Voluntary cooperation in terms of international financial supervision*

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Abstract

This paper analyses the issue of voluntary cooperation in terms of international financial supervision. We provide a simple modelling framework where financial supervision is an international public good and thus may be underprovided globally. We ask a simple question: would national supervisors cooperate and increase the level of global supervision, and by how much? We use coalition formation game theory to address this question. The main results are the following. If the situation is completely symmetric (identical sized countries and symmetric externalities), the amount of cooperation is relatively high and full cooperation could be achieved for particular numbers of the countries involved in the negotiations. However, in general full cooperation would not be an equilibrium because some countries have incentives to free ride on the cooperation of other countries. Introducing asymmetries in the size of the countries and/or in the externalities between countries reduces the scope for cooperation. Moreover, no-cooperation at all could emerge in equilibrium if asymmetries are sufficiently high. Nevertheless, cooperation would not be ruled out when the distribution of asymmetries has a particular shape, such that big countries are generating large asymmetric externalities on small countries, as it might be the case in reality.

Keywords: Financial supervision, Financial integration, Coalition formation, Cooperation

JEL Classification: C71, C72, F42, G15, H87

1 Introduction

One lesson from the global financial turmoil of August 2007 was clearly the deficiency of financial supervision in an increasingly integrated global financial system. The financial crisis started in the United States and quickly spill over in the rest of the world implying significant damage to the world economy (GDP growth forecasts were updated downwards all over the world after the crisis). Several major banks have defaulted and many distinguished economists wrote tribunes in financial newspapers trying to explain what was happening. For example, Paul de Grauwe (Financial Times, March 19 2008) stated:

*The views expressed here are those of the authors and do not necessarily reflect those of the Banque de France.

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> A massive overhaul of supervision and regulation of the financial system will be necessary, especially in the US, where a religious belief in the infallibility of markets has led regulatory authorities, especially the Fed while Alan Greenspan was chairman, to abdicate their responsibility of supervising and regulating markets.

In this paper we provide a simple modelling framework giving explanation to the fact that the amount of financial supervision supplied by national authorities would be insufficient in a global financial system. In fact, the level of financial supervision is sub-optimal because supervision has the feature of an international public good (Schinasi, 2007). However, here we show that this sub-optimality is (partly) offset by the incentives to cooperate and form coalitions in order to exploit mutually beneficial agreements (Ray and Vohra, 2001). In an environment allowing for such voluntary cooperation among countries[1], the fundamental question is then: *would supervisors cooperate and by how much*? We answer this question by using the theory of *endogenous formation of coalitions* developed by game theorists (Hart and Kurz, 1983, Bloch, 1996). We show that partial cooperation will emerge in equilibrium without any transfer being implemented among states (non-transferable utility, NTU). We find that if the situation is completely symmetric (countries have identical size and externalities between the countries are symmetric), the amount of cooperation is relatively high and full cooperation could be achieved for particular numbers of the countries involved in the negotiation. In general however, full cooperation would not be an equilibrium because there exist incentives to free ride on the cooperation of the others. We also show that introducing asymmetries in the game reduces the scope for cooperation and a fully inefficient coalition structure (that is, no-cooperation at all) could emerge in equilibrium if asymmetries are sufficiently high. More interestingly however, we find that cooperation would not be ruled out when the distribution of asymmetries has a particular shape, such that big countries are generating large asymmetric externalities on small countries, as it might be the case in reality.

The formal model is based on Ray and Vohra's (2001) public good coalition formation game. We adapt their model to the framework of international financial supervision. Compared to them we introduce non-transferable utility (NTU) and asymmetries, consisting in the introduction of heterogeneous sized countries and asymmetric externalities between the countries. In contrast with Ray and Vohra, we find that the fully inefficient structure of singletons could emerge in equilibrium *if asymmetries are sufficiently high*. We also find that smaller countries and countries that are net exporters of externalities will have greater incentives to free ride.

[1] We believe that in the European Union for instance such environment prevails by the possibility to write *Memoranda of understandings* and *Voluntary Specific Cooperation Agreements* among EU Supervisors. At a more global level, international bodies such as the FSF, IMF, BIS, OECD, G7 or G20 allow for cooperative action among national supervisors.

The paper is organised as follows. The next section presents the basic model and definitions. Section 3 presents the characterisation of the cooperation structure that will emerge in equilibrium. Section 4 presents a simulated game for G7 countries. Section 5 discusses some policy implications of the model.

2 The model

This section aims at providing a simple model of the political economy of financial supervision. We build on Ray and Vohra (2001) who analyse a game of pollution control. Basically, in this paper pollution could be thought as a financial crisis and pollution control could be considered as financial supervision. We slightly modify the model of Ray and Vohra by adapting it to the specific case of financial supervision discussed in this paper. However, the main mechanisms are preserved and we refer the reader to their paper for an excellent methodological discussion of the model.

The world (continent, etc.) is represented by a set of regions (or countries) denoted $N = \{1, \ldots, n\}$. Each country could produce a "public bad"—a financial crisis—the costs of which spill over to the rest of the world according to an externality matrix A (see below). There is an instantaneous probability, q_i, for financial crisis in country i. This probability is a decreasing function of the amount of financial supervision[2], r_i, supplied by country i, that is, $q_i(r_i)$ with $q' < 0$ and $q'' > 0$. One may ask why the probability q_i does not depend on the entire vector of financial supervision $(r_1, \ldots, r_n)$? More precisely, can financial supervision provided abroad be substituted to financial supervision provided at home? We believe that the answer to this question might be "No". To see this, assume that the amount of financial supervision in the rest of the world is high enough such that the probability for crisis abroad is zero. Would this mean that the home supervisory authority could abstract from supervising the home financial system? Clearly No, because the probability of default of the home system is directly related to the efforts of supervision of the home authority, which is the only one having control over the financial institutions located in the home jurisdiction. This is an important feature of the model as it will permit to have an additive representation of external effects.

A financial crisis is associated to a country-specific loss noted D_i. This summarizes mainly the slowdown of GDP growth and the fiscal costs of cleaning up the system[3]. We assume that these costs can be expressed as a fraction of country's GDP, that is $D_i = \beta_i GDP_i$ (with

[2] Financial supervision is then a public good.

[3] For instance, the fiscal costs associated to a financial crisis can be very large. The fiscal costs of cleaning up after Sweden's 1992 banking crisis were estimated to 6 percent of GDP, after Norway's 1987 crisis to 8 percent, after Spain's 1977 crisis to 16 percent and estimates for the Japan's bill after 1992 crisis exceed 20 percent of GDP.

