The minimum liquidity deficit and the maturity structure of central bank's open market operations: Lessons from the financial crisis

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1 Introduction

Since the start of the financial crisis in August 2007, the ability of central banks to steer money market rates has been put to the forefront of interest. As loss of confidence and uncertainty about counterparty credit risk increased, banks increasingly relied on central bank financing. During the early stage of the financial crisis, central banks have taken on a more and more active role as liquidity providers. For instance, they have altered the time-path of liquidity supply to the banking sector, relaxed their collateral standards and introduced a time-shift in the maturity structure of their operations. With the help of these policies, short term money market rates were by and large contained to the desired ranges – although at the cost of an increased volatility. During this period relatively quickly it became clear that the size of the liquidity providing operations was an important factor for steering money conditions. If an operation was "too small", relative to the prevailing stress level in the money market, the resulting price for central bank liquidity could be very high, thus driving up short term money market rates and adding further stress via this signal to the market.

In October 2008, faced with extreme stress levels in the wake of the bankruptcy of Lehman Brothers, central banks resorted to the provision of unlimited funds at relatively favourable conditions in order to prevent short term money market rates from exceeding desired levels. This policy rendered the amount of central bank liquidity endogenous, i.e. its supply depended only on the demand of banks and came at the price of persistent excess liquidity and consequently lower short term money market rates.

[1]Both authors: European Central Bank. Comments from Simone Manganelli, Flemming Würtz, and participants at the ECB workshop "Challenges to Monetary Policy Implementation" in December 2009 are greatly appreciated.

In this paper, we aim to provide both a theoretical foundation for the relation between the size of a central bank's liquidity deficit and auction rates in its open market operations in times of financial market stress, and an empirical analysis which aims to determine the size of open market operations needed to absorb large stress levels in interbank money markets and hence contain central bank auction rates.

In particular, the theoretical part of the paper aims at guiding the optimal institutional set up for situations of increased financial market distress such as the early phase of the financial crisis, from August 2007 to September 2008, when most central banks around the world tried to address tensions in money markets using more standard tools.

In the second phase of the crisis, starting with the bankruptcy of Lehman Brothers, central banks resorted to so-called non-standard measures. As a consequence, in many cases, banks were given the freedom to demand as much central bank liquidity as they needed. The liquidity provision of the ECB during this period serves as the basis for our empirical analysis. Using the example of the Eurosystem's tender operations since October 2008, we derive several measures of the minimum amount of liquidity needed to absorb demand from banks which are likely to display a very high marginal valuation for central bank liquidity, thus driving up marginal tender rates.

The results of the analysis can shed light on the appropriate choice of the liquidity deficit during the phasing-out of the extraordinary measures that were implemented during the financial turmoil of 2007-2010.

The second focus of the paper is the maturity structure of central bank refinancing operations. Let us again consider the example of the Eurosystem.[2] Since the last change to the operational framework in March 2004, the bulk of refinancing was provided to the market using operations of a short maturity (one week), while only a smaller fraction (about one-third) was provided with a maturity of three months. During the early phase of the crisis until August 2008, this relationship was roughly reversed, as the drying-up of liquidity in markets for term refinancing led the Eurosystem to enhance its three months refinancing at the expense of one-week operations and hence lengthen the maturity structure of its operations.[3] When the

[2] Also the Fed and the Bank of England did lengthen the maturity structure of their liquidity provision.
[3] This relationship held until September 2008 when in the wake of the further escalation of the banking crisis as a temporary measure fixed rate tender with full allotment in both MROs and LTROs were announced.

crisis intensified, demand in longer term refinancing operations increased further, bringing the share of standard one-week liquidity at times below 10%.

A sound theoretical basis for these choices (i.e. the policy choice of the central bank in the early phase and the endogenous choice of the banks in the later period of the crisis) has yet to be found. While this paper does not aim at developing a full-fledged analysis of the optimal maturity mix of open market operations, it does point out that a central bank that wants to avoid elevated marginal rates in its open market operations should take care that changes in the maturity structure of its operations do not augment tender spreads in existing operations. At the core of our analysis stands the observation that tender rates in the one-week operations sharply increased after the change of the maturity profile. This leads us to conjecture that there might be a lower size limit to open market operations under variable rate tender regimes (seen in relation to the banking sector's aggregate reserve requirement). At the same time, we elaborate on the intermediary function of central banks, which since the start of the market turmoil has increased in importance.

The paper is structured as follows. Section 2 considers a theoretical analysis of banks' demand for refinancing at a central bank auction, in an environment where banks face constraints of varying degrees when borrowing on the unsecured interbank market. The empirical analysis in section 3 derives several measures of the liquidity deficit needed to absorb high stress levels in euro area money markets, using participating banking groups' demand in the fixed-rate tender with full-allotment operations conducted by the Eurosystem since October 2008. The following section 4 extends the theoretical analysis of section 2 to a framework with two periods that allows the analysis of operations with different maturities. The empirical part of this section uses the results of an econometric analysis of the relationship between bidding rates at the ECB's tenders and the size of its operations, using the experience gained in variable rate tender operations between August 2007 and September 2008. The last section concludes.

2 The baseline model

We assume that the central bank wishes to supply a certain amount of refinancing to the banking sector, S, for one period. This amount is based on the aggregate liquidity needs of the banking sector.[4] There is a continuum of banks of measure one in the economy, and all these banks can participate in the central bank's open market operations.

We assume that banks experience two types of shocks: liquidity and solvency shocks. The liquidity shock implies that banks' demand for central bank funds π can vary between π_H (in this case, the bank is cash-short) and π_L (i.e. the bank is cash-long), where $\pi_L < \pi_H$, and $\pi_L < 0$ is possible. That is, cash-long banks can have a positive or negative overall liquidity demand.

