Stability Consequences of Fiscal Policy Rules*

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Abstract

Using an optimisation based model with endogenous labour supply and a proportional tax rate we compare the stabilising properties of different fiscal policy rules. The economy is affected by shocks from both government spending and technology. The fiscal policy rule can be based on government liabilities or the government budget deficit. As both are given as measures of fiscal policy performance in the Stability and Growth Pact (SGP), we also use a fiscal policy rule based on the combination of the two. We compare the accounting definition of deficit with the economic definition which takes inflation into account. The fiscal policy rule based on debt, wiht monetary policy consistent with the Taylor principle, results in an unstable solution. However, a fiscal policy rule based on deficit produces stable solutions with a wide range of fiscal policy parameters. Moreover, we find that putting more weight on the deficit than the debt in the fiscal policy rule creates less cyclical responses to shocks. Finally we find out that the SGP definition of deficit performs as well as the real deficit based on the government budget constraint.

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1 Introduction

The link between monetary and fiscal policy has become a growing part of the literature of monetary economics and public finance economics[1]. The discussion was revived by Sargent and Wallace's (1981) unpleasant monetary arithmetic. The basic finding is that, in the case where the government budget constraint is fulfilled, an active monetary policy, ie a policy that raise the nominal interest rate by more than inflation increases, can stabilise the economy and ensures the uniqueness of equilibrium. At the same time a passive monetary policy, ie a policy that underreacts to inflation by raising the nominal interest rate less than the inflation increases, destabilises the economy.

Sargent (1982) refined Barro's (1974) idea about Ricardian equivalence theorem applied on the public debt, and describes a Ricardian fiscal policy regime to be one where fiscal policy has to ensure that the government's intertemporal budget constraint is always in balance. An other case is one where the government's intertemporal budget constraint may not be satisfied for arbitrary price levels. The latter is called a non-Ricardian regime by Woodford (1995). The fiscal theory of the price level states that the government's intertemporal budget constraint is satisfied only at the equilibrium price level[2], and the public debt is critical in determining the price level. Analogically to Woodford's (1995) Ricardian and non-Ricardian fiscal policy regimes, Leeper (1991) calls them passive and active fiscal policies. In Leeper (1991) both monetary and fiscal policy can not be active or passive at the same time in order to a stable equilibrium to exist. This finding is also supported by Evans and Honkapohja (2002a), who also make the same distinction to the polar cases by assuming that fiscal policy is either active or passive ex ante. Leith and Wren-Lewis (2000 and 2002) claim that by losing the assumption about the fiscal policy regime in question ex ante makes the determination about the active or passive fiscal policy regime impossible a priori. We adopt the view that the fiscal policy regime can be determined only ex post.

In this paper we argue that particular monetary and fiscal policy regimes are consistent with the stability of the economy while others are not. What are the options for fiscal policy to fulfil the government budget constraint, and how fiscal policy can be judged to be active or passive? We form a simple closed economy New Keynesian model with endogenous labour supply and no capital. The only form of taxes in the model is income taxes, which

[1] See eg Woodford (1994, 1995, 1996, 2001) and Sims (1994), Benhabib, Schmitt-Grohé and Uribe (2001).

[2] Note that part the definition of an equilibrium is that the government budget constraint is satisfied.

are proportional and have distortionary effects. Price stickiness is introduced by using the Rotemberg (1987) approach. Monetary policy follows a Taylor (1993) type interest rate rule. In the economy, there are two different type of shocks, government spending and technology shocks, which are independent of each other.

