Can Lifecycle Plans Weather the Pension Storm? - A Micro - and Macroeconomic Perspective

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Abstract: To ensure the sustainability of pension systems, many governments are encouraging their citizens to save for retirement through funded DC schemes - both occupational and individual. This new trend in pension reform raises important questions at both the microeconomic and macroeconomic level. In this paper we aim to address two key questions. First, how should individuals invest in asset classes with different risk/return properties over the lifecycle ? Second, given the very large size of pension assets, what are the macroeconomic consequences of these investment strategies? At the microeconomic level, we conclude that lifecycle plans offer attractive features. Looking at the current market offer, these can be strengthened by greater focus on inflation linked bonds. Moreover, more work needs to be done on how to help individuals hedge their longevity risks. At the microeconomic level, the ability to adjust labour supply is of key importance. At the macroeconomic level, we also find lifecycle plans to hold a number of attractions. Flexible human capital (i.e. a flexible labour market) again appears as a key vector, this time of economic growth.

Contents

1 Introduction

In recent years, "a perfect storm" of increased longevity, disappointing capital market returns, low long-term interest rate and a tighter regulatory environment has ravaged the global pension's landscape. As a result, the traditional occupational funded defined benefit (DB) pension plans are being set aside in favour of funded defined contribution (DC) plans. Ageing populations have also eroded the state-run pay-as-you-go (PAYG) pension systems. To ensure the sustainability hereof, governments are now adjusting the plan parameters (retirement age, the level of benefits and the level of contributions) reducing the generosity of state run schemes. At the same time, many governments are encouraging their citizens to save for retirement through funded DC schemes - both occupational and individual. This shift towards funded DC plans raises important questions at both the microeconomic and macroeconomic level.

At the microeconomic level, the shift to funded DC plans entail that the risks previously carried by governments and/or corporations are now borne by the individuals; these include (1) longevity risk, (2) human capital risk, (3) inflation risk and (4)investment risk. Taking account of market information asymmetries and misaligned interests, the risk is that the DC model could result in both expensive and suboptimal choices. The key question is how individuals best can manage these risk, and how governments should regulate to ensure a well functioning pensions market; from the perspective of cost-efficiency, encouraging pension savings (a lack of pension savings is one of the most important issues today) and optimal asset allocation. Zooming in on the question of asset allocation, an important trend in pension reform has been a shift away from quantitative investment restrictions towards a prudent person approach. It is in this context that lifecycle investment strategies have gained ground.

The Lifetime Portfolio Selection (LPS) theory was first explored by Merton [1969] and Samuelson [1969], who initially showed that under several restrictive assumptions, the optimal portfolio structure is independent of age. Later, Bodie, Merton and Samuelson [1992], improved upon the initial framework by introducing human capital as a state variable. They found that the fraction of an individual's wealth optimally invested in equity should decline with age for two reasons: (1) young persons can adjust their labour supply with a greater flexibility and (2) young persons' human capital is greater than that of older persons and is generally less risky than many financial assets. This conclusion appears in line with the simple rule often advanced by financial advisors that an individual's equity exposure should be equal to 100 minus their age (i.e. a 20

In this paper we refer broadly to the lifecycle investment strategies as those strategies where allocation between different asset classes is - at least in part- determined by the investor age. In practic, a wide number of lifecycle funds are available year old should hold 80% of his financial assets in stocks).

After having reviewed the literature on LPS in section 3, we test the robustness of different lifecycle investment strategies proposed by the financial services industry, set in the framework of historical market episodes. The stress-tests approach gives for interesting insight on the two key questions ; 1 does a lifecycle asset allocation strategy offer a superior risk return outcome compared to constant mix portfolio and 2) what is the risk freee asset ? These observations allow us to consider alternative investment strategies, in particular those based on the inflation-linked bonds and other hedging products. The empirical analysis highlights in particular the key role of inflation linked bonds. in view of this result, we propose an extention of the LPS framework in section 5 to take explicit account of inflation.

At the macroeconomic level, pension fund assets are taking on increased importance representing just under 90% of GDP in the OECD in 2005. Moreover, this ratio is set to increase further over the coming years, not only in the OECD but at the global level. Intuitively, the choices in terms of asset allocation (money market, fixed income and equities) made at the microeconomic level must thus have incidence at the macroeconomic level. Indeed, it has been suggested from several sources that the on-going restructuring of DB pension plans in favour of fixed income solutions may be one of the explanations behind the very low level of long-term interest rates and the very flat yield curves on the major government bond markets today.

Assuming that the lifecycle approach becomes widespread in the future, we consider what this would imply in terms of key macroeconomic variables in section 6. In particular, we consider what strategies "excessively" favouring (1) nominal bonds, (2) indexed linked bonds, (3) equity (4) private equity, (4) real estate would entail at the macroeconomic level. In view of this discussion, we address the question of what widespread use of lifecycle strategies would imply and notably whether this would ultimately trigger financial stability as older age cohorts sell equities and buy bonds. We review this question both in the context of a closed economy and open economies with different demographic profiles. We also consider, whether governments should favour issuing more longer dated inflation-linked bonds

The basic model to analyze the impact of the long run on the macro-economy was developed by Solow [1956]. This model links physical capital, labour and technology to output, suggests that slower growth of the labour force as populations age, and as-

Source OECD, Global Pension Statistics, " Evolution of the size of pension funds relative to GDP " suming fixed savings, will result in a lower cost of capital (and hence lower real interest rate) and higher real wage. Diamond [1965] introduced the overlapping generations model (OLG), which rendered the savings decision endogenous, but reached similar conclusions to the Solow model. In our analysis, we apply a simple OLG model to illustrate the impact of the "safe" investment strategy of holding 100% government bonds.

2 The shift to funded DC plans

Funded defined contribution plans have taken centre stage as one of the key building blocks in the redesign of pension systems. Forced to reduce the generosity of PAYG or funded DB pension models, many governments are today encouraging their citizens to save for retirement through funded defined contribution (DC) schemes - both occupational and individual - to compensate for the lesser future pension payments from the state system. Turning to the private sector, corporations are increasingly abandoning the funded defined benefit (DB) plans in favour of DC plans.

According to the OECD pension statistics, total pension assets in the OECD area stood at just under 90% of the OECD in 2005. Given a rapidly ageing population and growing recourse to funding (pension reserves, etc) - not just in the OECD but at the global level - this market is set to grow strongly in the coming years. Moreover, we expect ithe on-going shift from DB to DC to continue; at least for the foreseeable future.

In the following sections, we consider the implications of this shift to DC plans, first at the microeconomic level of the individual plan beneficiary, and next at the macroeconomic level of incidence hereof on the broader financial markets and the real economy.

3 The microeconomic perspective on funded DC plans

At the microeconomic level, the shift to funded DC plans entail that certain risks previously carried by governments and/or corporations are now borne by the individuals; these include (1) longevity risk, (2) human capital risk, (3) inflation risk, and (4) investment risk. Consider each in turn.

Top of the list of risks transferred is **longevity risk**. Longevity risk in retirement is

PAYG systems provide a defined benefit stream and can hence be described as an unfunded (or notional) defined benefit plan (NDB). While a discussion hereof lies outside the scope of this paper, it is interesting to note that in addition to the trend to shift to funded DC plans; notional defined contribution (NDC) plans are also winning ground as a building block for pension reform. The resulting shift in risks shares a number of common points with the discussion above.

a so-called "pure" risk to the extent that it involves a probability of loss (the cost of living longer) but with no chance for a gain (assuming that the individual is no longer earning any labour income). While the "law of large numbers" allows a certain level of predictability of longevity for a total population, at the level of the individual it in near impossible to predict within any meaningful range. Hence, replacing a DB system with a DC system thus removes the "insurance" philosophy of the former.

To the extent that funded DC plans serve to compensate a reduction of state pension benefits that were independent of labour income, the shift will involve a transfer of **human capital risk**. Human capital is here defined as the discounted value of all future labour income. In a funded DC plan, contributions are defined as a share of labour income. As such, if there is a loss of labour income, for example due to a period of unemployment, contributions to the plan and thus future benefits will, all else being equal, be reduced. Note, that to the extent that the benefits of occupational DB plans are defined on labour income, the shift from occupational DB to occupational DC plans does not involve any transfer of human capital risk, which was already carried by the plan beneficiary.

The key variable for any pension plan beneficiary is not the nominal amount of the pension payment, but the purchasing power hereof, this is the **inflation risk**. Purchasing power can be viewed as an absolute notion measured as the ability to purchase a particular basket of goods or as a relative notion in terms of an individual's purchasing power relative to that of the overall society. In the first instance, the reference measure for protecting future pension payments is indexation to consumer prices. In the second instance, the reference measure is wage inflation. DB plans often offer some degree of protection to both risk to the extent that pension benefits are often calculated on the basis of a terminal salary, and then indexed to consumer prices in retirement. For DC plans, there is no explicit inflation protection and as such this risk, that can also be considered a "pure" risk, is fully transferred to the individual. It should be noted, that to the extent that contributions are made as a percentage of wage earnings under the DC scheme, there is some implicit protection of purchasing power.

Investment risk distinguishes itself from longevity, human capital and inflation risks in that this is a "speculative" risk as opposed to a "pure" risk, and can thus yield either a gain or a loss. For funded DB plans - either in the state or private sector regime - the investment risk is carried fully by the plan sponsor, but is transferred to the

Practices on cost of living adjustment for pension benefits vary greatly from country to country and in some instance from plan to plan.

individual under DC schemes. The question of how to manage the investment risk lies at the core of the discussion in the following sections. As a first observation, we note that successfully tackling investment risk requires some degree of financial education. This point was highlighted by the G8 meeting in June 2006, that acknowledged "*the importance of better financial education and literacy for improving the ability of people to use financial services and to make effective decisions with respect to their present and future welfare*." The issue of financial education is not just relevant for individuals in relation to funded DC plans, however, but more broadly so regulators, pension fund trustees and other actors involved with pension funds. Indeed, the UK Myners Review (2001), highlighted poor financial knowledge as one of the major weaknesses of institutional investment in the UK.