$0 < \beta_i < 1$). On the other hand, producing financial supervision has a fiscal cost that is expressed by a cost function $c_i(r_i)$, with $c' > 0$ and $c'' > 0$.

The objective function of a country is written simply as the expected cost of a crisis plus the current cost of financial supervision:

$$L_i = q_i(r_i)D_i + \sum_{j \in N \backslash i} \alpha_{ij} q_j(r_j) D_j + c_i(r_i). \tag{1}$$

Note that, because countries are integrated, a crisis abroad damages the home country. This is taken into account by the parameter $\alpha_{ij} \in [0, 1]$ measuring the spill over (or externality) from country j to country i. The global map of externalities can be represented by the matrix:

$$A = \begin{pmatrix} 1 & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & 1 & \dots & \alpha_{2n} \\ \vdots & & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & 1 \end{pmatrix} \tag{2}$$

The parameter α_{ij} will be typically positive when there exist cross-boarder financial investments between countries i and j, e.g. large cross-boarder banks, but also because of trade links between the countries transmitting macroeconomic shocks. In particular, the parameter α_{ij} will be larger for highly economically integrated countries (such as the Member States of the EU). A parameter equal to 1 accounts for complete integration. This is trivially the case inside a country ($i = j$). We assume that integration is incomplete across countries, that is $\alpha_{ij} < 0$ for $i \neq j$. In general it could be that $\alpha_{ij} \neq \alpha_{ji}$ (asymmetric integration).

It would be more convenient to rewrite the model in terms of benefits rather than losses. Let $p_i(.) \equiv 1 - q_i(.)$,with $p' > 0$ and $p'' < 0$, be the probability to avoid a financial crisis (i.e. to preserve financial stability). We can rewrite the loss function as:

$$L_i = \text{Const}_i - \underbrace{\left[p_i(r_i)D_i + \sum_{j \in N \backslash i} \alpha_{ij} p_j(r_j) D_j - c_i(r_i) \right]}_{V_i}$$

The minimization of L_i is then equivalent to the maximization of V_i, which represents the benefit (or payoff) function.

While the benefit of each country depends on the amount of financial supervision provided at home *and* in the rest of the world, a country has control only on the amount of supervision provided by its national supervisory authority. Then, for example, if country i observes that the amount of supervision in country j is low and if i receives a large externality from j, then i would feel some pressure to negotiate an agreement with country j, such that j's supervision is improved. Then, some form of coordination-cooperation between countries would emerge.

Before turning to the analysis of this coordination-cooperation, we study the benchmark case of autarky (i.e. $\alpha_{ij} = 0$ for all $i \neq j$), which can be seen as the initial state of the global system in a story of financial integration.

2.1 Autarky

The objective of each national supervisor is to maximise his payoff:

$$\max_{r_i} \{V_i = p_i(r_i)D_i - c_i(r_i)\}. \tag{3}$$

This gives the standard first order condition equating marginal benefit and cost (of supervision):

$$p_i'(r_i)D_i = c_i'(r_i). \tag{4}$$

For the sake of the illustration (see Figure 1), let the probability function $p_i(.)$ be the same across all countries as well as the benefit D_i. Then the only difference between national amounts of supervision will be determined by the differences in the marginal costs of supervision, with higher marginal cost implying lower provision of supervision. On the other hand, if we set the marginal cost of supervision to be the same across countries but the benefit of financial stability, D_i, to be proportional to the economic size of the country, then the larger the economy the larger the amount of supervision. This is simply because larger economies have larger financial systems to supervise.

2.2 An integrated financial system

We now turn to the case of an integrated financial system. Formally, we make the following simplifying assumptions. Let the proportionality between the loss associated to a financial crisis and the economic size of a country be the same across all countries, so that heterogeneity in economic size is accounted by D_i ($D_i = \beta GDP_i$). Let the cost of financial stability be given by a linear function, $c_i(r_i) = r_i$, identical for all countries. Finally, let the function $p(.)$ be identical across all countries (N.B. this does not mean that the probabilities for crisis are identical across countries).

We have a well defined game situation where the payoff of one country depends on the action taken by all other countries. The basic equilibrium concept in such a situation is the Nash equilibrium, presuming non-cooperative interaction among the players, that is, the

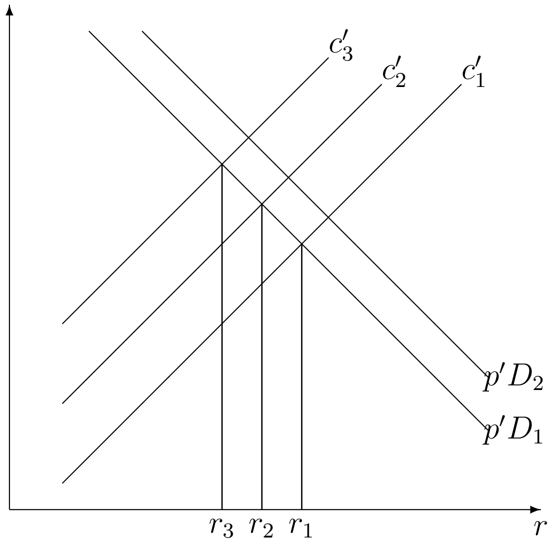

Figure 1: Equilibrium in autarky

action chosen by one individual is *unilateral best reply* to the actions chosen by all other individuals. The maximising problem of the Nash equilibrium is the following:

$$\max_{r_i}\{V_i = p(r_i)D_i + \sum_{j\in N\setminus i} p(r_j)\alpha_{ij}D_j - r_i\}, \quad i = 1,\ldots,n \tag{5}$$

implying the first order condition:

$$p'(r_i)D_i = 1, \quad i = 1,\ldots,n. \tag{6}$$

This condition is the same as in autarky. Thus, the passage from autarky to an integrated financial system would *as a minimum* preserve the amount of financial supervision provided in autarky[4].