The solvency shock is modelled in the following way: We assume that banks receive correct signals about each other's solvency prior to the central bank auction. These signals indicate a bank's probability of being solvent, and, more specifically, its probability to repay a loan obtained in the (unsecured) interbank market. For simplicity, we assume that signals reveal the true type of the banks. It is assumed that the probability of being solvent v is uniformly distributed on the interval $(0,1]$. Thus, $f(v)=1$ for $v \in (0,1]$. Different types of banks are charged different rates on the interbank market: banks that are surely solvent pay the risk-free rate $1+r$, while troubled banks need to pay a premium $p(v)$ which compensates lenders for the additional risk they take when lending to more risky institutions. We assume that the interbank market is a lender's market and that interest rates are such that lenders are indifferent between lending to borrowers with different probabilities of repayment. This requires $1+r \overset{!}{=} v(1+r+p(v))$ for all v, which leads to a premium

$$p(v) = \frac{1-v}{v}(1+r).$$

As is intuitive, $p(v)$ is decreasing in v – that is, the higher the probability of solvency of a bank, the lower the premium it needs to pay on the interbank market.

We furthermore assume that the liquidity shock by a certain bank is correlated with its solvency: in particular, the lower the probability of being solvent, the higher is the liquidity shortage of a bank. To achieve this, we assume a simple relationship between v and a bank's liquidity demand

$$d(v) = \pi_H - v(\pi_H - \pi_L) \qquad (1)$$

[4] One determining factor for aggregate liquidity needs are reserve requirements. Also, so-called autonomous factors are considered, i.e. factors which are not under direct control by the central bank, such as the demand for banknotes.

This implies that banks which are surely solvent *(v=1)* have a low liquidity demand π_L, while those that are almost certainly bankrupt *(v→0)*, face high demand for liquidity, π_H. Banks can obtain liquidity at the central bank's open market operation, or by trading liquidity in an interbank market. We assume that banks obtain funds in these ways only to satisfy their own liquidity needs – that is, they do not borrow more than needed from the central bank in order to lend some of the funds to other banks in the interbank market. Moreover, we assume that there is no stigma associated with borrowing from the central bank or in the market.

Consider the following timing:

- the central bank decides upon the refinancing volume S
- banks receive liquidity shocks $\pi \in [\pi_L , \pi_H]$
- peer banks receive a perfect signal about each individual banks' solvency probability v, which is correlated with π
- the central bank auction takes place (where banks bid $b(v)$, depending on their type v, and the marginal interest rate is denoted $1+b^*$)
- the interbank market takes place (at the risk-free interest rate $1+r$ plus the appropriate premium $p(v)$).

2.1 Central bank liquidity supply

We assume that there is no aggregate uncertainty and that the central bank is perfectly able to forecast aggregate liquidity needs. The central bank chooses its liquidity supply to match the aggregate (average) demand, i.e. $S \overset{!}{=} D$ where

$$D = \int_0^1 d(v) f(v) dv = \frac{1}{2}(\pi_H + \pi_L) \equiv \overline{\pi}$$

corresponds to the midpoint between π_L and π_H (i.e. the average individual liquidity demand). It is assumed that the system operates in a liquidity deficit so that $\overline{\pi} > 0$.

Furthermore, we assume that no liquidity shocks occur after the central bank auction. In this case, since the central bank will always supply just the amount of liquidity that is needed to satisfy aggregate liquidity demand, demand and supply are balanced, and the risk-free interest rate $1+r$ in the interbank market (i.e. that rate charged to banks that are considered to have a zero probability of default) is at the same, predictable level.[5]

The central bank conducts its open market operation as a variable rate tender, in which banks bid one pair of a quantity and the interest rate they are willing to pay. The highest bids are served first, until the allotted quantity equals the total supply *S*. The marginal rate of the auction, *1+mr*, is the lowest bid at which banks obtain liquidity.[6]

2.3 Liquidity demand at the central bank auction

When entering the central bank auction, banks' opportunity cost of funding is given by their ability to obtain funds in the interbank market, which in turn is a function of their probability of solvency, *v*. The rate they need to pay in the interbank market, $1+r + p(v)$, determines their bid rate at the central bank auction,[7]

$$b(v)=1+ r + p(v).$$

In order to determine the marginal bid rate at that auction, the aggregate demand curve needs to be established. Aggregate demand is given by

$$D(\bar{v}) = \int_0^{\bar{v}} d(v) f(v)\, dv = \bar{v}\pi_H - \frac{\bar{v}^2}{2}(\pi_H - \pi_L)$$

because of (1) and $f(v)=1$ (uniform distribution). This implies

$$D(b) \equiv D(v(b)) = \frac{1+r}{b}\pi_H - \frac{(1+r)^2}{2b^2}(\pi_H - \pi_L)\,. \qquad (2)$$

The marginal bid rate is determined by the bid rate that equalizes demand and supply, i.e. by the b that satisfies

$$D(b^*) \overset{!}{=} S = \bar{\pi}\,.$$

It is easy to see that this quadratic equation has two solutions, b_1 and b_2, given by

$$b_1 = 1+r$$
$$b_2 = \frac{\pi_H - \pi_L}{\pi_H + \pi_L}(1+r) \qquad (3)$$

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[5] If we abstract from the need to post collateral in central bank operations, the risk-free rate corresponds to the minimum bid rate of the auction.

[6] In real-life auctions, with step-wise bid schedules, banks might be rationed on a pro-rate basis at the marginal rate. In the present model, the demand curve is continuous, so no rationing is needed.

[7] This might be simplifying, as distortionary factors such as bid shading are not considered.