As seen from the discussion above, the move to funded DC schemes entails an important shift of risks to the individual. Three of these risks are "pure" risks while the final risk factor, investment risk, is a "speculative" risk. This distinction is important as it is the efficient management of the latter that will serve to cover the former.

The role of the liabilities

In retirement, an individual thus requires a stream of cash flows capable of covering his desired working life salary replacement rate in retirement, over the span of his lifetime. As shown below, drawing up such a profile of cash flows would look very much like the cash flow of a fixed income instrument. Considering the liabilities implicit to any pension plan, these will thus contain an interest rate risk. As retirement approaches, the interest rate risk will decline. However, if the individual lives longer, the interest risk will increase anew. This is the longevity risk. The chart also clearly illustrates the important of inflation - in the calculation an inflation rate of a very moderate 2% was applied. Over the 50 years work life and 20 years of retirement, even 2% inflation becomes very significant. This is the inflation risk.

Before entering the discussion on how the best manage investment risk, it is important to recognize the importance of pension regulation, which in a broad sense aims to (1) protect the financial security of pension plans, i.e. avoid systemic risks, mismanagement and fraud, (2) ensure proper incentives, transparency and competitiveness in order to encourage saving and control costs, and (3) ensure appropriate investment rules with the aim to protect investment returns. One of the greatest challenges in a voluntary funded DC system is simply to ensure savings. Moreover, taking account of market information asymmetries and misaligned interests, the risk is that the funded DC model could result in both expensive and suboptimal choices. The OECD guide-

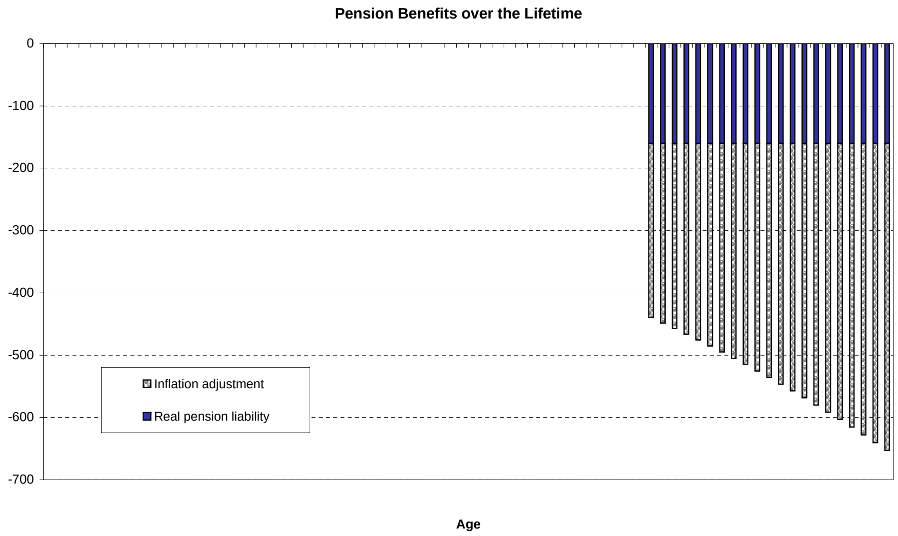

Figure 1: Pension Benefits over the Lifetime

lines on pensions are an important example of how such regulation may take shape . While a detailed analysis of optimal pension fund regulation and system design lies is beyond the scope of this paper, certain aspects of regulation will be discussed in relation to the macroeconomic implications of lifecycle plans. In analysing the issues from the micro perspective, the question of costs will be considered.

3.1 Managing investment risk

As outlined above, the objective in funded pension plans is to manage the investment risk in order to cover longevity, inflation (or purchasing power) and human capital risks. In the following section, we review some of the key findings of the academic literature addressing this problem.

The wealth of an individual at time t (W_t) is given by saving (or contributions) made over time, here defined as a percentage share (π_t) of wage income and any benefits that he may receive from the government $g_t = (Y_t + SB_t)$, the drawdown on the plan (i.e. consumption the plan, C_t) and the net investment return (after deducting costs and taxes) earned on the invested portfolio (r_p). The portfolio return is given as the sum of the net return earned on the individual asset classes (r_i) weighed by the share of the portfolio allocated hereto (w_i).

$$W_t = W_{t-1}(1 + r_t^p) + \pi_t(Y_t + SB_t) - C_t$$

$$where\ r_t^p = \sum_i w_i r_i\ and \sum_i w_i = 1, C_t \geq 0, Y_t \geq 0 g_t \geq 0$$

Four key questions for each period t over the lifetime result from this framework: (1) how to allocate wealth across individual asset classes (w_i), (2) how much to contribute (π_t),(3) how much to work to make income (Y_t) and (4) ultimately how much to consume (C_t).

3.1.1 The Basic Framework of Lifetime Portfolio Selection

In his seminal paper on portfolio choice, Markowitz (1952) showed that for a given level of expected return ($E[R_p]$), the optimal portfolio in the framework of a single time period, is found as the one yielding the lowest level of portfolio risk (σ_p), defined by the variance of portfolio return, thus implying that investors are risk adverse.

$$min\ \sigma_p^2 = min\ \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij}$$

$$sc\ E[R_p] = \sum_i w_i E[R_i]$$

$$\sum_i w_i = 1$$

The Markowitz efficient frontier of portfolios (or Minimum Variance Frontier) is defined by the set of portfolios that for any given level of risk offer the maximum level of expected return. Key to mean-variance is the observation that the correlation between individual asset classes is often less than 1 and even negative at times. As such, a portfolio of assets will yield a diversification gain. The expected diversification gain ($E[DG]$) from the single period implicit to the Markovitz framework will be a function of the intra-portfolio correlation (ρ_{IP}) and can be expressed as follows:

$$E[DG] \approx \frac{1}{2}\left(\frac{\sum_i w_i \sigma_i}{K}\right)(1 - \rho_{IP})$$

$$where\ \rho_{ip} = \frac{\sum_i \sum_j w_i w_j \rho_{ij}}{\sum_i \sum_j w_i w_j}$$

As seen, when the intra-portfolio correlation is 1, i.e. the securities in the portfolio are perfectly correlated, the gain from diversification is nil. Conversely, if the intra-portfolio correlation is -1, i.e. the securities in the portfolio are perfectly uncorrelated;

Erb and Harvey (2006)

the gain from diversification will be equal to the average of the standard deviations of the individual securities.

In the Markowitz framework, the individual maximizes the expected utility of return of investment ($E[R_P)$ and minimize risk of the portfolio (σ_P). The utility function $\hat{U}$ thus expresses the trade off between risk and return taking account of the individual's preference for risk or risk aversion (γ). Utility functions can take on different forms. The utility function below is written in the quadratic form that is generally used with $(\gamma) > 0$ indicating that the individual is risk averse.

$$U(E[R_p]) = E[R_p] - \frac{1}{2}\gamma\sigma_p^2 \ with \ \gamma > 0$$

By introducing a risk free rate (i.e. an asset with a return variance of nil) at which investors can both lend and borrow, the Markowitz efficient frontier will change shape so that all efficient portfolios now become a combination of the tangency portfolio (the portfolio which is tangent to the line that departs from the risk free rate) and the risk free asset class. Hence investors would leverage or de-leverage the tangency portfolio to obtain the desired level of risk. This line is referred to as the Capital Market Line (CML).

The most unsatisfactory aspect of the Markowitz framework is that this framework only considers a single-period framework and hence only considers one of the four question raised above, namely the question of how to allocate wealth across individual asset classes. In addition, variance of returns is considered the correct measure of risk, returns are assumed to be normally distributed, and financial markets are assumed to be frictionless (i.e. individuals can borrow and lend at the same rate, no transaction costs and perfect liquidity).

Building on the Markowitz framework, Merton [1969] and Samuelson [1969] extended the model to include multiple periods and, rather than maximizing the expected utility of return on investment, set out to maximize consumption (C) over the lifetime (from 0 to T), given the subjective time preference for consumption (ρ) and subject to the budget restriction. This framework thus describes the lifetime planning of consumption and investment decisions, presenting "Lifetime Portfolio Selection" (LPS). For each period t, the individual must decide how much to consume and how much to allocate across the available asset classes.

$$max \int_0^T e^{-\rho t} U(C_t)$$

$$sb \ C_t = W_t - W_{t-1}(1 + r_t^p)$$

$$where\ r_t^p = \sum_i w_i^t r_i^t,\ \sum_i w_i = 1\ and\ C_t \geq 0$$

Both papers show that under the two restrictive assumptions of (1) frictionless financial markets and (2) constant relative risk aversion (CRRA), the decision to invest between a risk-free and risky asset with stochastic returns for multiple periods does not introduce extra risk tolerance as a function of the stage of an individual in his lifecycle. The allocation to the risky asset at time t (w_t) is given only as a function of its expected return ($\tilde{r}$) over the risk free rate (r), its risk ($\tilde{\sigma}$) and the individual's relative risk aversion (γ).

$$w_t = \frac{\tilde{r}_{it} - r}{\tilde{\sigma}(1-\gamma)}$$

Moreover, it can be shown that the decision to consume is a function of the wealth stock (W_t), the remaining lifetime, assuming there is no bequest, and a variable v that is a function of expected return, risk aversion and the subjective time preference for consumption. In the special case of $v = 1$, current wealth will be divided evenly over the remaining lifetime.

$$C_t = \frac{\nu W_t}{T-t}$$

Note, that compared to the budget equation given in (3.1), the absence of labour income and hence contributions in this formulation of the LPS, corresponds to the retirement phase of the individual's life cycle assuming that he has no recourse to labour income in retirement, and receives no benefits

3.1.2 Extensions of Lifetime Portfolio Selection

Taking their point of departure from the initial LPS framework, successive authors have aimed to expand it to take account (1) labour income, (2) social benefits, (3) housing wealth (4) stochastic returns, (5) market frictions, (6) taxation and (7) inflation. In the following sections, we brebriethly consider how impacts the conclusions.