However, financial supervision is underprovided at a Nash equilibrium. This is because when optimizing their individual objectives, countries have not taken into account the externalities that their individual provision of the public good implies on the others. Accordingly, a *social planner* would require that every country's provision of financial supervision to be increased. Formally, we have:

[4]This is due to the additivity of external effects specified in the model. If external effects enter the payoff function non-linearly, the passage from autarky to an integrated financial system would be associated to unilateral decreases of supervision because countries would substitute home supervision with foreign supervision.

$$\max_{r_1,\ldots,r_n} \{V_N = \sum_{i=1}^{n} p(r_i)D_i + \sum_{i=1}^{n} \sum_{j\in N\backslash i} p(r_j)\alpha_{ij}D_j - \sum_{i=1}^{n} r_i\}, \tag{7}$$

implying a system of first order conditions:

$$p'(r_i)D_i \sum_{j\in N} \alpha_{ji} = 1, \quad i = 1,\ldots,n. \tag{8}$$

It is easy to see that, for all i, the optimal amount of financial supervision, r_i^O, is higher than the amount of supervision resulting at the Nash equilibrium, r_i^N, (see Figure 2).

This could be seen as a rather pessimistic result. However, non-cooperative game theory and the Nash equilibrium are not the appropriate modelling concepts in an environment allowing for voluntary cooperation among players, which may prevail in the context of financial supervision, particularly for developed countries (G7) or for the Member States of the EU. For instance, the scope for international cooperation in terms of financial supervision has increased after the financial turmoil of August 2007. For example, we can read in the Financial Times (April 3 2008):

> "The present crisis has really sharpened minds of European members about how we would handle a cross border financial crisis", Charles McCreevy, Europe's commissioner for financial affairs told the Financial Times (...) The EU agreement envisages the creation of "cross-border stability groups", voluntary structures bringing together relevant supervisory authorities, central banks and finance ministers.

The study of such a voluntary cooperation in terms of international financial supervision is pursued in the next section.

2.3 Voluntary cooperation in terms of financial supervision

In an environment allowing for voluntary cooperation among countries, the appropriate modelling framework is the one of *endogenous formation of coalitions* (see the seminal papers of Hart and Kurz, 1983, and Bloch, 1996). In this framework, a coalition is defined as a subset of players deciding to maximise their joint payoff (cooperative action). After deciding to form a coalition, the members of a coalition have to decide how the joint payoff will be allocated among them. There are two main branches of cooperative game theory in this respect: (i) *transferable utility* games (TU games) where it is possible to negotiate over the division of the payoff of the coalition, which implicitly assumes that it is possible to implement transfers among players; and (ii) non-transferable utility games (NTU games) where such transfers

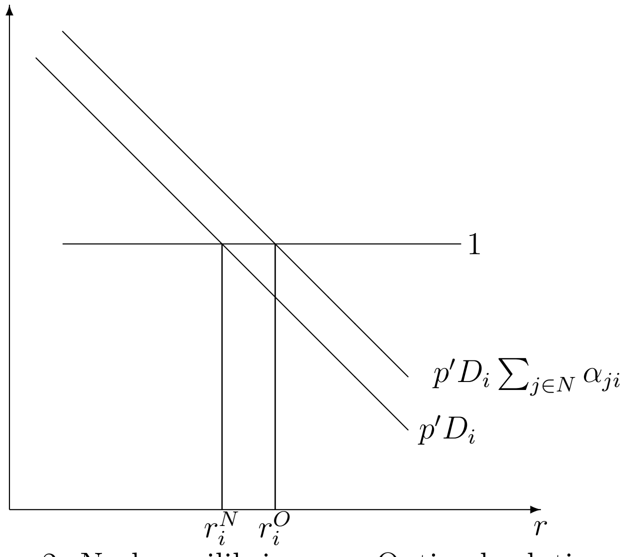

Figure 2: Nash equilibrium vs. Optimal solution

are not allowed and the payoff of the coalition is exogenously (according to the intrinsic characteristics of the players) divided among its members. In the context of international financial stability discussed here, it is more relevant to assume a NTU game because there is no existing institutional framework allowing for cross-border payments among national supervisors and there is little attempt from politicians to implement fiscal transfers among states, even in a highly integrated system such as the EU.

A second dimension of classification for cooperative games is the presence or not of *externalities* among coalitions. Without externalities, the payoff of a coalition depends only on the composition of the coalition. In the presence of externalities, the payoff of a coalition depends not only on the composition of the coalition but also on the composition of all other coalitions, that is on the entire coalition structure. Clearly, in the framework discussed here we have a game with externalities. To give an example, if France and Germany decide to cooperate by mutually enhancing their financial supervision this will benefit all other countries that have economic involvement with France and Germany (i.e. presence of positive externalities). On the other hand, it is not possible for the French government to finance the German supervisory authority or vice versa (i.e. non transferable utility). To summarize, we build a NTU coalition formation game with externalities which to our assessment accounts at best the existing institutional framework in terms of international financial supervision.

Now we specify the protocol of coalition formation. In this respect, the theory of endogenous coalition formation can be classified in two main fields: (i) simultaneous formation of coalitions (Hart and Kurz, 1983) where players simultaneously announce lists of partners with whom they would like to form a coalition (matching the lists according to a given rule determines the coalition structure); and (ii) sequential formation of coalitions (Bloch, 1996, Ray and Vohra, 1999) that mimics Rubinstein (1982)'s offer-counteroffer bargaining process where players move in a predetermined order and propose coalitions that can be accepted (then retiring from the game) or rejected (then followed by a counteroffer). Based on informal evidence about the conduct of international negotiations where an initiating country usually proposes an agreement that may be rejected or amended by the others, we assess that the sequential model of coalition formation is more relevant for the issue discussed here.

The coalition formation protocol is described simply as follows (see also Bloch 1996). Imagine players are grouped in a room where they are ordered according to an exogenous rule of order, noted ρ, (for example according to the alphabetical order of the countries). The first player in this order proposes a list of partners with whom he would like to form a coalition (he naturally includes himself in the list). Formally, this list is a subset of players (a coalition), noted S. All players in the list then sequentially respond to the proposal. On one hand, if *all* players in S accept the proposal, coalition S forms and leaves the room. The game continues among the remaining players in the room, $N \setminus S$, with a rule of order ρ updated for the exit of S. The first player in $N \setminus S$ then makes a proposal to the players present in the room, etc. On the other hand, if one player in the list rejects the offer, the coalition is not formed and the player who has rejected makes a counteroffer (that is, becomes the proposer) and the game goes to the next round. Note that in this protocol, if a player does not want to form any coalition, he will (when having the right to move) propose only himself as a partner and thus exit the game. We say that he remains singleton in that case.