Maintaining price stability requires not only commitment to an appropriate monetary policy rule, but an appropriate fiscal policy rule as well (Woodford, 2001). The fiscal policy rules can based on government liabilities as in Leeper (1991). Woodford (2001) concludes that the fiscal policy rule based on the government budget deficit, with the Taylor rule type monetary policy, results in more attractive monetary-fiscal policy regime than the fiscal policy rule based on debt. Both debt and deficit are given as a measures of fiscal policy performance in the Maastricht treaty, and in this paper we formulate a general fiscal policy rule based on combinations of both. The definition of budget deficit is based on the Stability and Growth Pact (SGP) definition (accounting definition). We also compare the accounting definition of deficit to the real economic definiton which takes the effects of inflation into account. The fiscal rule used is of an error-correction type relating the changes of the tax rate to either debt, deficit, or both. The fiscal policy rule based on debt results in an unstable solution with monetary policy that is consistent with Taylor principle, ie with active monetary policy. However, the fiscal policy rule based on the SGP definition on deficit produces stable solutions with a wide range of fiscal policy parameters. Furthermore, we find that setting more weight on deficit than debt in the fiscal policy rule creates less cyclical responses to shocks than if the weight on debt is higher than on deficit. Finally we find out the SGP definition of debt performs as well as the real deficit based on the government budget constraint and, hence, using the SGP deficit in the fiscal policy rule is appropriate.

The paper is organised as follows. Section 2 goes through the optimisation problems for the household and the firm. In this section we also formulate the government sector excluding tax rules. In section 3 we formulate and represent the stability properties of the deficit, debt, composite and real deficit fiscal policy rules. We also show the impulse responses of the demand and supply shock with the different fiscal policy rules. The conclusions are drawn in section 4.

2 The Model

2.1 The Household

We begin by specifying an optimization based model with no capital. We assume that the representative agent in the economy owns the economy's representative firm. We use the money in the utility function approach to model money in the general equilibrium model like in Sidrauski (1967). A typical household seek to maximise a utility function[3]

$$E_t \sum_{t=0}^{\infty} \delta^t u\left(c_t, m_t, l_t\right) \tag{1}$$

subject to household's real budget constraint

$$c_t + m_t - (1 - \pi_t)m_{t-1} + b_t \leq \left(1 + r_{t-1}\right) b_{t-1} + w_t l_t (1 - \tau_t), \tag{2}$$

where c_t is real private consumption, m_t is real money balances, l_t is households' labour supply, b_t is government bonds hold by the household, w_t is the gross wage rate and τ_t is the tax rate[4]. The household's discount factor is δ and E_t is the expectation operator conditional on information available in period t. We assume that the utility function $u\left(c_t, m_t, l_t\right)$ is continuous, increasing and concave.

The first order conditions with respect to private consumption, real money balances and labour are

$$u_c\left(c_t, m_t, l_t\right) - \xi_t = 0, \tag{3}$$

$$u_m\left(c_t, m_t, l_t\right) - \xi_t + \delta E_t\left[\xi_{t+1}\left(1 - \pi_{t+1}\right)\right] = 0, \tag{4}$$

$$u_l\left(c_t, m_t, l_t\right) + \xi_t w_t (1 - \tau_t) = 0, \tag{5}$$

$$\xi_t = \delta E_t \xi_{t+1}\left(1 + r_t\right), \tag{6}$$

where ξ is the Lagrangean multiplier and subscripts note partial derivatives. Combining equations the first order conditions yield

$$\left[\frac{u_c\left(c_t, m_t, l_t,\right)}{E_t u_c\left(c_{t+1}, m_{t+1}, l_{t+1}\right)}\right] = (1 + r_t)\delta, \tag{7}$$

[3]The utility of the household depends on private and public consumption, which are assumed to be separable. See Railavo (2003) for further analysis.

[4]Inflation π is defined to be $\frac{P_t - P_{t-1}}{P_t} = \pi_t$, which implies that $1 - \pi_t = \frac{P_{t-1}}{P_t}$. The nominal interest rate R_t is $1 + R_t = \left(1 + r_t\right) / \left(1 - E_t \pi_{t+1}\right)$, where r_t is the real interest rate and $E_t \pi_{t+1}$ is the expected inflation rate.

$$u_m\left(c_t, m_t, l_t,\right) = u_c\left(c_t, m_t, l_t,\right)\frac{R_t}{1+R_t}, \tag{8}$$

$$u_l\left(c_t, m_t, l_t\right) = -\left[u_c\left(c_t, m_t, l_t\right) w_t(1-\tau_t)\right]. \tag{9}$$

Equation 7 is the Euler condition for optimal intertemporal allocation of consumption. Equation 8 states that the marginal rate of substitution between money and consumption is equal to the opportunity cost of holding money. The opportunity cost is directly related to the nominal interest rate. Equation 9 is the households labour supply function, which states that the marginal rate of substitution between labour supply and consumption is equal to the real net wage rate.