Labour Income

Introducing labour income, we can consider the discounted value of all future labour income as an individual's stock of human capital (H). We can then find total wealth (TWt) at any given time as the stock of financial wealth (Wt) plus the stock of human

The CRRA utility function is given by $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$ for $\gamma > 0$ and $\gamma \neq 0$ otherwise $U(C) = lnC$ for $\gamma = 1$

Merton [1969]

capital wealth (Ht). Over the lifetime, the value of human capital would intuitively be expected to decline, while financial wealth increases.

$$TW_t = W_t + H_t$$

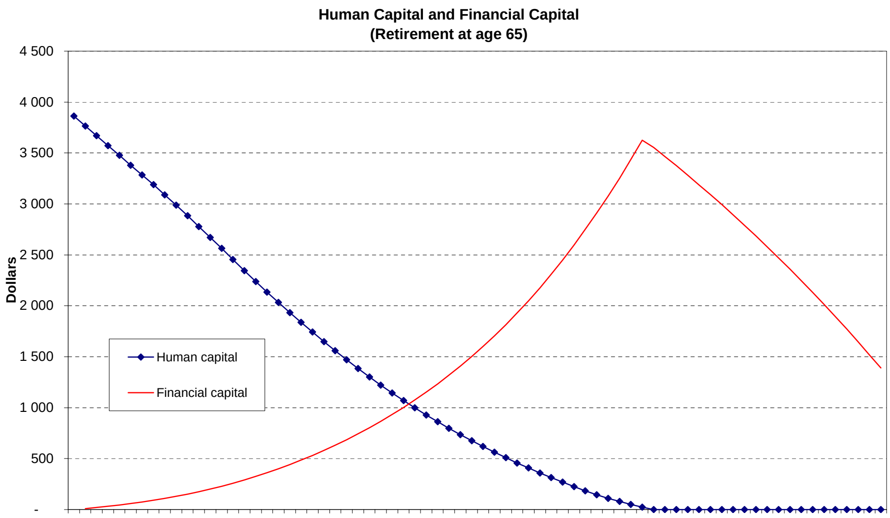

Figure 2: Human Capital and Financial Capital (Retirment at age 65)

Human capital is a non-tradable asset, however if an individual's wage income can be hedged using financial securities it can be viewed as a tradable asset. We can thus consider this equivalent to introducing an additional asset class in the LPS framework. In 1992, Bodie, Merton and Samuelson extended the initial LPS framework by introducing human capital and allowing individuals to supply labour in a flexible manner. Including flexible labour supply, the previous maximization problem can be restated as shown below. For each period t, the individual must now decide how much to consume (C_t), how much leisure (L_t) to take and how much to allocate across the available asset classes (w_i).

$$max \int_0^T e^{-\rho t} U(C_t, L_t)$$

$$sb\ C_t = W_t - W_{t-1}(1 + r_t^p) + (1 - L_t)$$

$$where\ r_t^p = \sum_i W_i^t r_i^t,\ \sum_i W_i = 1\ and\ C_t \geq 0$$

As in the initial LPS framework, financial markets are assumed to be frictionless and complete, and returns on the risky asset to be stochastic. Bodie, Merton and Samuelson consider both the case of a nonstochastic wage and a stochastic wage, and the case of inflexible and flexible labour supply.

Consider first the case of a nonstochastic wage yielding the same return as the risk-free asset (r) and inflexible labour supply. In this case, human capital is a substitute to the risk free asset. Given that the labour supply is inflexible, the individual can only choose the amount to consume and the amount to allocate between financial asset classes at each point in time. This becomes near analogous to the original LPS framework and in this case, we find the well known result that multiple periods do not introduce extra risk tolerance as a function of the stage of an individual in his lifecycle. What is important to note in is that what is constant is the allocation of total wealth to sum of two risk free assets. The share of total wealth held in the risky asset will be unchanged by the passage of time, however, as the value of the stock of human capital declines over time, the share of total wealth allocated to the risk free financial asset will increase.

$$TW_t = w_t W_t + (1 - w_t) W_t + H_t$$

Taking next the case where the wage is stochastic. In this case, human capital can be considered as some combination of the risky and risk free asset. Since labour supply is still fixed, this is analogous to the case of m-risky assets as illustrated by Merton [1969]. In this case, the allocation to the risk free and risky financial asset as a share of total wealth will both evolve over time . In the case where the correlation between wage income and the risky financial asset is perfect, the decline in human capital over time will entail an increasingly higher allocation to the risky financial asset, while the allocation of total wealth to the risk free financial asset will remain unchanged. In addition, note that for both nonstochastic and stochastic wages if in one period equities suffer a sell-off, the individual should add more to equities in the subsequent period, i.e. thus defining a contrarian investment strategy.

The key question thus becomes to determine the risk-return profile of labour income. The chart below shows the real fed funds rate, real monthly wage growth and real monthly equity returns in the US over time, as seen wage income appears to hold relatively low risk - at least for the US.

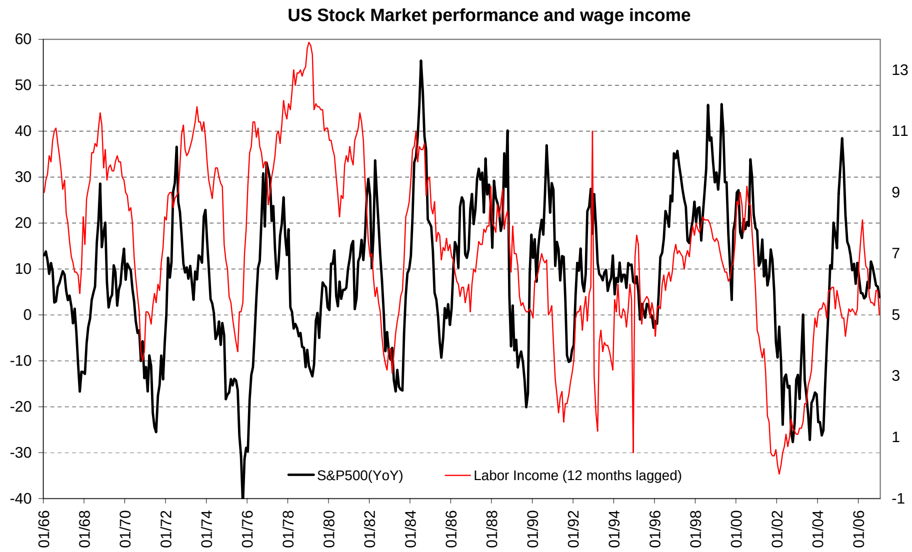

Figure 3: United-States: Wage and Equity Market performance

Introducing labour flexibility, individual now in addition chose how much leisure (L_t) to consumer in each period. Bodie, Merton and Samuelson find that this will at any point in time induce greater risk taking in allocation to financial assets. The logic being that if an adverse financial outcome were to occur, an individual has the opportunity to recoup some of this loss by working additional hours in the future. The ability to vary the supply of labour ex-post, thus leads to greater risk taking ex-ante - labour flexibility can as such be consider a type of "insurance" against a negative investment outcome.

In conclusion, extending the LPS to include labour income, the fraction of an individual's wealth optimally invested in equity should decline with age for two reasons: (1) young persons can adjust their labour supply with a greater flexibility and (2) young persons' human capital is greater than that of older persons and is generally less risky than many financial assets. This conclusion appears in line with the lifecycle investment approach promoted by many financial advisors that an individual's equity exposure should decline over the life time.

Social benefits

Following on from the discussion of the role of labour income above, it should be kept in mind that in addition to labour income, many individuals receive social benefits, such as pension benefits during retirement age years. In the case of pension benefits that are means-tested, these may encourage an individual to take on more investment risk, since benefits can thus be seen to act as an insurance against a negative outcome. This yields the counter intuitive result that individuals with fairly low financial wealth and thus potentially closer to the trigger level to receive benefits should assume more financial risk. These results can be derived from the LPS framework above and are very important ones for policy makers to keep in mind.

Housing wealth

In reality, housing wealth is one of the most important asset classes for individuals. In 2006, US households housing wealth accounted almost 40% of total wealth compared to just short of 25% for equities . Moreover, housing is one of the rare asset classes that individuals significantly indebt themselves to acquire. In addition, housing is at the same time and investment good and consumption good. Note, finally, the housing can either be bought or rented.

In considering the impact of housing on lifetime asset allocation, Cocco (2000) finds that in the presence of housing, the allocation to the risky asset tends to be lower, notably for younger persons that take out mortgages to finance housing acquisitions. As households age, and mortgages are paid off, the allocation to the risky financial asset class increases, all else being equal.

The role of special role of housing raises an interesting question in whether individuals should be allowed to take of cash from the pension plan before retirement to acquire their principal home. In a system with voluntary contributions to the pension saving plan, the argument in favour of hereof is that individuals may otherwise shun pension plans that are generally locked in until retirement, in favour of shorter term savings options. In theory, this should be neutral in the absence of any tax distortions. In practice, encouraging interest in the pension plan and the discipline of saving herein at an early stage of the working life may prove beneficial.

In debating the impact of housing, the structure of the mortgage market becomes all the more import - this is true both at the level of the individual and at the level of the macro economy. Recalling the interest rate risk inherent to the pension plan, there is a clear link to the financing of housing. Individuals, who finance their housing

Us Federal Reserve, Flow of Funds

using adjustable rate mortgages, gain a second level of exposure to interest rate risk. Conversely, individuals who finance housing through fixed rate mortgages will win a partial hedge on the interest rate risk. If the mortgage in addition contains a prepayment option, the hedge on interest rate risk if further improved. A final point to note in relation to both the pension investment decision and the house financing decision is that taxes and transaction costs will play a significant role. Moreover, both decisions financial knowledge is very important.

Stochastic returns

In the LPS framework, returns on the risky asset class were defined as stochastic, a growing amount of academic research have shown that risky asset prices do not follow a random walk process . This is all the more the case over longer horizons, where risky asset prices exhibit mean-reverting properties.

Compared with the LPS framework described above, the presence of mean-reversion in stock prices should have two implications. First, the fact that risky asset price returns have become "predictable" over longer horizon should encourage greater allocation hereto by younger individuals with longer horizons before retirement. As individuals age, and the investment horizon shortens, the allocation to the risky asset will decline. Second, high returns in mean reversion should result in lower allocation in the future, thus yielding a negative hedging demand for the risky asset. Conversely, when returns are stochastic, higher returns will all else being equal increase the demand for the risky asset.