We do not assume discounting of time but require that the game could not be played infinitely by normalising to zero the payoff of each player in that case. Because all players have a positive payoff when being member of a certain coalition, this guarantees that they will not bargain at infinitum. Then, at the end of the coalition formation process, all players have leaved the room, that is, they belong to a coalition (that may well be a singleton). Outside the room we have a partition of the set of players into coalitions, called a coalition structure. Next, coalitions (being considered as single players) play among them a non-cooperative game, that is, they chose strategies that are best replies to other coalitions' strategies. This gives a payoff to each coalition which is exogenously (NTU assumption) divided among members. As a result, before coalition formation is played, each player can calculate his individual payoff corresponding to a given coalition structure. This situation describes a large non-cooperative game where each plays seeks maximising his individual payoff by trying to influence the coalition formation process in such a way that his preferred coalition structure is obtained. A strategy for player i in this game is to propose a coalition S (when he is a proposer), or to respond 'Yes' or 'No' (when he is a responder).

3 Equilibrium coalition structures

With all this in hand, we can study the coalition structure that would emerge in equilibrium. To do this we have to specify the equilibrium concept. It is standard in the literature of sequential formation of coalitions to use the stationary perfect equilibrium concept (SPE). This equilibrium concept restricts attention to stationary strategies, that is, strategies depending only on the current state of the game[5] but not on the entire history of the game. The concept also uses the notion of subgame perfection requiring that there exists no history of the game at which a player benefits from a deviation from his stationary strategy[6]. Without restricting attention to stationary strategies, the set of equilibrium strategies in a sequential game can be very large (see Chatterjee et al., 1993, Sutton, 1986) and support virtually every end of the game as an equilibrium outcome.

When a given coalition S forms, it maximises the joint payoff of all its members, that is:

$$\max_{r_i \in S} \{V_S = \sum_{i \in S} p(r_i) D_i + \sum_{i \in S} \sum_{j \in N \setminus i} p(r_j) \alpha_{ij} D_j - \sum_{i \in S} r_i\}, \tag{9}$$

implying the first order condition:

$$p'(r_i) D_i \sum_{j \in S} \alpha_{ji} = 1, \quad i \in S. \tag{10}$$

We first note that the optimal amount of supervision decided by a coalition does not depend on what other countries are doing (N.B. this does not mean that the payoff of a coalition does not depend on the action of the other countries). This follows from the additivity of external effects[7]. Second, *the formation of the grand coalition* (the coalition of all players, N) will guarantee the provision of the optimal amount of global financial supervision. In other words, any coalition structure different from the grand coalition will result in an amount of global supervision lower than the optimal amount. Moreover, only the coalition structure of singletons will be equivalent to the non-cooperative Nash equilibrium outcome. Thus, we have two benchmark coalition structures in the game: (i) the grand coalition corresponding to the fully efficient outcome and (ii) the coalition structure of singletons corresponding to the fully inefficient outcome.

[5]In the model, the current state of the game is described by the coalitions formed outside the room and the players still present in the room.

[6]A subgame perfect equilibrium is a refinement of the Nash equilibrium used in sequential games. A strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game.

[7]See Ray and Vohra (2001) Section V for a discussion how this property influences the results.

3.1 Fully symmetric case

To begin the characterisation of equilibrium structures, we first consider the fully symmetric case (that is, all countries have identical sizes; and $\alpha_{ij} = \alpha$ for all $i \neq j$, i.e. externalities are symmetric and uniform across all peers of countries.). For simplicity we normalise $D_i = 1$ for all $i \in N$. As countries are completely identical, the only fact that matters for the characterisation of the coalition structure is the size of the formed coalitions (but not their composition). As in Ray and Vohra (2001) we will represent the coalition structure by the partition of the integer n into a sequence of ordered positive integers $t = (t_1, \ldots, t_m)$ with $\sum_{i=1}^{m} t_i = n$ and $t_i \leq t_{i+1}$. For example, in a three-person game, $(1, 2)$ denotes a partition where one player stays alone and the other two players form a coalition.

To ease the analytics, we specify the function $p(.)$ as $p(r) = 2r - r^2$ (this allows us to generate numerical examples but has no impact on the qualitative results of the model)[8]. We therefore can derive precisely the amount of supervision supplied by a country when being member of a s-size coalition[9] :

$$r(s) = 1 - \frac{1}{2[1 + (s-1)\alpha]} \tag{11}$$

where $s = |S|$ is the size of a coalition. In turn, given a coalition structure $t = (t_1, \ldots, t_m)$, the individual payoff for country i is uniquely determined:

$$V_i(t) = p(r(t_i)) + \alpha \left[(t_i - 1)p(r(t_i)) + \sum_{t_j \in t \setminus t_i} t_j p(r(t_j)) \right] - r(t_i) \tag{12}$$

where t_i is the coalition to which i belongs.

When a player proposes a coalition he might predict the entire structure in which this coalition will be embedded. As suggested by Ray and Vohra (2001) the solution of the game can be found by a backward induction argument: if only two countries are left in the room the problem of prediction is elementary (they either remain singletons or form a two-player coalition); the solution of the two-country case will, in turn, provide predictions for the three-country case, and so on. We illustrate this by a generic example.

Example 1. Fix $\alpha = 0.5$ (this is done without loss of generality). We can compute the numerical expression for the proposer's payoff corresponding to different coalition structures. These are summarised in Table 1. The reasoning is as follows. If we have only two countries

[8]By definition, $q(r) = 1 - p(r) = 1 - 2r + r^2$. This function is such that when $r = 0$ the probability for crisis is 1; when $r = 1$ the probability for crisis is zero; for $r > 1$ we set $q(r) = 0$ to respect the definition of a decreasing function. Consequently, $p(r) = 1$ for $r \geq 1$.

[9]Under our assumptions r will be included between $1/2 \leq r < 1$.