Now we assume a periodical utility function expressed in form $u\left(c_t, m_t, l_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\Gamma m_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda}$. This is a CRRA utility function, where $\sigma \geq 0$ is the measure of risk aversion and Γ is a positive constant. $\lambda \geq 0$ is the inverse of the labour supply elasticity. Using the periodical utility function, the first order conditions can be rewritten as

$$c_t^{-\sigma} = E_t c_{t+1}^{-\sigma}(1+r_t)\delta, \tag{10}$$

$$\Gamma m_t^{-\sigma} = c_t^{-\sigma}\frac{R_t}{1+R_t}, \tag{11}$$

$$-l_t^{\lambda} = -\left[c_t^{-\sigma} w_t(1-\tau_t)\right]. \tag{12}$$

To log-linearise equations 10 and 11, we first take natural logarithms[5] and rearrange to yield

$$\ln c_t = E_t \ln c_{t+1} - \frac{1}{\sigma}\ln(1+r_t) - \frac{1}{\sigma}\ln\delta, \tag{13}$$

$$\ln m_t = \ln c_t - \frac{1}{\sigma}\ln\left(1+\frac{1}{1+R_t}\right) + \frac{1}{\sigma}\ln\Gamma. \tag{14}$$

The equation 13 holds at the steady state with values $\overline{c}_t$ and $\overline{r}_t$ and equation 14 holds also at the steady state with values $\overline{m}_t$, $\overline{c}_t$ and $\left(1+\overline{R_t}\right)$. We note the steady state values of variables with bar and variables with hat defines logarithmic fractional deviations from steady state values. Subtracting the steady state values and using the definition that the logarithmic deviation from steady state, for example for consumption, is $\widehat{c}_t = \ln\left(\frac{c_t}{\overline{c}_t}\right)$, we can rewrite equations 13 and 14 to yield

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \frac{1}{\sigma}\widehat{r}_t, \tag{15}$$

[5] Note that $\frac{R_t}{1+R_t} = 1 - \frac{1}{1+R_t}$.

$$\widehat{m}_t = \widehat{c}_t - \frac{1}{\sigma}\widehat{R}_t. \tag{16}$$

Like in Walsh (2003, Chapter 5), the government purchases final output g_t in addition to the consumption by households. We use the economy wide recourse constraint to eliminate private consumption $\widehat{c}_t$ from equations 15 and 16. The economy wide resource constraint is

$$y_t = c_t + g_t. \tag{17}$$

We follow the representation of Uhlig (1999) to obtain log-linear approximations. Log-linearisation of the equation 17 around steady state yields

$$\widehat{y}_t = \frac{\overline{c}}{\overline{y}}\widehat{c}_t + \frac{\overline{g}}{\overline{y}}\widehat{g}_t. \tag{18}$$

Using the log-linearised recourse constraint equation 18, we can write equations 15 and 16 as deviations from steady state

$$\widehat{y}_t = E_t\widehat{y}_{t+1} + \frac{\overline{g}}{\overline{y}}\left[\widehat{g}_t - E_t\widehat{g}_{t+1}\right] - \frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}\widehat{r}_t, \tag{19}$$

$$\widehat{m}_t = \frac{\overline{y}}{\overline{c}}\widehat{y}_t - \frac{\overline{g}}{\overline{c}}\widehat{g}_t - \frac{1}{\sigma}\widehat{R}_t. \tag{20}$$

Now we want to write equations 19 and 20 in (log) levels. We use again the definition of the logarithmic deviations and the steady state versions of equations 13 and 14 in order to write

$$\ln y_t = E_t \ln y_{t+1} + \frac{\overline{g}}{\overline{y}}\left[\ln g_t - E_t \ln g_{t+1}\right] - \frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}r_t - \frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}\ln\delta, \tag{21}$$

$$\ln m_t = \frac{\overline{y}}{\overline{c}}\ln y_t - \frac{\overline{g}}{\overline{c}}\ln g_t - \frac{1}{\sigma}R_t + \frac{1}{\sigma}\ln\Gamma. \tag{22}$$