Utility functions

The CRRA utility function applied in the LPS framework is of central importance for the conclusions reached. Intuitively speaking, one would expect both wealth and age to play a role in the utility function. Ameriks and Zeldes (2004) show a simple model in which consumption only occurs in the final period. Given W* as the minimum subsistence level of terminal wealth, then the individual must place $\tilde{W}(1+r)^{t-T}$ in the risk free asset to ensure that this wealth level is protected. The remainder of the wealth would then be invested in the same manner as would be given as optimal in the CRRA framework. In this simple model, the utility function takes the form below.

$$U(W_T) = \frac{(W_T - \tilde{W}_T)^{1-\gamma}}{1-\gamma} \; for \; W_T > \tilde{W}_T$$

We can thus concluded that if the minimum level of subsistence is the same for all individuals, then the rich can adopt more risky investment strategies than the poor.

Conversely, if what is considered the minimum level of vary, one could imagine that the risk may consider their minimum subsistence level higher than for the poor, leading to the counterintuitive view that the risk should have less risky strategies than the poor. A second point is that if we consider the minimum subsistence level in retirement as a future flow of liabilities, the value hereof will all else being equal increase as individual age, hence suggesting that as an individual ages, he should reduce the share of his portfolio allocated to the risky assets.

Taxes and Transaction Costs

In discussing housing wealth, taxation was already mentioned as an important factor in financial planning. From a lifecycle perspective, most people expect to have lower income in retirement than during their working life as such deferred taxes on pensions will be a motivating factor to subscribe to pensions, notably for high income individuals. Taxation can also be important if individual asset classes are taxed differently. In any planning situation, the key variable will always be after-tax return.

Turning to transaction costs, these can be considered from a more narrow perspective covering fees and trading costs or more broadly to include information cost. The latter factor is directly linked to financial education. In designing pension systems, it is important that regulators focus on both types of cost.

Inflation

The final variable we discuss with regards to possible extensions of the model is inflation. As already mentioned, the key variable is future real pension wealth - not nominal wealth. This is the inflation risk. The chart below show the correlation between annual consumer price inflation in the United States and annual asset class returns. As seen, even on these annual data, correlations change significantly over time and from asset class to asset class. This chart clearly illustrates that only inflation linked bonds provide a good hedge against inflation (note, that in the chart the inflation linked bond is the same synthetic one a previously defined).

In the following section we take a closer look at the implications of inflation when explicitly included in the LPS framework.

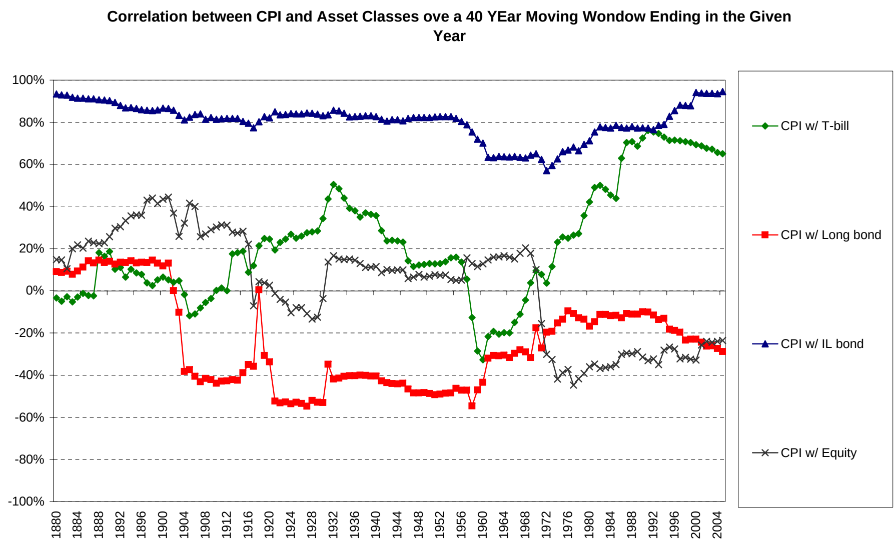

Figure 4: Correlation between CPI and Asset Classes over a 40 Year Window Ending in the Given Year

4 The current market offer

Looking across the available offer in financial markets today, a number of different products addressing the question of LPS are available. An important share hereof falls within the category of deterministic life-styling. The most well know of these is an allocation to the risky asset class equal to 100 minus a persons age, such that a person of 15 years of age holds 85% of the "risky" equity portfolio and 15% of the "safe" fixed income portfolio. These products have grown in popularity in recent years. In the US, 48.5% of 401(k) plans in the US offered a life-styling fund in 2005 - up from a mere 12.1% in 1996 . Moreover, the new Pension Protection Act of 2006 envisages life-styling funds as the qualified default option.

In the following, we take an empirical approach using historical data for the US since 1840 to compare the performance of (1) a deterministic life styling portfolio (2) a constant mix portfolio (i.e. a fixed share of the risky asset, as suggested by the initial papers by Merton and Samuelson) and (3) a pure fixed income portfolio. In the US today, many 401K plans still offer a "safe" short-maturity fixed income (i.e. money market) solution as the default option.

The starting point for our analysis is to consider an individual whose real wage income over his lifecycle follows the same path as the instantaneous picture given by the US median weekly earnings data from the Bureau of Labour and Statistics. As the individual gains work experience, he thus benefits from a "seniority" premium. As seen from the chart below, the seniority premium turns negative towards the end of the active life, indicating that employers are willing to pay less for older employee in the final stages of their work life. Although not the subject matter of the current paper, it should be kept in mind that the debate on how to encourage both employees and employers to extend the working life is an important area for research in the pension debate. Indeed, working longer is the most efficient and the fairest solution to the challenges posed by increased longevity. In the simulation, the working life of the individual is set to span 50 years from age 15 to 65.

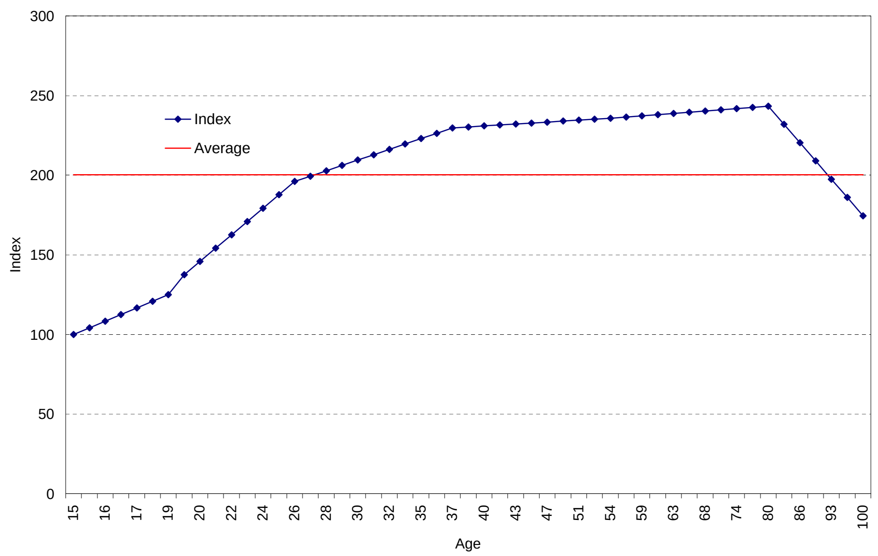

Figure 5: Real Wage Profile Over the Working Life

In each period, the individual is assumed to contribute 10% of wage to pensions. Applying the three different strategies outlined above, the individual invests in an equity fund and a fixed income fund according to the predetermined rules of the investment strategy; (1) the life-styling strategy is defined such that the individual starts with 85% equity at 15 and this then falls proportionally to 0% at retirement, (2) the constant mix portfolio is set as the classic 60% equity 40% bonds traditionnaly considered appropriate for US pensions funds, and (3) the final strategy is 100% pure fixed income. For each strategy, we consider three different fixed income securities; (1) a short term money market instrument, (2) a long dated nominal government bond and (3) a long dated inflation-linked government bond. We thus consider a total of nine strategies.

Starting in 1840, we thus apply each successive 50 year period of market history to the individual, given the lifetime earning profiles and applying a constant contribution rate of 10%. Hence, labour supply is assumed to be fixed with no options to alter pension savings in response to financial market performance. The simulation is done in real terms across all periods. In terms of the wage profile, only seniority is taken into account, the simulation does not give any reward to employees in terms of real wage gains (i.e. productivity gains). The data used in the model are historical data for the US equity market, long-term nominal government bonds and short-term money market bills. Given the long history of the data, earlier observations may be of lesser quality. Moreover, it should be kept in mind that the functioning of economic policy and the financial markets has changed significantly over the period. Inflation-linked bonds are a much more recent invention, appearing in the US only in 1997. Consequently, the inflation linked bond used in the model is a synthetic one, where the real yield is set as a 15 year average of real GDP growth multiplied by a factor of 75%. This admitedly highly approach approach was used to approximate the real yields on actual TIPS since issue and to avoid negative real yields. The authors recognize that this approach is open to sharp criticism, and hope in future work to develop an econometrically more sound approach.

The overall idea is thus to test how the described investment strategies did over all the 50 year periods of market data from 1840 to 2005 ; given that the final 50 year period starts in 1955, the simulation covers a total of 116 overlapping periods. As seen from the tables and charts below, results for all nine strategies vary significantly from period to period. Unsurprisingly, the fixed income strategies offer the safest option where bonds are used. However, the money market option appears decidedly less "safe" than intuition may have suggested. This is because money market instruments provide no interest rate hedge due to their short maturity nor do they offer any inflation hedge.While their return over the full period actually outperforms the other fixed income solutions, this is not the case when looking merely at the post-WWI period.

The more surprising outcome of the simulation is that the life-styling strategy does not appear superior to the constant mix in a significant way measured in return. This result must be interpreted with a certain precaution. As, the simulation does not adjust for financial market volatility. As seen from chart 2 we note that the financial market risk of properties of lifestyling appears superior to the constant mix. Note that the data used is annual - lower frequency data - would likely accentuate the results. The key point of the life styling strategy is that financial market risk exposure declines over the lifecycle as the share of equities declines. This is clearly not the case for the constant mix strategy. Intuitively, adjusting for this factor should yield the life styling strategy superior to the constant mix strategy. This is clearly an area that merits further research on our part.