Figure 1 below illustrates two examples of the demand curve (or better: its inverse, the bid curve, given that bid rates b figure on the vertical axis, and demand D on the horizontal one).[8] In the first panel, an example with $\pi_L > 0$ is chosen. In this case, the bid curve is downward sloping until it reaches the risk-free rate $1+r$ at bid rate b_1. This is the marginal rate of the auction, since at this rate, all bids above the marginal rate are successful. All bids below this rate are unsuccessful. Therefore, the other bid rate for which demand equals supply, b_2, is irrelevant in this case.

The second panel depicts an example with $\pi_L < 0$. Since $\bar{\pi} > 0$, this corresponds to the case of a very unequal distribution of liquidity needs – compared to the size of the deficit, $\bar{\pi}$. Again, the bid curve is downward sloping. In contrast to the other example, here, the interest rate at which demand first equals supply is b_2, so that b_2 is the marginal rate for the auction. Thus, in this case, the marginal rate is higher than the risk free rate.

From (2), it is easy to see that

$$b_1 < b_2 \quad \Leftrightarrow \quad \pi_L > 0.$$

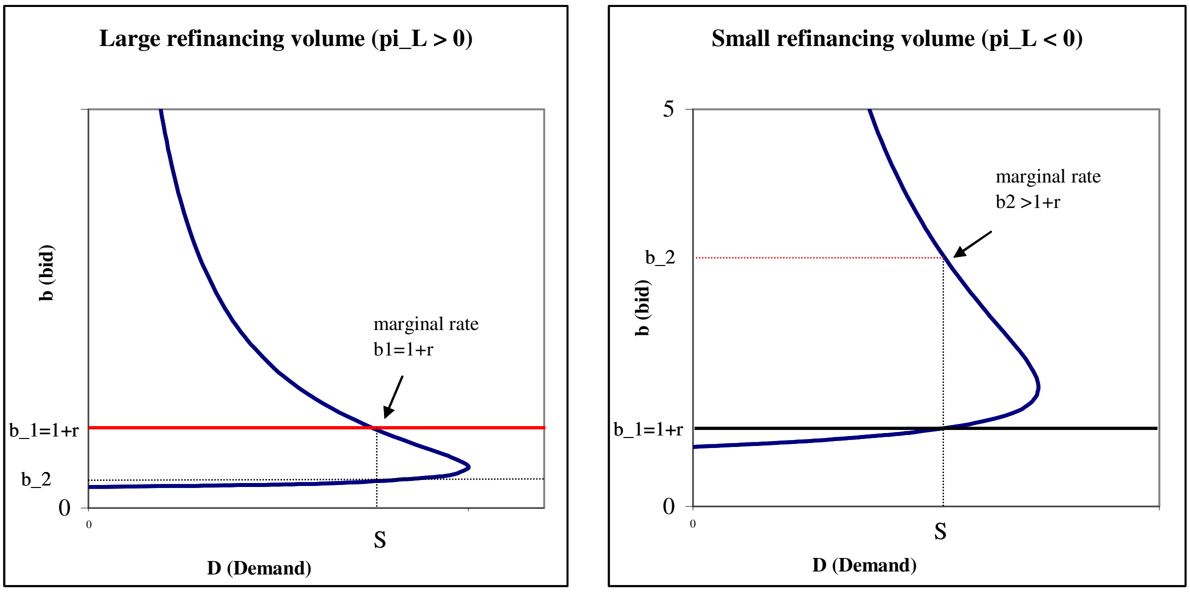

Figure 1: The bid curve for positive and negative π_L.: in the left-hand panel (π_L>0), the marginal rate is b_1, the risk-free rate. In the right-hand panel (π_L<0), the marginal rate equals b_2, higher than the risk-free rate.

[8] Note that the somewhat unusual shape of the bid curve, which is backwards bending, is a result of allowing for negative individual liquidity demand. In reality, banks with excess liquidity bid an amount of zero. The results of this section would not change if we allowed for zero bidding.

In other words, if both cash-short and cash-long banks have a positive liquidity demand, then the marginal rate will be low (and equal to b_1). In this case, demand will be composed of both types of banks, including also surely solvent banks (for which v is close to or equal to 1), who are the marginal bidders in the auction, and whose bid $1+r$ determines the marginal rate

If, however, the demand of cash-long banks is negative (e.g. these banks have excess liquidity), then the marginal rate increases to b_2. This is because in this case, demand at the auction will be dominated by cash-short banks, which are more likely to have solvency problems. These banks bid above the risk-free rate, and cause the higher marginal rate.

The single condition for this to happen is to have banks with a liquidity surplus $\pi_L > 0$. Comparing b_1 and b_2 in equation (3), it becomes evident that this is the case when

- The total liquidity deficit $\bar{\pi} = 1/2(\pi_H + \pi_L)$ is low (close to zero), and when
- Liquidity needs are very divergent across banks ($\pi_H - \pi_L$ is large).

In particular, the relation between the two is crucial: a larger divergence between liquidity needs does not lead to an elevated marginal rate if it is accommodated by a large liquidity deficit. Thus, if a central bank wants to avoid that bid rates increase, it should take care that the liquidity deficit (or the size of the auction) is not too small in relation to the divergence of liquidity needs of banks.

The above results were derived for the specific case of a uniform distribution of solvency types, and for a linear relationship between solvency and liquidity needs. These assumptions are of course simplifying, however, not essential to generate the results. Still, some relationship between solvency and liquidity is necessary for the above to hold. If solvency and liquidity were uncorrelated, the marginal rate would always equal the risk-free rate.

One further step in the analysis would be to introduce specifically a variable that would capture the state of the economy – for instance, the ability of banks to raise funds in the interbank market. In the above model, banks are always able to obtain funds in the interbank market, provided they are willing to pay the appropriate interest rate. During the ongoing financial turmoil, however, some banks (at times: most banks) were unable to raise any funds in the interbank market and were completely dependent on central bank refinancing.[9] In this sense, the model captures a rather conservative case compared to the turmoil period.

