it can be noticed (in proportion) that the immigrants are overrepresented in the low-skilled worker category, which quite obviously reinforces the observation of the stylised facts of Part I.

	TQ	TQI	Q	QI	NQ	NQI	K
Number of workers (in thousands) and quantity of capital (en milliards of €)	7741.681	404.581	11355.385	474.202	5710.177	637.974	6888900000
Share of the factors (in %)	0.273	0.013	0.201	0.008	0.071	0.007	0.426

Annex 2: The impact of immigration on high, intermediate and low-skilled natives

Returning to the equation (12) with $i=(Q,NQ,QI,NQI)$ and knowing that $dW_{TQ}=dW_{TQI}$, we obtain a system in matrix form. Employing Cramer's method and formula which enable the expression of the solutions of a 5×5 system according to two determinants, we end with:

$$\frac{dTQ}{dTQI}=\frac{\begin{vmatrix} -f_{Q,TQI} & f_{Q,Q} & f_{Q,NQ} & f_{Q,QI} & f_{Q,NQI} \\ -f_{NQ,TQI} & f_{NQ,Q} & f_{NQ,NQ} & f_{NQ,QI} & f_{NQ,NQI} \\ -f_{NQI,TQI} & f_{NQI,Q} & f_{NQI,NQ} & f_{NQI,QI} & f_{NQI,NQI} \\ -f_{QI,TQI} & f_{QI,Q} & f_{QI,NQ} & f_{QI,QI} & f_{QI,NQI} \\ -(f_{TQ,TQI}-f_{TQI,TQI}) & f_{TQ,Q}-f_{TQI,Q} & f_{TQ,NQ}-f_{TQI,NQ} & f_{TQ,QI}-f_{TQI,QI} & f_{TQ,NQI}-f_{TQI,NQI} \end{vmatrix}}{\begin{vmatrix} f_{Q,TQ} & f_{Q,Q} & f_{Q,NQ} & f_{Q,QI} & f_{Q,NQI} \\ f_{NQ,TQ} & f_{NQ,Q} & f_{NQ,NQ} & f_{NQ,QI} & f_{NQ,NQI} \\ f_{NQI,TQ} & f_{NQI,Q} & f_{NQI,NQ} & f_{NQI,QI} & f_{NQI,NQI} \\ f_{QI,TQ} & f_{QI,Q} & f_{QI,NQ} & f_{QI,QI} & f_{QI,NQI} \\ f_{TQ,TQ}-f_{TQI,TQ} & f_{TQ,Q}-f_{TQI,Q} & f_{TQ,NQ}-f_{TQI,NQ} & f_{TQ,QI}-f_{TQI,QI} & f_{TQ,NQI}-f_{TQI,NQI} \end{vmatrix}} \qquad (13)$$

According to the Hicks partial elasticities of complementarity:

$$c_{ij}=\frac{f\cdot f_{ij}}{f_i\cdot f_j} \text{ with } f_i=\frac{\partial f}{\partial x_i} \text{ and } f_{ij}=\frac{\partial^2 f}{\partial x_i \partial x_j}$$

Retaining the properties of constant returns to scale production functions[16]:

$$c_{ij}=\left(\frac{dW_i/W_i}{dx_j/x_j}\right)/s_j \text{ and } x_i=\frac{VAB.s_i}{W_i},$$

We deduce that: $f_{i,j}=\frac{W_iW_j}{VAB}.c_{i,j}$ (14) and $s_i=\frac{W_i.x_i}{VAB}$ (15)

From here it is possible to express $\frac{dlnTQ}{dlnTQI}$, for this we use the mathematical definition of a determinant of a matrix A size 5×5: $detA=\sum_{\sigma\in S_5}\varepsilon(\sigma)\prod_{i=1}^{5}a_{i\sigma(i)}$ where $\varepsilon(\sigma)$ is either -1 or 1, depending on the number of permutations of the symmetry σ. Re-employing the equations (14) and (15), without attempting to clarify all the terms, we are going to replace $f_{i,j}$ by $\frac{W_iW_j}{VAB}.c_{i,j}$ The first index means that the calculation of $a_{1,\sigma(1)}a_{2,\sigma(2)}a_{3,\sigma(3)}a_{4,\sigma(4)}a_{5,\sigma(5)}$ sweeps the five lines exactly once. Consequently, the five factors in W_i linked to the line of interest which are going to appear are: W_Q, W_{NQ}, W_{NQI}, W_{QI} and $(W_{TQ}-W_{TQI})$. We can