Finally, we note that in all instances, the use of the inflation linked bond yields a superior result. Given that this instrument provides a hedge both against inflation risks and real interest rate risks, the outcome is very pleasing. As mentioned above, however, these results are obtained via a synthetic inflation linked bond and as such, should be taken with a good deal of precaution.

Simulation 1840 to 1955

Years of retirement (Pension wealth at 65 / Average salary over life time)

	Mean	Minimum	Maximum	Standard deviation (STD)
Years of retirement (final 10 years of salary)	12.3	5.8	21.2	4.39
Life styling w/nominal bonds	14.4	8.8	19.8	2.94
Life styling w/Inflation linked bonds	12.3	5.5	24.1	5.07
Life styling w/money market	12.1	5.3	21.3	4.34
Constant mix (60/40) w/ nominal bonds	14.1	9.0	18.3	2.37
Constant mix (60/40) w/Inflation linked bonds	12.3	6.3	25.3	5.22
Constant mix (60/40) w/Money market	8.5	3.1	17.2	3.82
Pure nominal bonds	10.3	7.4	13.7	1.53
Pure Inflation linked bonds	8.9	3.9	23.4	5.30

Figure 6: Simulation results 1840 to 1955

Longevity bonds

In our simulation, we cit inflation linked bonds as a good risk free asset. A still better risk free asset would be an inflation linked longevity bond. A longevity bond pays coupons that a proportional to the survival rate of a given population. Although this would not provide any protection for an individual's longevity, it would offer a partial hedge if an individual's longevity is assumed to be in part a function of the overall population's longevity. While inflation linked bonds have won some popularity as part of the governments fixed income issue, longevity bonds are still far and few between. Recently, however, there has been some renewed interest surrounding this asset class.

What is evident from the simulation above is that in the simplest form, life styling

Simulation 1920 to 1995

Years of retirement (Pension wealth at 65 / Average salary over life time)

	Mean	Minimum	Maximum	Standard deviation (STD)
Life styling w/nominal bonds	12.6	6.4	18.6	3.6
Life styling w/Inflation linked bonds	14.7	12.9	17.0	1.4
Life styling w/money market	11.3	9.6	12.8	0.9
Constant mix (60/40) w/ nominal bonds	11.2	5.3	18.7	4.1
Constant mix (60/40) w/Inflation linked bonds	14.0	11.7	17.7	1.5
Constant mix (60/40) w/Money market	10.3	7.0	15.6	2.4
Pure nominal bonds	7.3	3.1	12.7	3.1
Pure Inflation linked bonds	9.7	8.8	10.4	0.5
Pure Money market	6.4	4.7	7.9	1.2

Figure 7: Simulation results 1920 to 1995

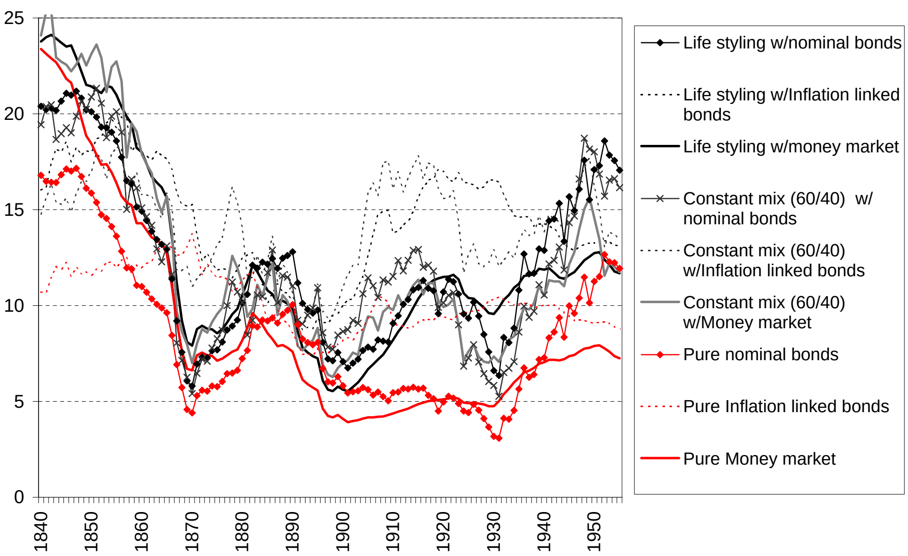

Figure 8: Simulation results

funds (as is also the case for constant mix) appear to still leave plan beneficiaries exposed to non-negligible risks. At the same time, opting for pure inflation-linked income solutions poses a dilemma in turns of lower return. Moreover, from a practical point of view the very low level of supply of inflation linked bonds raises a scarcity problem (cf. the discussion in the macro section). This is particularly obvious for the post WWI period where the mean outcome in the number of years of retirement is significantly lower than in the two alternative strategies. These results thus suggest

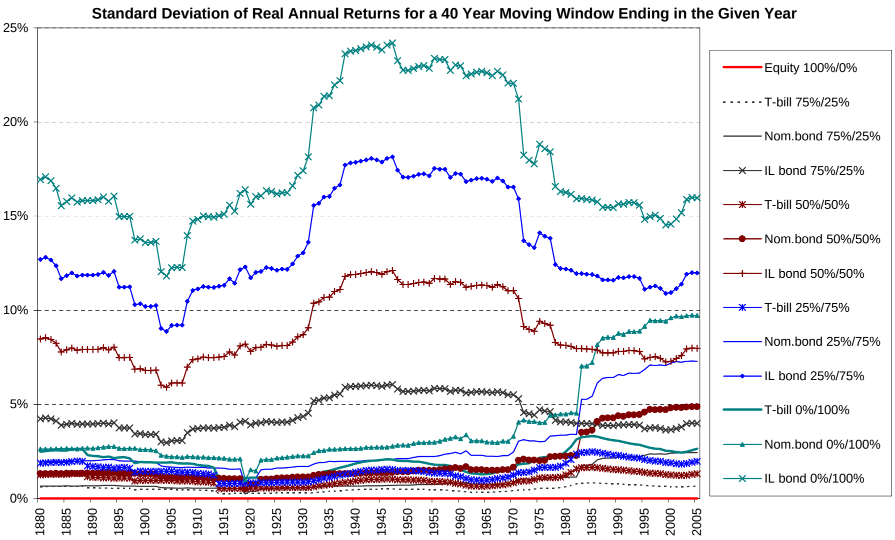

Figure 9: Standard deviation of Real Annual Return for a 40 year Moving Window Ending in the Given Year

that there is room for new investment strategies to address the LPS problem. Three avenues of research appear particularly attractive (1) using the sophisticated financial market instruments available today to structure more appropriate solutions, in terms of protection that they over (2) to apply active management to the strategies outlined above and (3) developping new financial instruments better suited to hedge the risk inherent to DC plans. In future research, we plan to extend the strategies tested above to contain a tactical overlay as a function of the equity risk premium, taking account of the mean reverting nature hereof.

Wealth and risk tolerance

The insurance market deserved mention in this context. It is important to distinguish between two different types of life insurance; (1) insurance against mortality risk, i.e. the insurance premium is paid upon the death of the individual and (2) insurance against longevity risk, i.e. the insurance premium (or annuity) is only paid out as long as the individual is alive. In relation to pension plans, longevity insurance is of key interest, although there are clear ties between the two forms of insurance. A simple solution to the problems raised by transferring from funded DB to funded DC plans would be fore each individual to by annuity life insurance products. In doing so, risks would again be mutualised as under the funded DB schemes. Moreover, such products are indeed available, but come at a very high cost.

5 Extending the framework to include inflation

In our opinion the important role of inflation is still too often set aside both by accademic research and practicioners. In this section we extend the LPS framework to include inflation and consider the implications for the results.

5.1 The structure of the model

We consider a simple framework where the individual aims to set the optimal asset allocation of his wealth during his working life, choosing between a risky financial asset to which he allocates α_t in each period and a risk free asset to which he allocates $(1-\alpha_t)$. As seen from the discussion before, over the long term horizon of the individual - both in term of working life and retirement life - price dynamic plays an important role. Indeed, most of the LPS model consider nominal wealth and do not explicitly consider price dynamics in the definition of the investment strategy. We extend the LPS framework to include price dynamics. We consider inflation linked bonds as the risk free asset and we integrate consumer price into the salary dynamics.

Let P_t be the consumer price is described by :

$$dP_t = P_t \pi dt \tag{1}$$

We assume that, the instantaneous inflation rate is mean reverting and thus follows an Ornstein-Ulhembeck process:

$$d\pi_t = \lambda_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dZ_{\pi t} \tag{2}$$

where λ_π represents the degree of mean reversion, $\bar{\pi}$ the long term inflation rate, σ_π the inflation rate volatility and $Z_{\pi t}$ the brownian motion.

The price of the risky asset is set as a stochastic variable that follows an Itô process:

$$dS_t = S_t(r_t + \pi_t + \xi_t\sigma_1)dt + \sigma_S S_t dZ_{St} \tag{3}$$

where r_t is the real interest rate, π_t the inflation rate, ξ_t represents the **time varying** market price of risk (the risk premium) σ_S is the stock index volatily (the difusion parameter of the stochastic differential equation), and Z_{St} is a standard browninan

ξ_t can also be viewed as the Sharpe ratio i.e $\xi = \frac{\mu_t - r}{\sigma_1}$, (Bolliger, [2005]) motion.

We note that the excess return parameter ξ_t evolves with time. In particular, we suppose that ξ_t is caracterized by a **mean reverting process**. It follows an Ornstein-Ulhembeck process:

$$d\xi_t = \beta_\xi(\bar{\xi} - \xi_t)dt - \sigma_\xi dZ_{St} \quad (4)$$

where β_ξ represents the degree of mean reversion in the excess return, $\bar{\xi}$ is the long run equity risk premuim and σ_ξ is the excess volatility parameter.