The analysis shows that not only the congruence of aggregate liquidity supply and demand matters for the determination of interest rates at central bank auctions. Instead, the ability of the interbank market to effectively distribute liquidity across the banking sector is also important. If this function is impaired, for instance because of asymmetric information, liquidity hoarding or (as in this model) heightened counterparty credit risk, then marginal bid rates at open market operations can increase.

The model illustrates that a surge in counterparty credit risk in the interbank market does not necessarily imply that the marginal rate in the central bank auction is affected. This is only the case if the distribution of banks' liquidity needs is wide compared to the refinancing volume. If the liquidity deficit however is large enough, then the marginal rate of the operation will be determined by those banks that are able to borrow at the risk free interest rate.

This analysis suggests that central banks that are eager to tightly control the marginal rate of their operations should take into account the state of the interbank market and the level of credit risk of banks for the design of their balance sheet and thus their refinancing volume. The more impaired the market, the higher should be this volume. Otherwise, the central bank risks the emergence of a large spread between marginal rates and its target rate.

It should be stressed at this point, that these considerations do not take into account potentially adverse effects stemming from large refinancing volumes. First, a too large intermediation role of the central bank may crowd-out money market activity. Secondly, larger refinancing operations are, ceteris paribus, associated with a more significant liquidity insurance function of the central bank, which may increase moral hazard. Lastly, absorbing financial market stress levels through large auctions works through the central bank taking credit risk on its balance sheet, which may socially not be desirable. Ideally, this credit risk should be appropriately priced.

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[9] Two theories explain the breakdown of the interbank market. First, banks may have been unwilling to lend out funds because they were uncertain about own future liquidity needs (see, e.g. Eisenschmidt and Tapking, 2009). Second, adverse selection in the interbank market, caused by asymmetric information about counterparties' solvency may have induced banks to hoard liquidity (see Heider et al, 2009).

3 Empirical Analysis

We now turn to the data. To this end, a panel containing bidding data for 1653 banking groups active in Eurosystem refinancing operations or using Eurosystem standing facilities for the period from January 2007 until November 2009 was used.

3.1 The liquidity deficit, central bank refinancing and tender design

The liquidity deficit is the quantity of central bank reserves the banking system is in short supply each reserve maintenance period, due to refinancing needs arising from reserve requirements and autonomous factors[10]. In other words, the size of the liquidity deficit provides a lower bound for the aggregate demand for central bank reserves and hence the size of the refinancing operations. The evolution of both, the size of the liquidity deficit and the overall size of the refinancing operations for the euro area from January 2007 until July 2009 are depicted in figure 2 below.

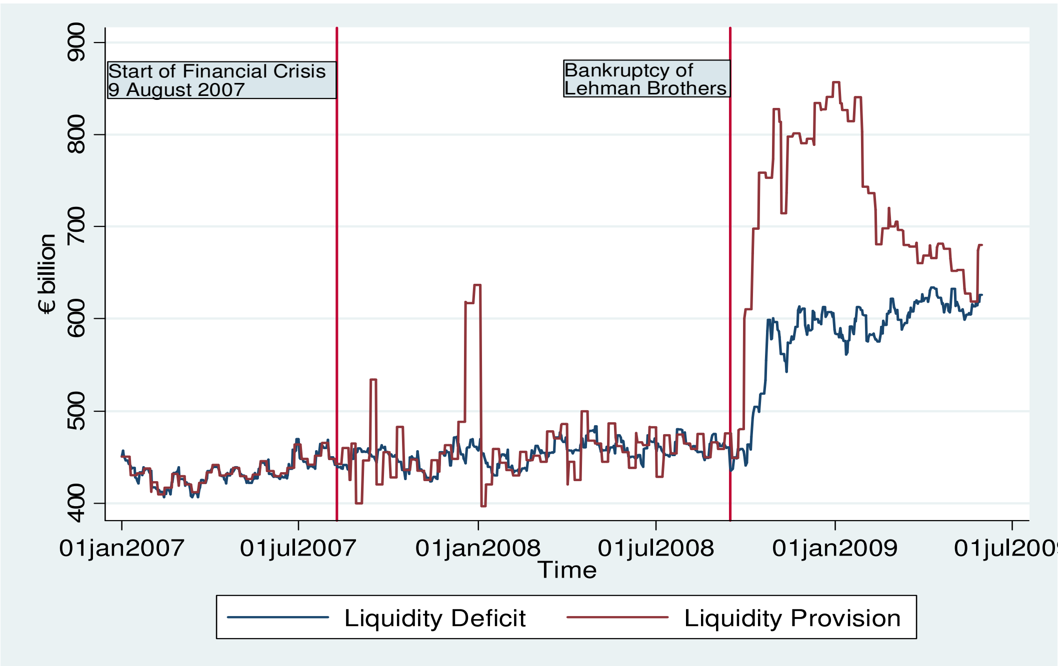

Figure 2: Aggregate Eurosystem liquidity provision and liquidity needs of the banking sector, January 2007 to May 2009

It can be seen from figure 2 that during normal times, e.g. in terms of the above chart from January 2007 until August 2007, both variables are closely aligned. With the start of the financial crisis and the increasing tensions in euro area money markets, the ECB changed the supply pattern of liquidity throughout the maintenance period, i.e.

[10] The most important autonomous factors in the Eurosystem are banknotes in circulation and government deposits. Generally, autonomous factors are refinancing needs which are not influenced by the central bank.

more liquidity than needed was provided at the beginning of the reserve maintenance period reducing the excess allotments gradually during the course of the period. This pattern is clearly visible from the chart above and ends with the bankruptcy of Lehman Brothers and the subsequent introduction of fixed-rate tenders with full allotment in all refinancing operations of the Eurosystem. From 14 October 2008 onwards, banking groups themselves determined the amount of central bank liquidity in euro area money markets. As is visible from figure 2 above, this endogeneity of euro area liquidity provision resulted in an extended period with substantial amounts of aggregate excess liquidity, which, of course, was reflected in persistent and large recourses to the deposit facility of the Eurosystem.