16 Grossman (1982), p. 601

write $\frac{dTQ}{dTQI}$ using $c_{i,j}$ and employing the formula $s_i = \frac{W_i . x_i}{VAB}$, therefore we obtain: $\frac{d\ln TQ}{d\ln TQI} = \frac{dTQ}{dTQI} \times \frac{TQI}{TQ}$ which gives the equation (16)[17]:

$$\frac{d\ln TQ}{d\ln TQI} = \frac{s_{TQI}}{s_{TQ}} \times \frac{\begin{vmatrix} -c_{Q,TQI} & c_{Q,Q} & c_{Q,NQ} & c_{Q,QI} & c_{Q,NQI} \\ -c_{NQ,TQI} & c_{NQ,Q} & c_{NQ,NQ} & c_{NQ,QI} & c_{NQ,NQI} \\ -c_{NQI,TQI} & c_{NQI,Q} & c_{NQI,NQ} & c_{NQI,QI} & c_{NQI,NQI} \\ -c_{QI,TQI} & c_{QI,Q} & c_{QI,NQ} & c_{QI,QI} & c_{QI,NQI} \\ -\left(c_{TQ,TQI}-c_{TQI,TQI}\right) & c_{TQ,Q}-c_{TQI,Q} & c_{TQ,NQ}-c_{TQI,NQ} & c_{TQ,QI}-c_{TQI,QI} & c_{TQ,NQI}-c_{TQI,NQI} \end{vmatrix}}{\begin{vmatrix} c_{Q,TQ} & c_{Q,Q} & c_{Q,NQ} & c_{Q,QI} & c_{Q,NQI} \\ c_{NQ,TQ} & c_{NQ,Q} & c_{NQ,NQ} & c_{NQ,QI} & c_{NQ,NQI} \\ c_{NQI,TQ} & c_{NQI,Q} & c_{NQI,NQ} & c_{NQI,QI} & c_{NQI,NQI} \\ c_{QI,TQ} & c_{QI,Q} & c_{QI,NQ} & c_{QI,QI} & c_{QI,NQI} \\ c_{TQ,TQ}-c_{TQI,TQ} & c_{TQ,Q}-c_{TQI,Q} & c_{TQ,NQ}-c_{TQI,NQ} & c_{TQ,QI}-c_{TQI,QI} & c_{TQ,NQI}-c_{TQI,NQI} \end{vmatrix}}$$

In order to simplify the demonstration, we have used the *TQ* variable but we can use the same approach to obtain each relation $\frac{d\ln x_i}{d\ln x_j}$, with x_i=(*TQ,Q,NQ,TQI,QI,NQI)* and x_j=(*TQ,Q,NQ,TQI,QI,NQI)*. Lastly, to evaluate the adjustment by the prices resulting from the admittance of immigrant workers we can deduce that $\frac{\mathrm{dlnW_i}}{\mathrm{dlnx_j}} = \frac{\mathrm{dW_i}}{\mathrm{dx_j}} \times \frac{\mathrm{x_j}}{\mathrm{W_i}}$, or:

$$\frac{\mathrm{dlnW_i}}{\mathrm{dlnx_j}} = \left(\frac{\mathrm{x_j}}{\mathrm{W_i}}\right) \times$$

$$\left[\mathrm{f_{i,TQ}} \frac{\mathrm{dlnTQ}}{\mathrm{dlnx_j}} \frac{\mathrm{TQ}}{\mathrm{x_j}} + \mathrm{f_{i,Q}} \frac{\mathrm{dlnQ}}{\mathrm{dlnx_j}} \frac{\mathrm{Q}}{\mathrm{x_j}} + \mathrm{f_{i,NQ}} \frac{\mathrm{dlnNQ}}{\mathrm{dlnx_j}} \frac{\mathrm{NQ}}{\mathrm{x_j}} + \mathrm{f_{i,TQI}} \frac{\mathrm{dlnTQI}}{\mathrm{dlnx_j}} \frac{\mathrm{TQI}}{\mathrm{x_j}} + \mathrm{f_{i,QI}} \frac{\mathrm{dlnQI}}{\mathrm{dlnx_j}} \frac{\mathrm{QI}}{\mathrm{x_j}} + \mathrm{f_{i,NQI}} \frac{\mathrm{dlnNQI}}{\mathrm{dlnx_j}} \frac{\mathrm{TQI}}{\mathrm{x_j}}\right]$$

By replacing $f_{i,l}$ by their formula of (14), then grouping the elements of the formula (15) in each term, we obtain:

$$\frac{d\ln W_i}{d\ln x_j} = s_{TQ} c_{i,TQ} \frac{d\ln TQ}{d\ln x_j} + s_Q c_{i,Q} \frac{d\ln Q}{d\ln x_j} + s_{NQ} c_{i,NQ} \frac{d\ln NQ}{d\ln x_j} + s_{TQI} c_{i,TQI} \frac{d\ln TQI}{d\ln x_j} + s_{QI} c_{i,QI} \frac{d\ln QI}{d\ln x_j} + s_{NQI} c_{i,NQI} \frac{d\ln NQI}{d\ln x_j}$$

with i=(*TQ,TQI,K*) and x_j=(*TQ,Q,NQ,TQI,QI,NQI)*.

(17)

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17 To simplify the demonstration, we have used the variable TQ, but we can use the same approach to obtain each relation $\frac{\mathrm{dlnx_i}}{\mathrm{dlnx_j}}$, with: $\mathrm{x_i}$=(TQ,Q,NQ,TQI,QI,NQI) and $\mathrm{x_j}$=(TQ,Q,NQ,TQI,QI,NQI).