The risk free asset can be compared to an inflation linked bond that delivers a fixed returns equal to the sum of the real interest rate and inflation rate. The risk free asset is dynamics is given by the following deterministic process:

$$\frac{dL_t}{L_t} = (r_t + \pi_t)dt \quad (5)$$

We now introduce the labour income as a stochastic variable that follows an Itô process

$$dY_t = (r_t + \mu_{Yt} + \pi_t)Y_t dt + \sigma_Y Y_t dZ_{St} \quad (6)$$

We can see that the labour income process exhibits a drift μ_{Yt} which can be interpreted by a deterministic function of time. This can be interpreted as the existing relation between age and salaty income. Indeed, the salary grows faster when the individual is young rather than when he is old, as illustrate in the chart before. The salary process is affected by the brownian motion dZ_{St}. It traduces the existing links between the salary and the risky financial asset. Empirically, in the United States, there is a lagged correlation between the salary income and returns on equity market (cf the chart US equity and wage income).

5.2 Case 1: excess returns are not mean reverting ($\xi_t = 0$)

The individual is not interested in maximizing his wealth per se, but the ratio $W(t)/Y(t)$ which can be viwed as the purchasing power. Indeed the maximization of the purchasing power is more pertinent than real wealth. Note that this ratio corresponds to the number of years of retirment that the pension wealth can cover as a function of pay. To solve this problem, we first write the financial wealth and we define the process $d(W(t)/Y(t))$, and thus we calculate the optimal weight α_t.
Let $W(t)$ the individual wealth at time t. We define $W(t)$ as follow:

As in Watcher 2002, we set a negative correlation between the stock index and the excess return processes in order to expressed the mean reversion idea. Indeed, when the stock return are high, they are followed by weaker stock return

$$dW_t = \alpha_t W_t \frac{dS_t}{S_t} + (1-\alpha_t) W_t \frac{dL_t}{L_t}$$

$$dW_t = W_t(\alpha_t \sigma_S \xi_t + r_t + \pi_t)dt + \alpha_t \sigma_S dZ_{St}) \tag{7}$$

Let $X_t = W_t/Y_t$ be the nominal wealth divided by the individual's income. Using the Itô lemma, we obtain

$$dX_t = \frac{dW_t}{Y_t} + d\left(\frac{1}{Y_t}\right) W_t + d\left\langle W_t.\frac{1}{Y_t}\right\rangle$$

$$dX_t = X_t[\alpha_t \sigma_S \xi - \mu_{Yt} + \sigma_Y^2 + \alpha_t \sigma_S \sigma_Y]dt + X_t(-\sigma_Y dZ_{St} + \alpha_t \sigma_S dZ_{St}) \tag{8}$$

We assume ξ is constant, i.e there is no mean reversion. We define the individual's terminal utility as

$$U(X(T)) = \frac{1}{\gamma}(X(T))^\gamma$$

where $\gamma < 1$ and $\gamma \neq 0$. If $\gamma = 0$ the utility function becomes logarithmic.

$$\underset{\alpha_t}{max} E[U(X(T))]$$

To solve this maximization problem, we set the form of the indirect utility function and then use the Bellman principle. Let $J(t, X_t, \pi_t)$ be the indirect utility function :

$$J(t, X_t, \pi_t) = \frac{1}{\gamma}(X(t))^\gamma exp(h(t, \pi_t)) \tag{9}$$

where $h(T, \pi_T)) = 0 \ \forall \pi_T$. To perform our calculation we use the Itô lemma for the indirect utility function:

$$dJ = J_t + J_X dX + J_\pi d\pi + \frac{J_{xx}}{2} d\left\langle X^2 \right\rangle + \frac{J_{\pi\pi}}{2} d\left\langle \pi^2 \right\rangle + \frac{J_{X\pi}}{2} d\left\langle dX d\pi \right\rangle$$

Where the partial derivatives are :

$J_t = \frac{X_t^\gamma}{\gamma} \dot{h}(t, \pi_t) exp(h(t, \pi_t))$

$J_X = X_t^{\gamma-1} exp(h(t, \pi_t))$

$J_\pi = \frac{X_t^\gamma}{\gamma} h'(t, \pi_t) exp(h(t, \pi_t))$

$J_{XX} = (\gamma - 1) X_t^{\gamma-2} exp(h(t, \pi_t))$

$J_{\pi\pi} = \frac{X_t^\gamma}{\gamma} exp(h(t,\pi_t))[h''(t,\pi_t) + h'(t,\pi_t)^2]$

$J_{X\pi} = X_t^{\gamma-1} h'(t,\pi_t) exp(h(t,\pi_t)$

Keeping in mind that $E[dZ_{St}] = E[dZ_{\pi t}] = 0$, we directly find $\underset{\alpha_t}{max} E[dJ(t, X_t, \pi_t)] = 0$

$$\underset{\alpha_t}{max}x = \dot{h} + \gamma(\alpha_t\sigma_S\xi_t - \mu_{Yt} + \sigma_Y^2 + \alpha_t\sigma_S\sigma_Y) + h'(\beta(\bar{\pi} - \pi_t)) + \tfrac{1}{2}(h'' + h'^2)\sigma_\pi^2$$

$$+\tfrac{1}{2}\gamma(\gamma-1)(\alpha_t\sigma_S - \sigma_Y)^2 + h'\gamma(\alpha_t\sigma_S\sigma_\pi \ - \sigma_\pi\sigma_Y)dZ_{St}dZ_{\pi t}$$

where $dZ_{St}dZ_{\pi t} = \rho_{S\pi}$ the covariance between the two brownian motions dZ_{St} and $dZ_{\pi t}$. By differentiatting the previous equation by α_t we obtain

$$0 = \gamma\sigma_S(\xi_t + \sigma_Y) + \gamma(\gamma-1)\sigma_S\sigma_Y(\alpha_t\sigma_S\sigma_Y - \sigma_Y) + h'\gamma(\sigma_S\sigma_\pi\sigma_Y)\rho_{S\pi}$$

and thus the optimal solution is

$$\alpha_t = \underbrace{\frac{1}{1-\gamma}\frac{\xi_t}{\sigma_S}}_{Part1} - \underbrace{\frac{\gamma}{1-\gamma}\left(\frac{\sigma_Y}{\sigma_S}\right)}_{Part2} + \underbrace{\frac{1}{1-\gamma}\frac{\sigma_\pi\rho_{S\pi}}{\sigma_S}h_\pi^{'}}_{Part3} \tag{10}$$

The optimal solution comprises three parts:

1. The first part can be interpreted as the hedging demand relative to the changes on the risky market. This result corresponds to the solution of the Merton model (cf section 3). We observe that the higher the market price of risk (or the risk premium) , the higher the hedging demand vis a vis an unfavourable change on the risky market becomes.

2. The second part corresponds to the hedging demand relative to the negative changes in the individual income. As in Blake and Bolliger, we find that the hedging demand depends on the risk aversion parameter γ. If individual is risk averse (a low value for γ), the hedging demand due to unfavourable changes in income increases and hence increases the allocation to the risk free asset.

3. The third part introduces the inflation risk. Several observations can be made. Even if the inflation rate does not appear explicitly in the solution, its volatility is taken in account. This result is a key issue because it raises the question of the risk free asset in the lifecycle plans. The important role played by nominal bonds in the lifecycle plans appears to be limited in order to hedge inflation risk while inflation linked securities provides such protection. The inflation hedging

demand depends also on the covariance between the stock market and the inflation rate. Based on historical observations, we note that depending on the period in question this correlation has been positive or negative, which can modify the demand vis a vis inflation hedging assets if predictable

We next to extent our first analysis, studying the case where the stock returns are mean reverting. Intuitively, the introduction of a mean reverting process on the stock return should reinforce the hedging asset demand vis a vis risky financial market when the market price of risk dynamic is unfavourable.

5.3 Case 2: Excess return are mean reverting ($\xi_t \neq 0$)

The first case deals with the inflation risk, wage risk, and investment risk. In order to improve our simple framework, we assume that the stock market returns are mean reverting, in other words ($\xi_t \neq 0$), which is in our opinion a more realistic framework. The resolution of the maximization problem is unchanged. The individual's wealth is the same as before. The main change comes from the indirect utility function. Indeed, we need to take account of the fact that return index evolves stochastically. Formally, the indirect utility function is $J(t, X(t), \pi_t, \xi_t)$

$$J(t, X_t, \pi_t) = \frac{1}{\gamma}(X(t))^{\gamma} exp(h(t, \pi_t, \xi_t)) \tag{11}$$

where $h(T, \pi_T, \xi_T)) = 0\ \forall \pi_T \quad \forall \xi_t$. To perform our calculation we use the Itô lemma for the indirect utility function:

$$dJ = J_t + J_X dX + J_\pi d\pi + J_\xi d\xi + \tfrac{J_{\xi\xi}}{2} d\left\langle \xi^2 \right\rangle + \tfrac{J_{XX}}{2} d\left\langle X^2 \right\rangle + \tfrac{J_{\pi\pi}}{2} d\left\langle \pi^2 \right\rangle$$
$$+ \tfrac{J_{X\xi}}{2} d\left\langle dX d\xi \right\rangle + \tfrac{J_{X\pi}}{2} d\left\langle dX d\pi \right\rangle + \tfrac{J_{\pi\xi}}{2} d\left\langle d\pi d\xi \right\rangle$$

We add the partial derivatives implied by the variable ξ_t

$J_\xi = \frac{X_t^\gamma}{\gamma} h'_\xi(t, \pi_t, \xi_t) exp(h(t, \pi_t, \xi_t))$

$J_{\xi\xi} = \frac{X_t^\gamma}{\gamma} exp(h(t, \pi_t, \xi_t))[h''_\xi(t, \pi_t, \xi_t) + h'_\xi(t, \pi_t, \xi_t)^2]$

$J_{X\xi} = X_t^{\gamma-1} h'_\xi(t, \pi_t, \xi_t) exp(h(t, \pi_t, \xi_t))$

$J_{\pi\xi} = \frac{X_t^\gamma}{\gamma} h'_\xi(t, \pi_t, \xi_t) h'_\pi(t, \pi_t, \xi_t) exp(h(t, \pi_t, \xi_t))$

The solution comprises $h'_\pi(t pi_t)$ which is the partial derivative of the function $h(t, \pi_t)$ to π_t. The implementation of this solution requires to find a functionnal form of the function h. Bolliger provides the solution to this problem.