Table 1: Equilibrium coalition structures (symmetric case)

	Structure 1	Structure 2	Equilibrium
$n = 2$	(1,1) 0.63	(2) 0.67	(2)
$n = 3$	(1,2) 1.14	(3) 1.13	(1,2)
$n = 4$	(2,2) 1.56	(4) 1.60	(4)
$n = 5$	(1,4) 2.17	(5) 2.08	(1,4)
$n = 6$	(2,4) 2.59	(6) 2.57	(2,4)
$n = 7$	(1,2,4) 3.059	(7) 3.063	(7)
$n = 8$	(1,7) 3.70	(8) 3.56	(1,7)
$n = 9$	(2,7) 4.11	(9) 4.05	(2,7)
$n = 10$	(1,2,7) 4.58	(10) 4.55	(1,2,7)

($n = 2$), the proposer has to compare the payoff in the case where he remains singleton (0.63) to the payoff obtained in a two-player coalition (0.67). The proposer is then better off by proposing the formation of a two player coalition which (by symmetry) is accepted by the other player. Thus, the equilibrium coalition structure is (2). This result is used to compute the equilibrium structure in the three-country case ($n = 3$). If the proposer decides to stay alone, he rationally predicts that the two other players will form a coalition. Then, he only has to compare his payoff under the structure (1,2) with his payoff under the grand coalition (3). Computation shows (see Table 1) that he is better off staying alone and letting the two other players form a coalition. The equilibrium coalition structure is then (1,2). For $n = 4$, the proposer has to compare his payoff under the structure (2,2) and under the grand coalition (4) (a three-player coalition is ruled out from the previous case while a singleton coalition structure is ruled out from the case $n = 2$). The proposer is better off in the grand coalition which is proposed and unanimously agreed. For $n = 5$ the argument is the same as for $n = 3$. The proposer is better off staying alone and letting the others form a four-player coalition. The continuation of the reasoning is shown in the table. Full cooperation emerges again for $n = 7$, $n = 13$, etc. As stated by Ray and Vohra (2001) full cooperation must return (infinitely often) as n varies. These authors also show some general results for the symmetric case that we reproduce here after.

(R1) For any equilibrium structure $t = (t_1, \ldots, t_k)$, where $k \geq 2$, we have $t_i \neq t_j$ for all $i \neq j$. In other words, if the grand coalition does not form in equilibrium we could not have two identical sized coalitions in the equilibrium coalition structure. To see this, assume that in equilibrium $t_i = t_j$ for some $i \neq j$. The two coalitions t_i and t_j can then be considered as two symmetric players in a two-person game where we have shown by the previous example that they are better off forming a coalition among them. This contradicts the fact that t is an equilibrium coalition structure.

(R2) For any equilibrium coalition structure $t = (t_1, \ldots, t_k)$, where $k \geq 2$, we have $t_k > n/2$. In other terms, if the grand coalition does not form in equilibrium the largest coalition will embed more than half of the countries participating in the negotiation. This is an important result showing that a significant amount of voluntary cooperation will emerge in equilibrium. To see this, assume that $t_k \leq n/2$ in equilibrium. Then there exist a subset of players t_p with size equal to t_k such that t_p and t_k are better off forming a coalition among them (according to the previous result). So, there is a profitable deviation for a subset of players which contradicts the fact that t is an equilibrium coalition structure.

(R3) If k is the number of coalitions in the equilibrium coalition structure, then $k < \log_2^n +1$. This result puts a lower limit on the amount of cooperation generated in equilibrium. It follows from the previous one (the formal proof is in the appendix). For example, if we take 27 negotiating countries (to fit the case of the EU), no more than 5 coalitions will form in equilibrium with one big coalition embedding at least 14 countries. In other words, this

Table 2: Two-player asymmetric size case

	$|1|2|$	$|12|$
Big (1)	9.4	9.47
Small (2)	5.2375	5.2167

implies a significant amount of cooperation among countries.

The general result is that the fully inefficient outcome of singletons that corresponds to the non-cooperative Nash outcome would *not* emerge in equilibrium. Moreover inefficiency is bounded and full cooperation emerges for particular values of n. *In other words, full cooperation could be seen as a natural attractor of the negotiation process that is (partly) perturbed by the basic incentives to free ride.*

3.2 Asymmetric size

The previous result could be seen as rather optimistic. However, the fully symmetric case discussed above is highly stylised. It is commonly used in coalition formation game theory for its mathematical tractability. In the real world an important dimension of asymmetry is the heterogeneity in economic size. We now allow for heterogeneous sized countries, motivated by the ongoing political debate among EU Member states where an important issue is how to share the burden of common financial supervision among small and big countries (Goodhart and Schoenmaker, 2006; Schinasi, 2007). Below we show that the introduction of asymmetric size decreases the scope for cooperation. We analyse a series of simple examples.

Example 2. Consider a two-player case ($n = 2$) with one Big country, $D_1 = 10$, and one Small country, $D_2 = 1$. As in the previous example fix $\alpha = 0.5$. We have to change the notation of the coalition structure as coalitions' composition is presently a matter of concern. We denote by $|1|2|$ the structure of singletons and by $|12|$ the structure where player 1 and player 2 form a coalition. The corresponding payoffs for the two players are displayed in Table 2. We can see that the Big country would like to form the grand coalition but this proposal will be blocked by the Small country because cooperation implies a negative gain for the Small country. Thus, the equilibrium coalition structure is the one of singletons, $|1|2|$.

The result is intuitive. The Small country has greater incentives to free ride on the supervision provided by the Big country. If the Small country agrees to cooperate, the cost of increasing his national supervision will be greater than the benefit coming from increased supervision abroad. The Small country is then better off by saving this cost while still continuing to receive (freely) the relatively large benefits from the supervision provided by

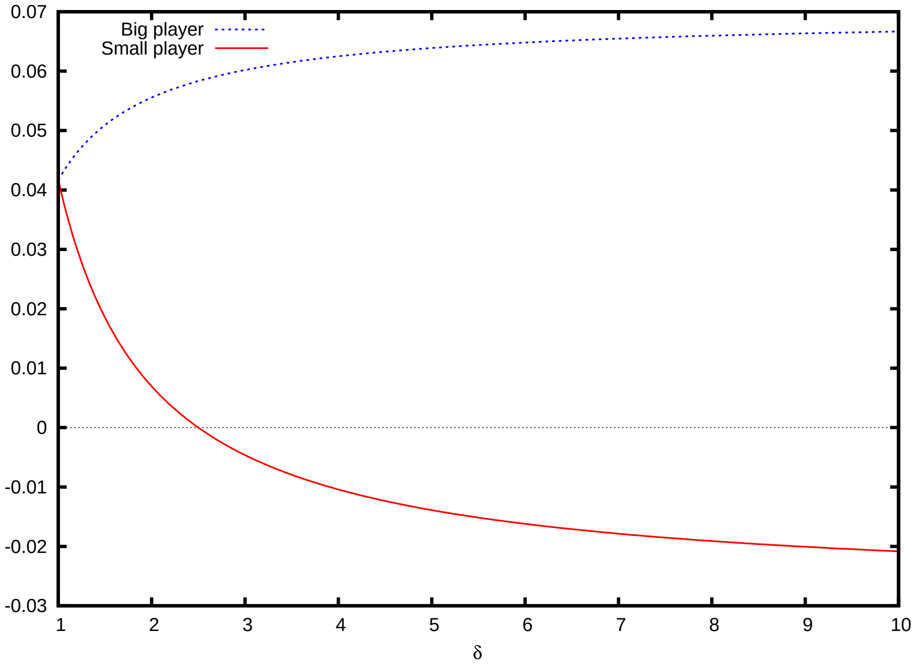

Figure 3: Gain from cooperation as a function of δ

the Big country. In other words, size matters, with big countries naturally protecting the relatively narrow financial system of small countries.