The liquidity deficit, between January 2007 and August 2008 hovering around EUR 450 billion (displaying only a slight upward trend), increased substantially in October 2008 and reached a value of more than EUR 600 billion. This mainly reflected an increase of banknotes in circulation and government deposits, with both developments being related to the intensification of the financial crisis after the bankruptcy of Lehman Brothers.

Recent research on the determinants of short term money markets (see Linzert and Schmidt 2008 or Cassola and Morana 2008 for empirical approaches and Valimäki 2006 for a theoretical model) finds that, during normal times, a large liquidity deficit may have unwanted side effects: with increasing size of the auctions, operational risks in these auctions increase and as a consequence tender spreads[11] (and via this channel short-term money market rates) may be biased upwards (see Eisenschmidt, Hirsch and Linzert (2009) for a discussion on the influence of the size of MROs and LTROs on the tender spread and Würtz (2006) and Banco de Portugal (2008) on the determinants of the tender spread in general). In the light of these results, a small liquidity deficit seems desirable from the perspective of optimal monetary policy implementation.

During times of financial crisis, however, this relationship seems to be reversed or at least is likely to be superseded by considerations relating to effective implementation as well as financial stability. Given increased levels of counterparty risk and elevated

[11] The tender spread is usually defined to be the difference between the marginal (or stop-out) rate of the tender and the minimum bid rate (or policy rate) prevailing at the tender.

uncertainty about liquidity shocks, money market intermediation of liquidity shocks, relying on thousands of unsecured interbank transactions every day, is plagued by an increasing level of frictions and may break down altogether. In such circumstances, the size of the liquidity deficit becomes a crucial input into the effectiveness of monetary policy implementation and the financial stability dimension of the monetary policy implementation framework.

The theoretical model of section 2, in which, depending on the share and liquidity demand of troubled or "ostracised" banks in the banking system, standard tender volumes can become too small and, as a consequence, strongly increasing marginal bid rates are observed, is capturing the essence of this argument. This finding mirrors closely the experience made in the first period of the financial crisis, under variable rate tender procedure with a pre-set amount.

An indication of the minimum size of the liquidity deficit needed to absorb heightened stress levels in euro area money markets is therefore a crucial lesson from the financial crisis. This should also be of use for the further development of the framework for monetary policy implementation, enhancing its effectiveness and strengthening its financial stability dimension.

3.2 Estimating the minimum liquidity deficit from fixed-rate tenders with full-allotment

The current episode of fixed-rate tenders with full allotment – in which banks themselves determine the amount of liquidity they need – provides an invaluable empirical basis to determine the minimum size of the liquidity deficit during times of intense money market stress.

A first (and relatively simple) approach to determine the minimum size of the liquidity deficit is to calculate the amount demanded by banks that have shown above average reliance on Eurosystem refinancing during the period of fixed-rate tender with full allotment procedure, *relative to their past demand behavior*. In order to obtain a measure of banking groups that are above average user in this sense we first standardize individual banks' bid volumes.[12] Let x_t^i denote the value of the bid volume at time *t* for banking group *i*, we transform the volume according to: $\tilde{x}_t^i \equiv (x_t^i - \mu^i)/\sigma^i$, where μ^i and σ^i denote, respectively, the sample mean and sample standard deviation for banking group *i*. Since our dataset contains 1565 active banking groups (out of a total of 1653) and their bid schedules in 148 main refinancing operations (MROs)[13], this transformation gives rise to 231620 different values $\tilde{x}_t^i$. Over time and within groups, each variable has a mean of zero and a standard deviation of 1.

We then proceed to compute two different time averages for the normalised values $\tilde{x}_t^i$ for each banking group for the period starting with the first fixed-rate tender with full allotment on 14 October 2008 until

1) the end of the sample on 3 November 2009 and
2) the first one-year tender operation was conducted, i.e. until 23 June 2009.

The restricted sample was chosen in order to control for the specific nature of the one-year supplementary longer-term operation (SLTRO) conducted on 24 June 2009, which may have attracted opportunistic bids from banking groups otherwise absent from Eurosystem operations.

We then rank the resulting time averages, i.e. the banking group with the highest average deviation from its behaviour before the introduction of the fixed-rate tender with full-allotment is ranked first and so on. Finally, we obtain the outstanding quantity of Eurosystem refinancing at each point of the sample for the 1st quartile of the so-ranked population of banking groups.

The resulting summary statistic for this group is contained in table 1 below. The refinancing volume demanded by the 25% of banking groups displaying the most

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[12] We are indebted to Simone Manganelli for the idea and intense discussions on the procedure.
[13] Main refinancing operations used to be (prior to the financial turmoil) the main source of refinancing to the banking sector. These are weekly operations with a one-week maturity. Longer-term operations used to have a maturity of 3 months, but were during the turmoil supplemented with 1, 6 and 12 month operations.

changed demand behaviour, varies substantially over time, with a stable tendency towards the end of the sample period. Not surprisingly, excluding the influence of the one-year SLTRO increases the quantity demanded by these top 25% of banking groups. There are two main reasons for this: the participation of many small banking groups in the first one year SLTRO, which is also born out by the very high number of bidders in this operation and a relatively high share of bidding driven solely by the very favourable conditions of the operation.