Using the Bellman principle, we derive the optimal solution

$$0 = \gamma\sigma_S(\xi_t + \sigma_Y) + \gamma(\gamma - 1)\sigma_S\sigma_Y(\alpha_t\sigma_S\sigma_Y - \sigma_Y) + h'_\pi\gamma(\sigma_S\sigma_\pi\sigma_Y)\rho_{S\pi} + h'_\xi\gamma(\sigma_S\sigma_\xi)$$

Thus we obtain

$$\alpha_t = \underbrace{\frac{1}{1-\gamma}\frac{\xi_t}{\sigma_S}}_{Part1} - \underbrace{\frac{\gamma}{1-\gamma}\left(\frac{\sigma_Y}{\sigma_S}\right)}_{Part2} + \underbrace{\frac{1}{1-\gamma}\frac{h'_\pi\sigma_\pi\rho_{S\pi}}{\sigma_S}}_{Part3} + \underbrace{\frac{1}{1-\gamma}\frac{h'_\xi\sigma_\xi}{\sigma_S}}_{Part4} \tag{12}$$

The solution of our problem can be divided now in four parts:

1. The first three parts represent the solution of the our first case i.e. when the the market price of risk is not mean reverting.

2. The fouth part of the solution corresponds to a hedging demand due to unfavorable changes in the excess return. Basically, when the market of risk (the risk premium) is getting more volatile, the individual will increase his hedging demand. The solution shows us that each source of risk is covered by an hedging demand.

Note moreover that we have only one source of covariance, between the inflation rate and the stock market. This framework can be extend considering explicit correlations in the model (between the risky financial market and the individual salary...)

5.4 Discussion and potential extentions

This simple framework we can be improved in several ways. In our analysis we have excluded the contributions made by the individual to the pension plan. However, these contributions represent a key feature which can significantly modify the investment decisions over the lifecycle. In the same vein, several studies show that there is a link between the individual risk aversion and the wealth level in other words, the more the wealth level is high, the more the investor is risk averse. In this case, a discussion about the utility function specification (different from the CRRA) could be a potential improvement.

In line with the previous section, other asset classes can be introduced in the model, in particular the real estate. Moreover, we have considered a determistic interest rate which is an important simplification in this type of model. It is more realistic to assume a stochastic process for the interest rate. The stochastic interest rate allows us to consider a more realistic bond market via the yield curve modelling (Vasicek [1977] - Cox Ingersoll Ross [1985])

In our model, the market price of risk and the inflation rate are supposed to be mean reverting, following Ornstein-Uhlembeck processes. Note that this type of processes comprises a long term parameters and a mean reverting coefficient. Economic models can help us to a better calibration of these parameters. For instance, we can wonder what will be the impact of the demographic trends on the long term inflation rate, interest rate and other fundamental variables.

6 The macroeconomic perspective on funded DC plans

According to the OECD, total pension fund assets in the OECD area stood at 87.6% in 2005. With many countries introducing new capitalized retirement schemes - both in the OECD and in the new quickly emerging countries - these assets are likely to see continued growth over coming years. As an example, the World Bank has published a study showing that Chinese pension assets could reach \$1.8trillion by 2030! Clearly how these assets are invested is of key importance to both the financial markets and the real economy.

6.1 Regulation on the investment policies of pension fund assets

In view of their importance, pension fund assets are generally the subject of regulation. The investment policy governing pension fund assets is defined by two board categories; those falling under the heading of a quantitative limit and those falling under a prudent person standard.

The most common types of quantitative limits are (1) limitations on the use of gearing (i.e. borrowing) by pension funds, (2) a maximum allocation to equities, (3) a minimum allocation to government bonds, (4) a maximum allocation to foreign assets and (5) restrictions on the maximum holding for a single company. The principle advantages of the quantitative portfolio limits are; (1) these are simple to define, (2) little financial knowledge is required to implement the rules and (3) this system tends to result in a "single portfolio", leading to fairly homogenous performance across pension assets.

While quantitative restrictions are still common in the OECD area, the trend is a shift is towards a prudent person standard. The prudent person rule requires that plan managers act with prudence, expert skill and due diligence in making asset allocation decision without imposing any quantitative restrictions. The prudent person rule originates with the notion of a "trust". The rule does not set any specific limits, but generates legal duties and responsibilities that seek to minimize principal-agent problems. The most important advantage of the prudent person rule is that it provides scope to take full account of the return objectives, given the long investment horizon of pension funds. The rule has two main drawbacks. The first is that it requires more sophisticated financial knowledge and expertise to implement than quantitative restrictions. Second, from a supervisory perspective, the prudent person rule requires jurisprudence on what constitutes prudent behaviour and what does not.

A final point to consider in this context is that regulation, not directly governing the investment of pension assets may nonetheless have a significant impact on allocations. One such example can be differential tax treatment of different financial asset classes, and for that matter real asset classes. In a system of voluntary pension savings, individuals may prefer to save through housing markets rather than through pension plans. Although there are numerous factors that can explain such choices, taxation may clearly be one.

A second example of how regulation may impact the allocation of pension assets are the new accounting rules introduced in many OECD member states in recent years, governing how corporations must account for post employment benefit schemes - including defined benefit (DB) pension plans. The key aspects of the new rules are that; (1) plan assets are now required to be marked-to-market as opposed to amortised at historic cost, (2) liabilities must be discounted with a market yield and (3) the plan balance of post employment benefits plans must now be included on the corporate balance sheet, as opposed to merely figuring in a footnote.

The basic problem of any pension plan is to invest assets (A_t) and contributions (C_t) to meet future liabilities. If the plan is able to generate return equivalent to the internal rate of return of the plan ($i_r r$), this problem is solved. The new accounting rule imposes an inter-temporal constraint that in each point of time, the value of plan asset must be equivalent to the value of accumulated future pension liabilities (AC) discounted by the market yield (and not the plans internal rate of return). While the new accounting rules thus bring welcome transparency, these also impose constraints on plan volatility that may be of little relevance to the overall economic health of the plan.

$$A_0 + \sum_t C_t (1 + irr_p)^{-t} = \sum_t L_t (1 + irr_p)^{-t}$$

$$\textit{subject to the constraint } A_t = \sum_t L_t^{AC} (1 + r)^{-t} \textit{ for } t \in [0, 1, 2, \ldots, n]$$

6.2 Pension plan investment and the macroeconomy

In the following sections we consider the implications of different investment strategies at macro-economic level. At the microeconomic level, we found justification for a lifecycle approach - preferably built with inflation-linked bonds (and ideally even inflation-linked longevity bonds). Given the dynamic nature of the lifecycle strategy, an overlapping generation (OLG) model provides the most suitable framework to study this question.

In the following discussion, we consider the allocation of pension assets to five asset classes; (1) nominal government bonds, (2) inflation-linked government bonds, (3) equity (4) private equity and (5) real-estate. For each asset class, we consider how an "excessive" allocation to this asset class may impact the overall macro-economy. Specifically we consider a simple OLG model to illustrate the potential negatives of the "safe" strategy of pension assets being 100% allocation to government bonds.

6.2.1 Nominal government bonds

Although our subject matter here is the allocation of funded DC plans, funded DB plans make a good starting point for our discussion in view of the new accounting rules outlined above. Indeed, from a macro-economic point of view, whether the funds allocated stem from DB or DC plans should have little impact, unless one considers a direct link to consumer behaviour (we shall ignore such links for the purpose of this paper, although this could be a topic for future research).

As discussed above, DB plans are increasingly subject to intertemporal constraints that are encouraging a shift of DB asset towards less volatile fixed income instruments, away from riskier assets classes, such as equities. DB pension plan demand for fixed income instruments (whether though cash bonds or swap markets) is often cited as one of - albeit far from the sole - cause behind the current level of low long term interest rates and flat yield curves on the major government bonds markets. To the extent that pension fund allocations push long-term government bond yields below fair value, we identify two potential negatives. Note, that for Dc plans, government bonds are still often viewed as the "safe" asset class, be it by regulators, plan beneficiaries or plan sponsors.

First, extensive academic literature has shown that the allocation of the capital stock in an economy to the public sector (through government bonds) rather than to the private sector (through equities) results in lower trend potential GDP growth as productive investment is crowded out through a hihger cost of capital. Ultimately, this effect will dampen productivity and thus potential trend growth of the economy. In this context, it is interesting to note that corporations can also use fixed income instruments to finance investment. Recent years have indeed seen strong demand for corporate bonds, which in turn has resulted in a significant compression of risk premium. The new risk then becomes that in an environment of easy financial conditions, corporations over invest, ultimately resulting in a capacity overhang.

Second, low government bond yields may tempt governments to adopt a less disciplined approach to fiscal policy. Against the backdrop of an ageing society - an hence a shrinking supply of labour in the absence of an increase in the retirement age - both of the factors described above become all the more critical.

6.2.2 A simple OLG model on the impact of higher government debt

We consider a standard overlapping generations model (OLG), developed by Crettez, Michel and Wigniolle, with young, old individuals and a government to illustrate the negative impact of the public debt increase on the economy. An increasing debt generates **a crowding out effect**. The young individual consumes c_t, work and earns a wage w_t whereas the older individual only consume d_{t+1}.We note s_t the saving rate and we assume a time separable log-utility function defined as follow:

$$U_t = (1-a)lnc_t + alnd_{t+1}$$

where $0 < a < 1$ and it parametrizes the time preference. By solving the maximization program of the consumer, we find the basic result of the Diamond model :

$$c_t = (1-a)(1-\tau_t)w_t$$

$$s_t = a(1-\tau_t)w_t$$

$$d_{t+1} = R_{t+1}a(1-\tau_t)w_t$$

We also consider a firm in the model and the technology is given by a standard Cobb Douglas production function $F(K_t, L_t) = AK_t^{\alpha}, L_t^{1-\alpha}$. The aim of the firm is to maximize its profit, in other words they equalize the marginal productivity of the input to their price:

$$w_t = (1-\alpha)AK_t^{\alpha}L_t^{-\alpha}$$

$$R_t = \alpha K_t^{\alpha-1}L_t^{1-\alpha}$$

We let B_t the government debt and it has a one period maturity. The debt B_t is issued at time $t-1$ and paid the same interest rate of the capital R_t. The government has to finance G_t in current expenditure and to pay $(1+r_t)B_{t-1}$ in debt servicing cost. The government withholds taxes on wages. Let τ_t be the taxe rate. The budget constraint of the government is

$$B_t = R_t B_{t-1} - N_t \tau_t w_t + \gamma Y_t$$

where B_t is the government debt at the date t, G_t represents government expenditures and the young individual (N_t) pays a tax to the government τ_t. We assume that government spending evolves like the output i.e. $G_t = \gamma Y_t$. We are interested in studying the dynamic of the economy in order to appreciate the impact of public debt on the capital accumulation.