When asymmetries decrease, cooperation will return as confirmed by the two-player symmetric case. Intuitively there might be a threshold such that cooperation emerges if asymmetry is below the threshold and *vice versa*. We explicitly calculate this threshold. We measure asymmetry by the relative size of the two countries noted $\delta = D_1/D_2$. We compute the net gain from cooperation for each player as a function of δ. The result is displayed in Figure 3. As it can be seen, in the case of identical countries ($\delta = 1$) the two countries obtain the same positive gain from cooperation. As the relative size of the big country increases (δ increases), the gain from cooperation increases for the Big country while it decreases for the Small country. It turns out that cooperation is no longer profitable for the Small country when the ratio of asymmetry exceeds a value of 2.5. Let note the value of this threshold by $\bar{\delta}$.

The result obtained for the two-player asymmetric case can be used to assess the general n-player asymmetric case. The fact that a significant asymmetry in the distribution of country sizes can generate no-cooperation in equilibrium is preserved. For instance, if the relative size of every peer of countries (D_i/D_j with i bigger than j) exceeds $\bar{\delta}$ obtained previously

Table 3: Three-player asymmetric size case

	$\|1\|2\|3\|$	$\|12\|3\|$	$\|1\|23\|$	$\|123\|$
Big (1)	9.8611	9.8812	9.9537	9.9792
Medium (2)	6.9445	6.9429	7.0046	7.0278
Small (3)	6.1945	6.2253	6.1898	6.1736

then the fully inefficient singletons coalition structure would emerge in equilibrium. This is illustrated by a simple three-player example.

Example 3. Consider a three-player case ($n = 3$) with one Big country, $D_1 = 9$, one Medium country, $D_2 = 3$, and one Small country, $D_3 = 1$; ($\alpha = 0.5$ as previously). Relative sizes are such that $D_1/D_2 > \bar{\delta}$, $D_1/D_3 > \bar{\delta}$ and $D_2/D_3 > \bar{\delta}$, so we must expect that the structure of singletons will emerge in equilibrium. This is confirmed by looking on Table 3 where players' payoffs corresponding to different coalition structures are displayed.[10] Is it easy to see that there could not be an agreement other than the structure of singletons. For instance, if the Big country proposes the grand coalition, the proposal will be blocked by the Small country who obtains his highest payoffs when being alone (either in the structure of singletons $|1|2|3|$ or in $|12|3|$). Then the Small country decides to stay alone whatever the proposal of the others. Consequently cooperation between the Big and the Medium country is blocked by the Medium country. Then the countries end up in the fully inefficient structure of singletons.

On the other hand, cooperation would be possible between peers of countries where the relative size D_i/D_j is less than $\bar{\delta}$. This does not mean that all countries where this condition is respected will cooperate (see the symmetric case analysed in the previous section). This however means that the coalitions that have been formed in the equilibrium structure would be composed by countries with similar sizes. We illustrate this by two more examples.

Example 4. Consider a three-player case ($n = 3$) with one Big country, $D_1 = 10$, and two Small countries, $D_2 = D_3 = 1$; ($\alpha = 0.5$ as previously). Players' payoffs corresponding to different coalition structures are displayed in Table 4. Cooperation is not possible between one Small country and the Big country (shown previously). On the other hand, cooperation is possible between the two Small countries. Looking on countries' payoffs we see that the two Small countries will in fact cooperate (they obtain a higher payoff under $|1|23|$ than under $|1|2|3|$). However, they will refuse the formation of the grand coalition that the Big country might propose. The equilibrium is then $|1|23|$.

The next example is more interesting and will also be analysed in the case of renegotiation presented in the next section. We have two Medium and one Big country such that

[10] The structure where the Big and the Small country form a coalition ($|13|2|$) is not displayed as it is always blocked by the Small country (see the two-player case above).

Table 4: Three-player asymmetric size case (bis)

	|1|2|3|	|1|23|	|123|
Big (1)	9.775	9.914	9.956
Small (2)	5.613	5.654	5.653
Small (3)	5.613	5.654	5.653

Table 5: Three-player asymmetric size case (ter)

	|1|2|3|	|12|3|	|1|23|	|123|
Big (1)	13.975	13.986	14.001	14.006
Medium (2)	11.513	11.514	11.521	11.528
Medium (3)	11.513	11.533	11.521	11.528

cooperation is not ruled out for any peer of countries. This offers the possibility to form the grand coalition in equilibrium. This however will not be the case here. The reason is the same as in the fully symmetric three-player case: there is always one country that has advantage to stay alone letting the two other players cooperate.

Example 5. Consider a three-player case ($n = 3$) with one Big country, $D_1 = 10$, and two Medium countries, $D_2 = D_3 = 5$; ($\alpha = 0.5$ as previously). Players' payoffs corresponding to different coalition structures are displayed in Table 5. The result is driven by the fact that a Medium country has an advantage to remain alone anticipating that the two other countries will form a coalition among them. We have multiple equilibria depending on the identity of the first proposer. If country 3 is the proposer the equilibrium is |12|3| and by symmetry if country 2 is the proposer the equilibrium is |13|2|. If the Big country is the proposer it will decide to stay alone by anticipating that the grand coalition (which is the best structure from the point of view of the Big country) will be blocked by one Medium country. The Big country then compares his payoff under |12|3| and |1|23| and decides to stay alone, letting the two Medium countries forming a coalition among them. As a result, the equilibrium is |1|23|.