2.2 The Firm

The cost minimising firm hires labour[6], produces and sells products in monopolistically competitive goods market. The firm produces a single good with labour l_t and pays wages w_t per unit of labour. In each period the firm minimise the cost function

$$\min w_t l_t \tag{23}$$

subject to the production technology

$$y_t = A l_t, \tag{24}$$

[6] We assume perfectly competitive labour markets.

where A stands for technological development and is defined $A = \mu e^{\zeta * Time}$. The cost minimisation implies the following real marginal cost

$$\frac{\partial}{\partial y_t}\left[w_t\left(\frac{y_t}{A}\right)\right] = w_t\frac{1}{A} \equiv mc_t. \tag{25}$$

The equilibrium wages are given by the labour supply $l_t^S = c_t^{-\frac{\sigma}{\lambda}} w_t^{\frac{1}{\lambda}} (1 - \tau_t)^{\frac{1}{\lambda}}$, where labour supply depends on consumption and net wage, and labour demand $l_t^D = \frac{y_t}{A}$ from production function. Substitute the equilibrium wages $w_t = c_t^{\sigma}\left(\frac{y_t}{A}\right)^{\lambda}(1-\tau_t)^{-1}$ into the marginal cost equation and take natural logarithms to yield

$$\lambda \ln y_t - (1+\lambda)\ln A + \sigma \ln c_t - \ln(1-\tau_t) = \ln mc_t, \tag{26}$$

where $\ln A = \ln\mu + \zeta * Time$. Lets note $\ln \mu \equiv z_t$ and assume that the productivity z_t follows the stochastic process

$$z_t = \rho z_{t-1} + \nu_t \tag{27}$$

with $0 \leq \rho \leq 1$ and the white noise supply shock $\nu_t = i.i.d.\left(0, \sigma_\nu^2\right)$. Equation 26 holds also in the steady state, so percentage deviation of the marginal cost is given by

$$\lambda\widehat{y}_t - (1+\lambda)\widehat{z}_t + \sigma\widehat{c}_t - (1-\widehat{\tau}_t) = \widehat{mc}_t. \tag{28}$$

Substitute the log-linearised resource constraint $\widehat{c}_t = \frac{\overline{y}}{\overline{c}}\widehat{y}_t - \frac{\overline{g}}{\overline{c}}\widehat{g}_t$ into equation 28 to get

$$\left(\sigma\frac{\overline{y}}{\overline{c}} + \lambda\right)\widehat{y}_t - \sigma\frac{\overline{g}}{\overline{c}}\widehat{g}_t - (1-\widehat{\tau}_t) - (1+\lambda)\widehat{z}_t = \widehat{mc}_t. \tag{29}$$

A monopolistically competitive firm sets its price as mark-up over marginal costs. In the long-run equilibrium, the real marginal cost is equal to the inverse of the mark-up. Consequently, we obtain from equation 26 and the recourse constraint that the (long-run) supply function is given by[7]

$$\ln y_t^* = \frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}} + \lambda}\ln g_t + \frac{1+\lambda}{\sigma\frac{\overline{y}}{\overline{c}} + \lambda}\zeta * Time + \frac{1}{\sigma\frac{\overline{y}}{\overline{c}} + \lambda}\ln(1-\tau_t) + \varepsilon_t^{y^*}, \tag{30}$$

where y_t^* is flexible price output, which we call potential output. Note that $\left[(1+\lambda)/\left(\sigma\frac{\overline{y}}{\overline{c}} + \lambda\right)\right] z_t \equiv \varepsilon_t^{y^*}$. Now the level of potential output in society is

[7] We assume that in the steady state equilibrium $\ln \overline{mc} = \ln\frac{1}{\kappa}$, where κ is the mark-up. See Railavo (2003) for details.

affected by fiscal variables, government consumption and taxation, together with technology. An increase in government consumption will expand production possibilities. A decrease in taxation will increase potential output, since the household is willing supply more labour. Potential output equation 30 holds also in the steady state.