Sample	Outstanding Eurosystem refinancing volume in EUR billion and share of total outstanding Eurosystem refinancing, 1st quartile of banking groups, ranked after their change in bidding behavior in terms of volumes					
	Min		Max		Mean	
	Volume	Share	Volume	Share	Volume	Share
14 October 2008 – 3 November 2009	202	27%	406	57%	334	46%
14 October 2008 – 23 June 2009	338	45%	504	70%	430	59%

Table 1: Minimum liquidity deficit, as defined by banks that are reliant above average on Eurosystem refinancing

Figure 2 below illustrates graphically the path of the two measures of the minimum liquidity deficit over time, from 14 October 2008 onwards.

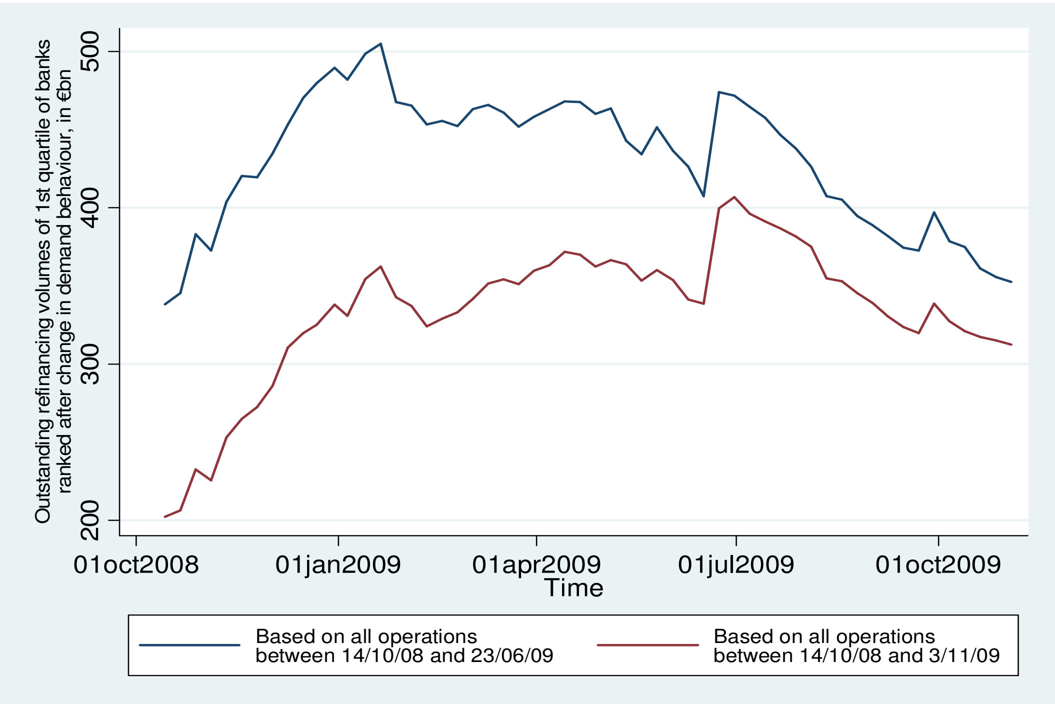

Figure 2: Outstanding Eurosystem refinancing for the 1st Quartile of banking groups ranked after changes in demand behavior, in EUR billion

Taking a prudent approach (i.e. using the maximum values from table 1), it emerges that the size of liquidity deficit before the start of the financial crisis was roughly sufficient to satisfy bids from the top 25% of banking groups, using the degree of their behavioural change after the introduction of the fixed-rate tender with full allotment relative to their previous bidding behaviour as a ranking criterion.

Although this simple mapping may include some bidding by banking groups taking advantage of the very low interest rates, or very risk averse acting banking groups, it is likely to include the bulk of the distressed bidders. Accordingly, using this metric and having in mind the theoretical model of section 2, a liquidity deficit smaller than EUR 400-500 billion may not be sufficient to fully absorb the demand for central bank refinancing of stressed bidders and therefore lead to very high marginal tender rates during episodes of money market tensions.

Note that, although we are using data from a period with fixed rate tender with full allotment, our results are applicable to a regime with variable rate tender procedure. First of all, we are interested in a size measure of banks displaying a high demand for central bank refinancing. The fixed-rate full allotment tender episode provides the best possible data, since banking groups have no incentive to bid strategically. Second, we are interested in a prudent measure, i.e. we want to obtain the size of the liquidity deficit that is large enough to absorb the highest stress levels observed in financial markets. There is little doubt that the period between October 2008 and March 2009 was associated with unprecedented levels of financial market stress.

Therefore, in following this approach, we are able to obtain an estimate of a lower size limit of central bank refinancing operations under heightened financial market stress. Note, however, that using this size measure to calibrate central bank auctions does not, however, imply that in reality tender rates during periods of financial market stress are always closely aligned to policy rates. There are other factors influencing tender spreads, like allotment uncertainty, asymmetric information and strategic bidding, all of which are likely to be elevated under conditions of overall financial market stress. Nevertheless, the size measure we obtain shoul be seen as a useful guide to avoid extreme tender rates solely due to heightened counterparty credit risk.

4 Refinancing operations of different maturities

4.1 Some theoretical considerations

During the financial crisis many central banks reacted by providing liquidity to banks at longer maturities than usual. For instance, the Eurosystem, which had prior to the turmoil provided the bulk of liquidity in one-week operations, introduced several supplementary longer-term operations with maturities of 1, 3, 6, and 12 months. This was done in an environment of increased demand for longer-term operations, be it for regulatory purposes or as insurance for banks to secure funds for a longer time horizon. The high demand for longer-term operations (compared to one-week operations) became even more evident when a full allotment policy was applied to those operations as of October 2008.

This section does not aim at explaining the reasons behind the increased demand for longer-term refinancing, or dispute the necessity of adjusting the liquidity supply to this demand. Instead, it points at some constraints when doing this adjustment (or when reversing it), along the lines set out in section 2.