We define a new variable $b_t = B_t/N_t$ which can be viewed as the debt per young individual. Note that the capital stock is given by the relation $K_{t+1} = I_t = N_t - B_t \forall t > 0 I_t \geq 0$. We can rewrite the government budget constraint dividing it by N_t.

$$\frac{B_t}{N_t} = R_t \frac{B_{t-1}}{N_t} - \frac{N_t \tau_t w_t}{N_t} + \frac{\gamma Y_t}{N_t}$$

$$b_t = \frac{\alpha A k_t^{\alpha-1}}{1+n} b_{t-1} + A k_t^{\alpha}(-tau(1-\alpha)+\gamma)$$

$$b_t = \frac{\alpha A k_t^{\alpha-1}}{1+n} b_{t-1} + \delta_t \tag{13}$$

In the same vein, the dynamic of the capital is given by

$$K_t + 1 = I_t = N_t - B_t \Leftrightarrow \frac{K_{t+1}}{N_{t+1}}(1+n) + b_t = s_t$$

$$(1+n)k_{t+1} + b_t = \alpha(1-\tau_t)(1-\alpha)A k_t^{\alpha} \tag{14}$$

To describe the dynamic of the economy let $x_t = b_t/s_t$. Substituting the different elements we thus obtain

$$x_t = \frac{1}{s_t}\left[\frac{R_t}{1+n} b_{t-1} + \delta_t\right]$$

$$x_t = \frac{\alpha}{a(1-\alpha)(1-\tau_t)}\left(\frac{x_{t-1}}{1-x_{t-1}}\right) + \frac{\gamma-(1-\alpha)\tau_t}{a(1-\alpha)(1-\tau_t)}) \tag{15}$$

and we can express the dynamic of the capital as a function of x_t

$$(1+n)k_{t+1} = a(1-\alpha)A k_t^{\alpha}(1-\tau_t)(1-x_t)$$

$$k_{t+1} = \frac{a(1-\alpha)Ak_t^{\alpha}(1-\tau_t)(1-x_t)}{(1+n)} \tag{16}$$

We can appreciate the impact of x_t on k_{t+1} just looking at the previous equation. Indeed there is a negative impact of x_t on the capital accumulation. The more the stock of debt per young there is, the more the capital accumulation is reduced. Intuitively, we can consider a situation where the government decides to increase the bond issuance in order to run an accommodative financial situation (low interest rate). Even if the cost of this new debt issue is lower, the cost of servicing the new debt will ultimately push up taxes in future and this burden will be supported by the coming generation. Thus the capital accumulation dynamic is reduced in the long run.

6.2.3 Inflation-linked government bonds

In previous sections, we have already highlighted the key role of inflation linked bonds within the context of pension plans. Supply of inflation linked bonds is very low raising a scarcity problem. Issuing more inflation linked bonds to meet the supply poses an interesting dilemma for governments. On the one hand, the real interest rate on these issues is quite low and provided that governments believe in the credibility of the explicit and implicit price stability targets of the major central banks, they should pursue a more aggressive issuance on inflation linked bonds. In doing so, however, they would be assuming the inflation risk in the economy. In our opinion, as the policy setter government should in theory be better armed to assume this risk. While the impact of ageing on inflation is a hotly debated topic, we favour the idea that all things being equal ageing will push consumer prices higher.

6.2.4 Equity and Private Equity

Much as a policy that drives and excessive demand for bonds is seen as a negative, so too would be a policy that drives excessive demand for equity. Risk capital in the economy is important for investment, innovation and economic growth. If demand for such products outstrips supply, the resulting compression in risk premium could result in poor decisions. An example hereof was seen with the equity bubble in the late 1990s.

One trend that we currently observed at the macro level is a strong demand for private equity. The classical argument for private equity in relation to pensions is that in buying listed equities, pension funds are paying for liquidity that they do not require due to their long horizons. As such, pension plans should favour private equity. The size of the private equity market is still relatively small compared to the size of pension savings and there is a risk that too many funds will chase too few assets, again compression risk premium.

6.2.5 Real estate

Following from our discussion on real estate at the micro level, the special role of housing as both an investment and consumption good is very important. At the macro economic level, a well functioning housing market is very important. In the discussion on investment for DC plans, there appears to be an interesting opportunity to link housing and pensions; first, by allowing younger persons to borrow against pension savings to acquire their principal residence, and second, by allowing older persons to "annuities" their how by extracting mortgage equity and transforming this into a stream of annuities.

6.2.6 Human capital

The final asset class we mention is human capital. In the micro analysis, the key role of human capital for pension investment decisions is evident. At the macro economic level, flexible labour markets are a well known condition for economic growth. This is perhaps one of the most important areas in which progress need to be made in the face of an ageing population.

6.2.7 Lifecycle Plans

Drawing together the points above, an investment policy that favours any one asset class alone appears to offer drawbacks at the macroeconomic level. In this context, lifecycle plans appear quite attractive in a society with a well balanced demography. In this situation, different individuals will be in different situations in the lifecycle, giving a fairly balanced macro allocation over time, thus avoiding excesses. In a quickly ageing population, however, there is a concern that the asset allocation of the overall economy will shift increasingly towards safe fixed income instruments. Two points deserve mention in this context. First, if labour markets are flexible we have seen those individuals are willing to take more financial risk than when labour markets are inflexible. Second, in an open economy what is important is less the demographics of the individual economy, but that of the global economy. As older Europeans sell equities into retirement, there will be an appetite for such assets elsewhere. While the overall world population is ageing, the pace hereof is much lesser that for the OECD area.

7 Conclusion

Drawing the points above together, this paper finds that the lifecycle approach, put forward by many financial advisors, offer a number of favourable elements for DC plans. The paper, nonetheless, identifies a number of axis for improvement including; (1) broader use of inflation-linked products, (2) products linking housing and pensions, and (3) the opportunity for longevity bonds. At the macroeconomic level, lifecycle plans appear to offer lesser drawbacks than alternatives, and notably of the "safe" strategy dominated by investment in government bonds. In this context we raise the importance of security flexible labour supply as a key element. Moreover, it would seem that a case can be made for governments to issue more inflation linked bonds to secure the credibility of price stability targets and take some role in assuming the pension risk currently shift onto individuals in the shift from DB based pension systems to systems that are becoming increasingly reliant on funded DC.

The shift to funded DC plans has sparked much debate on the optimal asset allocation for these plans over the lifetime of the individual plan beneficiary. Analysing this **microeconomic perspective**, the initial LPS framework suggested that portfolio choices over multiple periods do not introduce extra risk tolerance as a function of the stage of an individual in his lifecycle. Later expansions of this model, however, showed that in easing some of the very restrictive assumptions, the tolerance for risk does indeed change over the lifecycle. The role of human capital is of particular importance in this context. At the microeconomic level, flexible human capital offers an implicit insurance to the individual in the event of a bad outcome on the investment policy.

In this discussion, we placed our main focus on the investment policy of pension plan assets primarily up to the retirement age, i.e. during the working life. Tacking the management of the pension asset in retirement via "insurance" like solutions is an area that deserves a great deal more work - both in terms of academic research, but also from practitioners. One area that may merit exploration in this context is the asset liability management (ALM) techniques - including liability driven investment (LDI) - currently targeted to the DB industry. This is where financial engineering may well be able to add value not just to the management of DB plans, but also to that of DC plans. Although DC plans do not have "explicit" liabilities as is the case for DB plans, DC plans clearly have "implicit" liabilities. The most important challenge at the DC level is, however, to ensure investment solutions that can be managed at a reasonable cost and be understood by individuals with very little financial knowledge. Compared to other more complex solutions, this is where lifecycle plans hold one of their strongest advantages, generally fulfilling both these criteria.

At the **macroeconomic level**, the very size of pension assets makes the investment decisions hereof a key vector for economic growth. Regulation influences pension assets through many channels, and it is important that regulators be aware hereof. The main concern we raised is that the new accounting rules may be forcing pension asset to invest more heavily in fixed income solutions, pushing bond yields below fundamental fair value at the cost of risking fiscal discipline. One of the key points at the macroeconomic level again relates to labour market flexibility. In addition, we find a case for governments to issue more inflation linked bonds to ensure the credibility of price stability targets and carry some of the risks from DC plans, which individual citizens are generally very poorly to carry.

Looking ahead, we see three main axes for our future research. First is to better address the technical difficulties arising from the absence of long-term historical data for inflation linked bonds. The second is to further explore how a tactical overlay strategy can take advantage of the mean-reverting properties of asset classes over the long-term. This involves further exploration of the economic factors that driven asset classes over the medium to long term, and the study of the correlation here between. This also involves developing macroeconomic scenarios to take account of the impact of ageing, and other structural changes such as globalization, on the economy in aggregate. The final is to further explore the macroeconomic dimension via OLG models to analyse the different dimensions of lifecycle investment. In particular, we look to extend the model to include multiple interest rates (government bond yields, corporate yields, equity risk premium, etc.), rather than the usual single interest rate still very frequently used in macroeconomic models. This research should also consider optimal financing structures for corporations. Moreover, further research on the important of real estate is clearly a vector for further exploration in this context. The final area that we seen room to explore further is the behaviour.

In writing this paper, our most important idea was to foster links between practitioners, like ourselves, and academic research. We hope to strengthen these links in the future to better address the challenges of investing pension plan assets.

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