To find the pricing equation of the firm, we follow Rotemberg (1987). We assume that there exists costs to the firm when it changes prices. This assumption will introduce price stickiness and reflect the empirical aspect that individual price setting is lumpy. The forward looking firm sets prices by minimising a quadratic loss function

$$\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j\left[\left(\ln P_{t+j}-\ln P_{t+j-1}\right)^2+a\left(\ln P_{t+j}-\ln P^*_{t+j}\right)^2\right], \qquad (31)$$

where $\beta=\frac{1}{(1+r)}$, $r>0$ is discount factor and a is a adjustment cost parameter. The higher a is the more costly it is to the firm to change prices. Taking the first order conditions of equation 31 and replacing $(\ln P_t - \ln P^*_t)$ with marginal cost equation, we can write the Phillips curve in terms of deviations from the steady state

$$\widehat{\pi}_t=\beta E_t\widehat{\pi}_{t+1}+a\left[\left(\sigma\frac{\overline{y}}{\overline{c}}+\lambda\right)\widehat{y}_t-\sigma\frac{\overline{g}}{\overline{c}}\widehat{g}_t-(1-\widehat{\tau}_t)-(1+\lambda)\,\widehat{z}_t\right]. \qquad (32)$$

Rewriting equation 32 by using the notation $\widehat{y}_t=\ln\frac{y_t}{\overline{y}_t}$ and equation 30, we can write the expectations, technology and tax augmented Phillips curve as

$$\pi_t=\beta E_t\pi_{t+1}+a\left[\left(\sigma\frac{\overline{y}}{\overline{c}}+\lambda\right)\left(\ln y_t-\ln y^*_t\right)\right]. \qquad (33)$$

Current inflation depends on expected future values of inflation, not on past inflation. The model has a resemblance with Woodford (1999), where he points out, that there is an important dynamic link from expectations of the future to the present for both inflation and output. Leong (2002) finds support to the forward looking New Keynesian model from simulation exercises. Unlike Woodford (1999), we treat potential output y^*_t endogenously instead of assuming that it is an exogenous disturbance.

2.3 The Government

We construct the intertemporal budget constraint for policy authority, which links debt and policy choices. The consolidated real flow budged constraint of the public sector is

$$b_t + \tau_t y_t + \pi_t m_{t-1} + m_t - m_{t-1} = (1 + r_{t-1})\, b_{t-1} + g_t, \tag{34}$$

where b_t is the government bonds, $\tau_t y_t$ is the tax revenue, m_t is the nominal money balances, r_t is real interest rate and g_t is the public spending. The government balances its budged with new debt, taxes and seigniorage revenue $(\pi_t m_{t-1} + m_t - m_{t-1})$. The intertemporal government budget constraint is

$$\begin{aligned}(1+r)\, b_t \quad &\leq \quad \sum \left(\frac{1}{1+r}\right)^i (\pi_{t+i} m_{t-1+i} + m_{t+i} - m_{t-1+i} \\ &\quad +\tau_{t+i} y_{t+i} - g_{t+i}),\end{aligned} \tag{35}$$

which states, that the maximum level of outstanding debt including interest payments is determined by the discounted sum of seigniorage revenues and surpluses. Fiscal policy can rely on seigniorage funding to some extent. Schmitt-Grohé and Uribe (2002) show that even a small amount of price stickiness is sufficient to sustain low inflation tax, so that the government will rely more heavily on conventional taxes.

The government expenditure is characterised by

$$g_t = \rho^g \frac{g_{t-1}}{y_{t-1}} y_t + (1 - \rho^g)\, \overline{\gamma} y_t + \varepsilon_t^g, \tag{36}$$

where $\overline{\gamma}$ is a constant public consumption to GDP ratio, $0 \leq \rho^g \leq 1$ and $\varepsilon_t^g = i.i.d.\left(0, \sigma_{\varepsilon^g}^2\right)$. We assume that the two shocks hitting the economy, technology and government spending shocks, are independent of each other. Hence we do not need to specify the covariance of the two.