Consider a setup similar to the model described in section 2, but with a two period horizon. The central bank conducts a one-period liquidity auction in each period. We assume that banks' liquidity needs in both periods are completely independent - this assumption would reflect, inter alia, that both periods represent different reserve maintenance windows. In this case, banks' bidding behaviour would, in each period, be equivalent to the one described in section 2: whenever the distribution of liquidity demand is very dispersed relative to the size of the total refinancing at the central bank's auctions that some banks have a negative liquidity demand, the marginal rate of the auction increases.

Depending on the state of the financial sector at the time of each auction, the marginal rate at each point in time will differ. Let us consider the case where tensions in financial markets lead to an elevated marginal rate at both auctions, i.e. the marginal rates mr_1 and mr_2 in the two periods are given by

$$mr_t = \frac{\pi_H{}^t - \pi_L{}^t}{\pi_H{}^t + \pi_L{}^t}(1+r) \quad for \quad t = \{1, 2\}$$

Taking the policy rate (or risk free rate) as constant, time-varying conditions in the banking sector (e.g. in this model, π_H and π_L) will lead to differing marginal rates across time. As an example, let us consider the case that conditions at time 2 are already known at time 1, and that they are such that $mr_2 > mr_1$.

Suppose that the central bank was interested in smoothing the evolution of the marginal rate over time by using two different policy tools

1. The introduction of a longer term operation that spans both periods
2. The adjustment of the total liquidity supply from period to period

Let us consider the first option. A longer-term operation might be able to provide some stability in interest rates, as it could smooth demand in the one-period operations. To see whether this is the case, assume that total refinancing volume (liquidity supply) S stays constant over the two periods, and that a longer term (two-period) operation is launched with volumes S_L, which partially replaces supply in the one-period operations. Thus, $S_1 = S_2 = S - S_L$.

What is the marginal rate in this longer-term operation? When banks place their bids at time 1, they know that they can either bid in the one-period operations and pay a low interest rate at time 1, but a higher one at time 2. The interest rate they are willing to pay for the two-period operation will just be the weighted average of the two short-term interest rates, and the resulting marginal rate mr_L will satisfy $mr_1 < mr_L < mr_2$.

What is the effect on marginal rates in the one-period operations? Because the longer-term operation will reduce both demand and supply in these operations to the same extent, the effect will be a mere scaling down of the operation. From section 2, we know, however, that a reduced size of the operation can have an augmenting effect on the interest rate. To see this, consider the case that the demand for all banks types (i.e. for all v) is reduced by d.[14] Then,

$$\tilde{mr}_t = \frac{(\pi_H{}^t - d) - (\pi_L{}^t - d)}{\pi_H{}^t - d + \pi_L{}^t - d}(1+r) = \frac{\pi_H{}^t - \pi_L{}^t}{\pi_H{}^t \pi_L{}^t - 2d}(1+r) > mr_t$$

Thus, the introduction of a longer-term operation leads to an *increase* in the marginal rate in both periods. This outcome is of course based on the initial assumption that we start in a situation with an already low volume of the operation, for which $\pi_L<0$. If we

[14] A proportionate decline is not possible in the case of $\pi_L<0$.

considered the case of an appropriate volume in which all banks have a positive demand for liquidity, there would be no effect (or one that is less pronounced on the marginal rate). Thus, when a central bank introduces longer-term operations to accommodate the banking sector's demand for it, it should take care that the volume in the operations of shorter maturity is not too low. In particular, this implies that it may be beneficial not to conduct operations of too many maturities at the same time.

If the central bank was interested in smoothing marginal rates over time, it can instead resort to a different measure: the increase of the total refinancing volume. If the volume increases in times when liquidity conditions in the market are tenser (and liquidity needs are distributed more equally across participants), then the central bank can achieve a better smoothing of interest rates. If paired with an increase in the total volume, the introduction of longer-term operations should have a more limited effect on marginal rates in short-term operations.

4.2 Increasing LTROs, reducing MROs: Implications for MRO rates

In what follows one of the key findings of a recently conducted study on bidding behaviour in the ECB main refinancing operations during the financial crisis (see Eisenschmidt, Hirsch and Linzert, EHL 2009) is discussed. The authors of the paper find, in line with many other studies conducted on the subject, that the (increasing) size of the MRO auction (i.e. the benchmark amount) has a positive effect on marginal tender rates in the pre-crisis period. In previous studies, this effect has mainly been attributed to allotment uncertainty which appears to be increasing in the size of the operation and hence leads to higher bid rates (see, e.g., Valimäki 2008 or Würtz 2006).

At the same time, a new finding of EHL 2009 is that the higher the share of LTROs in total central bank refinancing, the higher will average bid rates (and hence MRO tender spreads) in MROs be.[15] This is an important finding which is relevant for the analysis presented in this paper, because a bigger size of LTROs implies a smaller volume of MROs. Therefore, the results of EHL 2009 can be directly used to "test" the empirical implications of the theory presented in section 4.1.

A banks' bidding behavior can be measured by its participation decision, its individual bid amount, the weighted average bid rate and the bid rate dispersion. Since a bank's

[15] Similarly, Donati (2010) finds that the size of MROs have been positively contributed to money market spreads during the financial turmoil.

bid amount or its average bid rate can only be observed if the bank actually participated in the MRO, the estimation may be subject to a selection bias, see Heckman (1979). Accounting for banks' participation decision, EHL 2009 employ panel sample selection estimation technique, which extent the cross sectional Heckman approach to the panel case.

The main focus of EHL 2009 are the determinants of banks’ bid rates in order to explain the sources behind higher tender rates during the turmoil. One may recall that the spread of the marginal bid rate of all banks over the minimum bid rate increased from 5 bps before the financial crisis to 16 bps, on average, in the period from August 2007 until October 2008.