These examples suggest a general result: *introducing heterogeneity in size reduces the scope for cooperation.* The main reason is that smaller countries obtain large benefits from the supervision provided by bigger countries and, as a result, have greater incentives to free ride. More interestingly, *the fully inefficient Nash outcome could result in equilibrium*, which we have shown to be impossible in the fully symmetric case. However, this will be the case under very strong assumptions about the asymmetry of country sizes that are virtually implausible in reality. Under some normal distribution of country sizes, partial cooperation will emerge in equilibrium between countries with similar sizes.

Table 6: Geographical breakdown of intra euro area exports (as a share of total)

	DE	FR	IT	ES	NL	BE	AT	GR	IE	FI	PT	LU
DE	1	0.24	0.16	0.11	0.15	0.12	0.13	0.02	0.01	0.02	0.02	0.01
FR	0.30	1	0.18	0.19	0.08	0.14	0.02	0.02	0.02	0.01	0.03	0.01
IT	0.30	0.27	1	0.16	0.05	0.06	0.05	0.05	0.01	0.01	0.03	0.00
ES	0.20	0.33	0.15	1	0.06	0.05	0.01	0.02	0.01	0.01	0.16	0.00
NL	0.40	0.15	0.09	0.06	1	0.21	0.02	0.01	0.02	0.02	0.01	0.01
BE	0.31	0.27	0.08	0.06	0.19	1	0.02	0.01	0.01	0.01	0.01	0.03
AT	0.59	0.08	0.17	0.05	0.04	0.03	1	0.01	0.01	0.01	0.01	0.00
GR	0.33	0.11	0.28	0.10	0.07	0.04	0.02	1	0.01	0.02	0.02	0.00
IE	0.21	0.15	0.10	0.08	0.11	0.31	0.01	0.01	1	0.01	0.01	0.00
FI	0.36	0.12	0.11	0.08	0.16	0.08	0.03	0.02	0.02	1	0.02	0.00
PT	0.22	0.19	0.07	0.37	0.06	0.06	0.01	0.01	0.01	0.01	1	0.00
LU	0.30	0.25	0.10	0.07	0.06	0.14	0.03	0.01	0.01	0.01	0.02	1

3.3 Asymmetric externalities

So far, economic and financial externalities between countries, measured by the parameters α_{ij}, have been assumed to be symmetric across countries. That is, the externality that country i implies on country j is the same as the externality implied by j on i. This is however not the case in general and in realty. For example, we display hereafter one possibility to approximate the externality matrix in the case of euro area countries, based on average intra euro area exports between 2000 and 2007.

For instance, the share of intra euro area exports of France going to Germany is 30% while the share of German exports going to France is 24%. That is, the economic (here only exports measured) externality of Germany on France is larger than the externality of France on Germany. The asymmetry is even more visible in the case of Germany-Luxembourg where Germany brings a 30% exports externality on Luxembourg, while Luxembourg only brings a 1% exports externality on Germany.

In this section we analyse the impact of such asymmetric externalities on the size and scope for cooperation. As previously, we start by studying a simple two-player example. Reconsider the case of two identical sized countries ($D_1 = D_2 = 1$) but now the externalities matrix is given by:

$$A = \begin{pmatrix} 1 & 0.1 \\ 0.9 & 1 \end{pmatrix} \tag{13}$$

That is, the externality that player 2 brings on player 1 (α_{12}) is 0.1 while the externality that player 1 brings on player 2 (α_{21}) is 0.9. In other words, player 1 is a *net exporter of*

Table 7: Two-player asymmetric externalities case

	|1|2|	|12|
Player 1	0.325	0.273
Player 2	0.925	1.086

externalities. The payoff matrix of this game is displayed in Table 7. Player 1 would refuse cooperation because in the cooperative solution he would be obliged to increase considerably his level of supervision, this being extremely costly for him.

On the other hand, we have shown previously that cooperation is an equilibrium in the symmetric case ($\alpha_{12} = \alpha_{21} = 0.5$). Then, by continuity it must be that cooperation between the two countries disappears as the asymmetry in externalities exceeds a certain threshold. We can show this numerically by defining a new variable, x, being the deviation from the symmetric case and then computing the gain from cooperation as a function of x:

$$A = \begin{pmatrix} 1 & 0.5 - x \\ 0.5 + x & 1 \end{pmatrix} \tag{14}$$

The result is displayed in Figure 4. When $x = 0$ (the symmetric case) both players have a positive gain from cooperation. As x increases, Player 1 becomes more and more a net exporter of externalities and then has less incentives to cooperate.

This result can rationalise the behaviour of the so called *non cooperative jurisdictions*. These are typically small countries hosting large investments from the rest of the world, implying large financial externalities on the rest of the world. Thus, these countries combine precisely the features—small size and net exporters of externalities—that reduce the most the incentives to cooperate. In this respect, our analysis points out to the difficulties to incite non cooperative jurisdictions to cooperate as their behaviour is the result of powerful equilibrium forces. One way to do this is to implement punishments for non cooperative countries, which in our model could be viewed as increasing the externality that the rest of the world could have on those jurisdictions.

On the other hand, a small open economy would be a net importer of trade-related externalities given its large dependence on the economic conditions in major economies. This will then increase the incentives to cooperate for small open economies, which would contribute to offset the incentives to free ride for small sized countries pointed out in Section 3.2. Then, if asymmetries in size and externalities are both high, this does not necessarily mean that cooperation between the two countries can be ruled out. It will actually depend on the distribution of asymmetries between the countries. This line is investigated in the next subsection.