We assume that the interest rate is set according to the Taylor (1993) rule. The interest rate is set based on the domestic economic conditions, placing a positive weight on inflation and real output. Taylor suggested, that increase in the nominal interest rate should be more than one-for-one in response to inflation. We write the interest rate rule with respect of inflation deviations from inflation target and output deviation from potential output.

$$R_t = \pi_t + r^* + \eta_1 (\pi_t - \pi^*) + \eta_2 (\ln y_t - \ln y_t^*), \tag{37}$$

where r^* is the real interest rate in steady state, π^* is the inflation target, y_t^* is potential output at time t defined by equation 30. The rule represents interaction between monetary and fiscal policy, since y_t^* is affected by fiscal policy variables. Taylor principle is $\eta_1 > 0$ and $\eta_2 > 0$. The larger values η_1 gets the tighter is monetary policy. In the literature the discussion about the form of the interest rate rule has emphasised the simple, robust rule and stabilisation properties. We use the contemporaneous time interest rate rule, which according to Bullard and Mitra (2002) is stable with larger range of parameter values than the forward looking rule. Edge and Rudd (2002) argued that distortionary taxation increases the value of Taylor rule parameter consistent with the stability in the economy. To complete the model we formulate fiscal policy by using the tax rule, which is based on deficit, debt and combination of the two.

3 Stabilising Properties of the Model

3.1 Parameters

The calibration sets parameter values used in stability analysis and in the standard simulation of the theoretical model to the level, which are in line with the literature[8]. We set the risk aversion coefficient σ to 0.5 and the interest rate coefficient of the IS curve is $\frac{\bar{c}_t}{\bar{y}_t}\frac{1}{\sigma} = 1.5$. By setting $\lambda = 1.5$, the labour supply elasticity with respect to real wages is 0.67. The output coefficient in the Phillips curve is $a\left(\sigma\frac{\bar{y}_t}{\bar{c}_t} + \lambda\right) = 0.043$, when the adjustment cost parameter a is 0.02. The ratio of government consumption to output is set to 0.25, so $\frac{\bar{c}_t}{\bar{y}_t} = 0.75$ and $\frac{\bar{y}_t}{\bar{c}_t} = 1.34$. The output coefficient in the Taylor rule η_2 is set to be equal to 0.4, which is smaller than in the original Taylor (1993) rule, representing that monetary authority is less interested on output than suggested by Taylor. The inflation target $\pi^* = 0.02$ and the long term real interest rate r^* is 0.03.

The household discount factor δ is 0.98, but the firms discount rate β is set to be equal to one. The income elasticity of money demand is 1.34 and the interest rate elasticity of money is 2. The ratio of money balances to GDP is set to be equal to 0.12 by setting the coefficient $\Gamma = 0.3$. The share of public consumption of GDP is 25 percent and hence we set $\overline{\gamma} = 0.25$. The Maastricht Treaty defines deficit and debt requirements. Following it we set the deficit to GDP ratio $\psi_1 = 0.03$ and real debt to GDP target $\psi_2 = 0.6$.

We calibrate persistence of the technology shock hitting the economy by using Cooley and Prescott (1995), who find that 95 percent of the shock remains after one quarter, so in annual terms we set $\rho = 0.81$. Blanchard and Perotti (2002) estimated that 95 percent of government consumption shock is still present after two years. Following them we set $\rho^g = 0.975$. The parameter values reflect the economic structure of a large economy, such as the euro area.

[8] For calibration see eg Rotemberg and Woodford (1999), Clarida, Galí and Gertler (2000) and Bullard and Mitra (2002).

3.2 Deficit Rule

Recently monetary policy literature has emphasised the link between the degree to which monetary and fiscal policy respond to inflation rate, debt, deficit and macroeconomic stability. Leeper (1991) studied the fiscal policy rule based on government liabilities. The fiscal rule based on debt is widely used in the literature, eg Evans and Honkapohja (2002a) used the debt based fiscal policy rule to study learnability conditions of fiscal and monetary policy. Real debt is used in the fiscal policy rule and as mentioned in Woodford (2001), monetary policy affects the real value of outstanding debt through its effects on the price level. Hence, monetary policy has effects on real debt as well.