EHL 2009 estimate a panel random effects regression explaining an individual bank’s bid rate (as the spread over the minimum bid rate) using a set of bank and auction specific characteristics as well as variables characterizing money market conditions. The estimations are conducted on a sample from April 2004 to October 2008, while the crisis refers to the period from August 2007 to October 2008. The sample ends with the introduction of fixed-rate tender with full allotment in all ECB refinancing operations on 14 October 2008.

Statistically, the turmoil period is accounted for by including a respective dummy variable from August 2007 onwards, and allowing this dummy to interact with all the explanatory variables in the model.

Table 1 in the appendix contains the regression results of EHL 2009. The first column contains the parameter estimates for the pre-turmoil period, spanning April 2004 until 7 August 2007. The second column reports the joint parameter estimates for the crisis (14 August 2007 until 7 October 2008) and the pre-crisis period. This joint effect has two components, the parameter estimate for the pre-crisis period, if significant, added to the parameter estimate for the crisis period (also only if significant).

If in the crisis period the statistical relationship between the dependent variable and explanatory variable did not change, the reported coefficients would be the same for both periods. On the other hand, if the explanatory variable was not significant in the pre-crisis period, an entry is still shown in the table but the joint effect will only consist in the (statistically significant) parameter estimate for the crisis period. The relavant parameter estimate, the expected size of the LTRO operation outstanding over the week following the MRO, is reported in table 2 below.

Variable	Pre-crisis	Crisis
Expected LTRO amount (log), β_1	0.0282	0.1508**

Table 2: LTRO-size related parameter estimate of the EHL 2009 model, the 95% significance level is depicted by **

The average volume of outstanding LTROs increased by about 120 billion to 270 billion euro from August 2007 to October 2008. Using the estimate of EHL2009, the shift to LTROs explains around 6.5bps of the total increase of around 16bps in average tender rates during the crisis period.[16] The panel regression of EHL2009 controls for a variety of other factors that also were shown to have driven up average bid rates. Among those, however, the LTRO variable is the second most important in quantitative terms.[17] Further, a variety of robustness checks was conducted, with the LTRO variable preserving sign and significance. From the empirical side, therefore, there are strong indications that the maturity shift of the ECB's refinancing operations may have come at the cost of higher tender rates in the standard main refinancing operations. This is in line with the simple extension of our theoretical model presented in section 2.

5 Conclusion

This paper argues that there is a minimum size limit for a central bank's open market operations needed to guarantee well-behaved auction rates, which is dependent on conditions in the money market. It develops arguments in favour of making the refinancing volume of a central bank dependent on the ability of the money market to distribute liquidity across banks. A situation is analysed in which a central bank

[16] To arrive at this number, divide the coefficient by 270 and multiply by 120, the result is in basis points, which is the specification of the dependent variable of EHL2009.
[17] Another important variables which EHL2009 show to have driven up average bid rates in MROs is the relative attractiveness of the collateral framework. Further, in another analysis on the same subject conducted by Cassola, N, Lazarov, V. and O'Brien, E. (2008), increased credit risk is found to have driven up tender rates.

conducts open market operations as variable rate tender auctions, and is concerned about possible deviations of the marginal tender rate at these auctions from its target rate. Such deviations may increase as result of imperfections in the interbank market, if the volume of open market operations is not adjusted upwards accordingly.

In particular, the analysis demonstrates that for larger refinancing volumes, the presence of imperfections in the interbank market does not have an effect on the marginal rate at the central bank auction (it affects intra-marginal bid rates, but not the marginal rate itself). The same is true in case the proportion of banks facing troubled in the interbank market is small.

If, however, the size of the operation becomes too small, then demand for central bank funds becomes dominated by banks with high liquidity needs who want to avoid having to pay a large rate in the interbank market. In this case, there is an upward effect on the marginal rate. The premium banks are willing to pay reflects banks' problems they encounter in the interbank market. It is shown that it is not the amount of liquidity supplied per se which is of importance, but rather its relation to the dispersion of liquidity shocks.

We conclude that the size of the refinancing volume of a particular central bank operation may have an impact on rates in this auction. In particular, we have demonstrated that for very small volumes, there can be upward pressure on interest rates. Of course, it does not follow from the analysis that the overall relationship between bid rates and refinancing volumes is necessarily negative, as we have only modelled a particular feature of bidding behaviour that is most likely to be observed for smaller volumes. Also, we have abstracted from other determinants of bidding rates, such as collateral issues or possible market manipulation. The present analysis instead establishes a lower bound for the amount of refinancing a central bank may wish to provide to the banking sector for each maturity bracket.

The empirical results broadly support these findings. Using the experience of the period of fixed rate tender with full-allotment and ranking all participating banks by the degree of change of their demand behaviour relative to their own past, possible measures of the minimum liquidity deficit were derived. To this end, the amount of Eurosystem refinancing demand displayed by the top 25% "demand changing" banking groups was used. Contrasting this amount with the liquidity deficit from before the start of the financial crisis in August 2007 shows that the EUR 450 billion liquidity deficit may be sufficiently large to absorb the needs of the aforementioned top 25% banking groups.

The paper also considered the issue whether the introduction of longer-term refinancing operations may help to smooth interest rates at open market operations over time. This was found not to be the case. Instead, when introducing operations with longer maturities, a central bank should take care that the volume in the shorter operations does not become too small relative to the diversity of banks, in order to avoid that an increase in interest rates is caused. Using the results from a recently conducted study on bidding behaviour in MROs of the Eurosystem during the financial crisis, it is shown that the increase of LTRO volumes at the expense of MRO volumes contributes to an average increase of 6.5bps in average MRO bid rates. Therefore, a central bank may, instead of shifting the maturity structure of its operations, opt to increase the total refinancing volume to banks. This would lower marginal interest rates, and leave room for the introduction of more operations with different maturities.

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