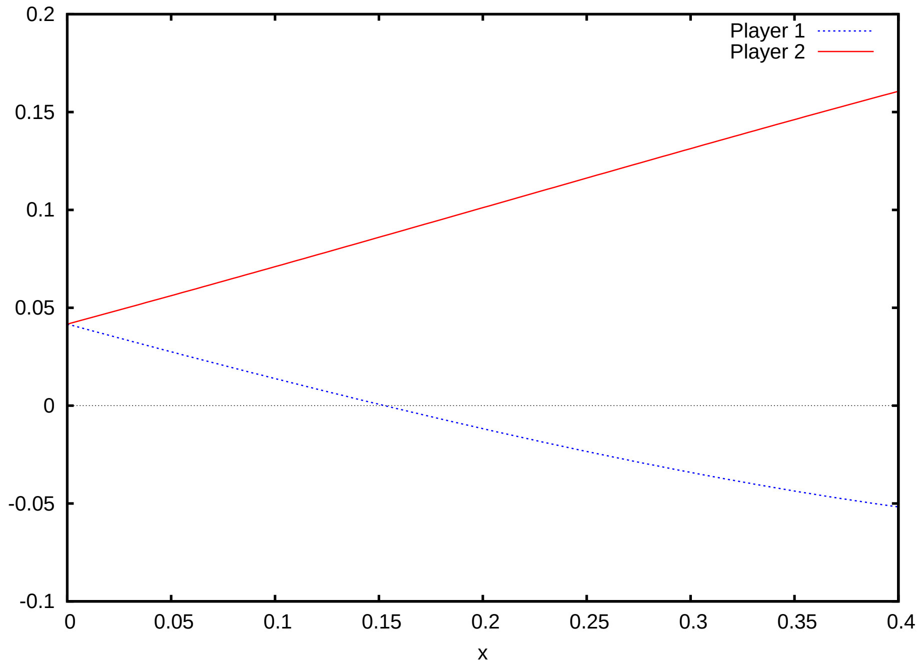

Figure 4: Gain from cooperation as a function of x

3.4 Both asymmetries

If large countries are net exporters of externalities and small countries net importers of externalities (as it might be the case if we consider trade-related externalities as presented in Table 6) then cooperation would be possible even in a very asymmetric situation. This is confirmed by a numerical analysis of the two-player case where we allow both asymmetries—size and externalities—to vary simultaneously.

We compute the gain from cooperation for each player (the difference between the payoff in the grand coalition and the payoff in the structure of singletons) as a function of both δ and x. To do this we define $F_i(\delta, x) = V_i(|12|) - V_i(|1|2|)$ to be the net gain from cooperation for player i. We then construct a function $F(\delta, x) = \min[F_1(\delta, x); F_2(\delta, x)]$, such that cooperation will emerge when F is positive. Indeed, when F is negative, at least one of the two players has a negative gain from cooperation and thus refuses to cooperate. Accordingly, we are interested by the values of δ and x for which F will be positive. We plot F in Figure 5 where δ vary from 1 to 100 and x from 0 to 0.5. The region of cooperation is situated between the two zero level lines depicted at the bottom of the gird. It is such that increasing asymmetry in externalities requires increasing asymmetry in size for cooperation to be preserved. For instance, if we take a situation where player 1 is 10 times bigger that player 2 (that is, $\delta = 10$) and player 1 triggers an externality of 0.8 to player 2 while player 2 triggers an externality of 0.2 to player 1 (that is, $x = 0.3$), then $F(10, 0.3) > 0$ and the two players will cooperate.

This result sends a rather optimistic message for the emergence of international cooperation as big countries tend to be naturally net exporters of externalities because of the large size of their economy.

4 Calibration and simulation for G7 countries

Under construction

5 Discussion

What fundamentally makes the countries cooperate? The basic incentive is the fact that in an integrated financial system a crisis starting in one country spills over to the rest of the world, which makes the amount of supervision supplied originally insufficient compared to the multiplication of the damage. As countries are aware of this fact (and the subprime crisis has only reinforced this understanding) they will have natural incentives to cooperate and increase their level of supervision. On the other hand, there are also natural incentives to free ride when a public good is provided internationally. The trade-off between these

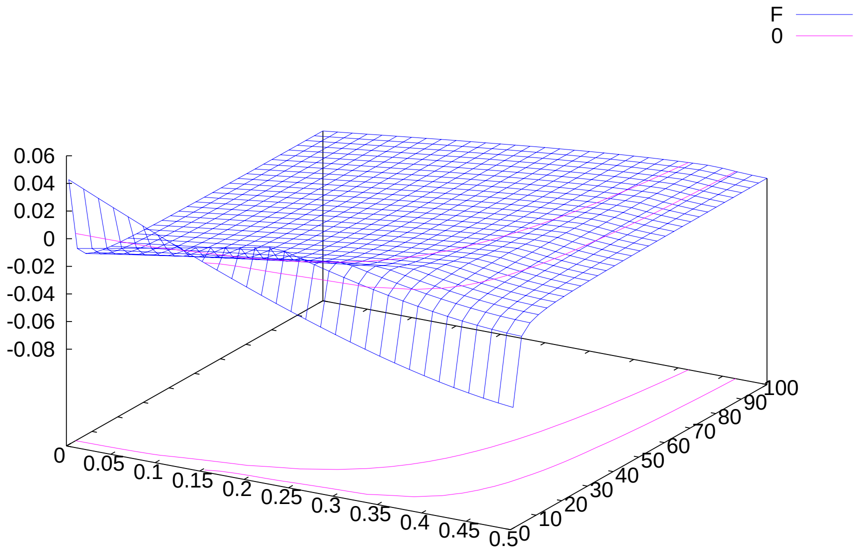

Figure 5: Region of cooperation as a function of δ and x

two forces will generate the emergence of some form of partial cooperation. This process has already started in the European Union with the production of the *Larosière Report* and the large consensus to implement the reforms of cross-border financial supervision that the report is proposing. However, we have shown that this process is not free from obstacles as there may be countries that would not be willing to cooperate: typically, small states that host large international investments.

A precondition for cooperation is to set an institutional framework allowing negotiating and writing agreements. This presupposes the existence of an international body that formally organises the process of negotiations by organising meetings and providing the necessary information. Globally, this role could be played by the IMF or the FSF. In addition, there must be an international body that coordinates actions among the members of an existing agreement, a sort of international supervisory committee that, once again, could be hosted by one of the existing international organisations (IMF, BIS, etc).

To be completed

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Appendix

Proof of R3

Let $(t_1, \ldots, t_k)$ be an ordered equilibrium coalition structure. Define $A_i \equiv \sum_{j=1}^{i} t_j$ as the number of players in a substructure taking the first i coalitions. By R2 we have:

$$t_i > \frac{1}{2} A_i, \quad i = 1, \ldots, k \tag{A-15}$$

Furthermore, $A_{i+1} = A_i + t_{i+1}$. Then,

$$t_{i+1} > \frac{1}{2} A_{i+1} \tag{A-16}$$

$$A_{i+1} > A_i + \frac{1}{2} A_{i+1} \tag{A-17}$$

$$A_{i+1} > 2A_i \tag{A-18}$$

Knowing that $A_1 = 1$ we can obtain by induction that $A_i > 2^{i-1}$ for $i > 1$. Finally, as $A_k = n$ we have $n > 2^{k-1}$ which is equivalent to $k < \log_2^n +1$.