Woodford (2001) finds that there is an analogue between fiscal policy rules based on government liabilities and the government budget deficit. He concluded that fiscal policy based on the government budget deficit is more attractive monetary-fiscal policy regime with the Taylor rule type monetary policy than fiscal policy based on government liabilities. He also finds that the fiscal policy rule based on both debt and deficit results in a stable solution for the price level. In addition, Woodford (2001) states that when fiscal policy is consistent with stable prices, the policy regime may not preclude other equally possible rational expectations equlibria. Alternative fiscal policy commitments may instead exclude these undesired deflatory equlibria.

Below, we study four cases with different fiscal policy rules. First we use the government deficit besed rule as recommended by Woodford (2001) with the Taylor rule type monetary policy. The government budget deficit used is the accounting definition following the convention of the Maastricht treaty and the Stability and Growth Pact (SGP). Second, we use a rule with real government liabilities like in Leeper (1991). Third, we formulate a fiscal policy rule using both deficit and debt. This imitates the SGP measures of the fiscal policy performance and we can study how much weight should be put on each of the two. Finally we use the real definition of and compare it with the SGP definition of deficit which we explored first.

In the first case the fiscal authority reacts according to the rule based on the accounting definition of the government budget deficit. The government budget deficit is defined as the difference between tax revenue $\tau_t y_t$, government spending g_t and interest payments on the real debt outstanding $R_t b_{t-1}$[9]. This is the accounting definition we use to imitate the SGP practice to calculate

[9] We use the nominal interest rate to approximate the interest payments of real govenment debt instead the correct $R_t - \pi_t R_t$. With low inflation and nominal interest rates the latter term is relatively small.

deficit. The fiscal policy rule with the SGP deficit can be written

$$\tau_t = \tau_{t-1} + \Omega\left[g_t - \tau_t y_t + R_t b_{t-1} - \psi_1 y_t\right]/y_t, \tag{38}$$

where parameter $\psi_1 \geq 0$ can be interpreted to be a constant target level for the debt to GDP ratio like in Woodford (2001). If $\psi_1 = 0$ then we have a special case of the balanced budget rule in the long run. Woodford (2001) concludes that adoption of the deficit target in conjunction of the Taylor rule for monetary policy would create a regime consistent with stable and low inflation.

Note that the rule 38 is of an error-correction or gradualist type, relating the change, not the level of the tax rate to the deviation of the deficit fron the target. We feel that this is much more realistic than assumption of the immediate adjustment would be.

In order to find the fiscal policy parameters values consistent with stable and low inflation, we analyse stability of the model by using methods by Blanchard and Kahn (1980). When the model is written in state-space form, Blanchard and Kahn requirement is that the number of roots inside the unit circle should be equal to the number of non-predetermined variables for a unique solution under rational expectations. Using the terminology of Evans and Honkapohja (2002b), whether under rational expectations the system possesses a unique stationary rational expectations equilibrium (REE), the system is said to be determinate. If the system is indeterminate then multiple stationary solutions, including sunspot solutions, may exist.

The system is given by output equation 19, real money balances equation 20, potential output equation 30, government consumption 36, inflation equation 33, government budget constraint equation 34, interest rate rule equation 37 and tax rule equation 38, and has 2 non-predetermined variables, output and inflation $\{\widehat{y}, \widehat{\pi}\}$. Defining

$$\widehat{X}_t' = \begin{bmatrix} \widehat{y}_t & \widehat{\pi}_t \end{bmatrix}, \tag{39}$$

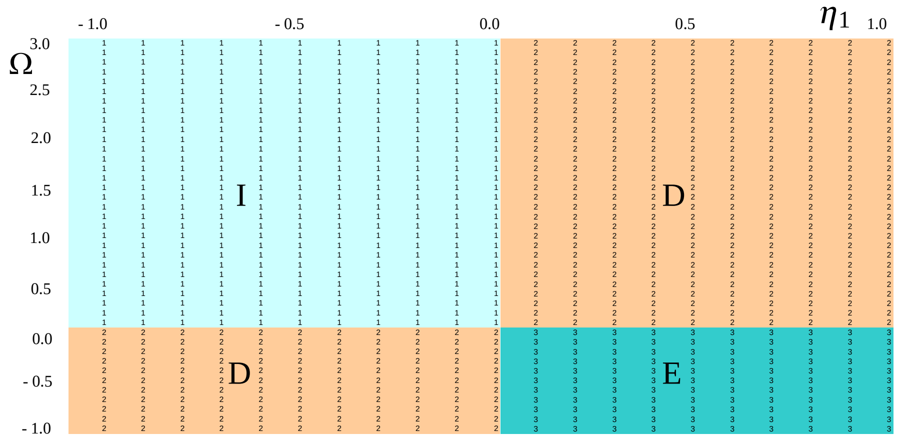
$$\widehat{x}_t' = \begin{bmatrix} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{bmatrix},$$

$$\epsilon = \begin{bmatrix} \varepsilon_t^{y^*} & \varepsilon_t^{g} \end{bmatrix},$$

where $\widehat{X}_t'$ is the vector of non-predetermined variables, $\widehat{x}_t'$ is the vector of predetermined variables and ϵ is the vector of shock variables we write the reduced form as

$$A\begin{bmatrix} \widehat{X}_t \\ \widehat{x}_t \end{bmatrix} = B\begin{bmatrix} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{bmatrix} + C\epsilon, \tag{40}$$

Figure 1: Determinate, indeterminate and explosive regions with the deficit rule.



which is equal to

$$\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = M \left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right] + N\epsilon, \tag{41}$$

where $M = A^{-1}B$. Matrix M is defined by suitable matrices A and B defined in Appendix A, whereas matrix $N = A^{-1}C$ is omitted. Matrix M is a 5×5 matrix and has 5 roots. We require the number of roots of matrix M inside unit circle to be two for determinacy.

Figure 1 shows the number of roots of matrix M inside the unit circle when then Taylor rule parameter η_1 for inflation runs from -1 to 1 and deficit rule parameter Ω runs from -1 to 3. Regions D, E and I are associated with parameter values of η_1 and Ω for which the solutions is determinate, indeterminate or explosive.

Following Leeper (1991) we define an active fiscal policy as such that it is not constraint by budgetary conditions, whereas passive fiscal policy must generate sufficient tax revenues to balance budget. The passive decision rule depends on government debt, summarised by current and past variables, while the active rule can be formed from more freely on past, current or expected future variables. The fiscal policy rule becomes more passive when the value of the fiscal policy parameter, relating taxes to debt or deficit, increases. Leith and Wren-Lewis (2000 and 2002) find that by excluding the assumption of a non-Ricardian regime means that the distinction between monetary and fiscal policy dominated regimes is difficult to make in advance. The introduction of distortionary taxation and endogenous labour supply links monetary and fiscal policy parameters in stability analysis in a way that it is impossible to make the distinction of dominant policy regimes a priori.

As we can see from figure 1, stable, determinate, regions in our modelare in the upper right hand side and in the lower left hand side corners. On the right hand side there exist a unique solution with the Taylor rule parameter η_1 larger than zero, which is Taylor (1993) requirement for the interest rate to react more than one-for-one to inflation, when the fiscal rule parameter is also larger than zero. The positive values of the fiscal policy parameters are reasonable in sense that policy authority reacts by increasing income taxes when the budget deficit increases. In the right hand side stable region monetary policy is always active, but the degree of the nominal interest rate response can vary. However, fiscal policy changes from active to passive as the value of Ω increases. Now fiscal policy can be active together with active monetary policy and still be consistent with the dynamical stability of the economy. This contradicts findings in Leeper (1991) and Evans and Honkapohja (2002a) who claim that fiscal policy can not be active together with active monetary policy and result in determinate solutions. Hence, we conclude that the distortionary tax rate together with supply side channel changes the interpretation and makes it possible to have an active monetary-fiscal policy regime with the fiscal policy rule based on the government deficit.

The other determinate area is found in the region where the Taylor principle is no longer valid. In the lower left hand corner, the fiscal policy parameter gets negative values. The negative fiscal policy parameter values are less sensible then the positive values, since with negative values the rise in deficit will lower the tax rate and the adjustment would happen through debt or inflation. In the upper left hand corner region with the negative Taylor rule parameter and the positive fiscal parameter larger than zero is indeterminate and there is no unique solution. The lower right hand corner displays parameter values for region with explosive solution.

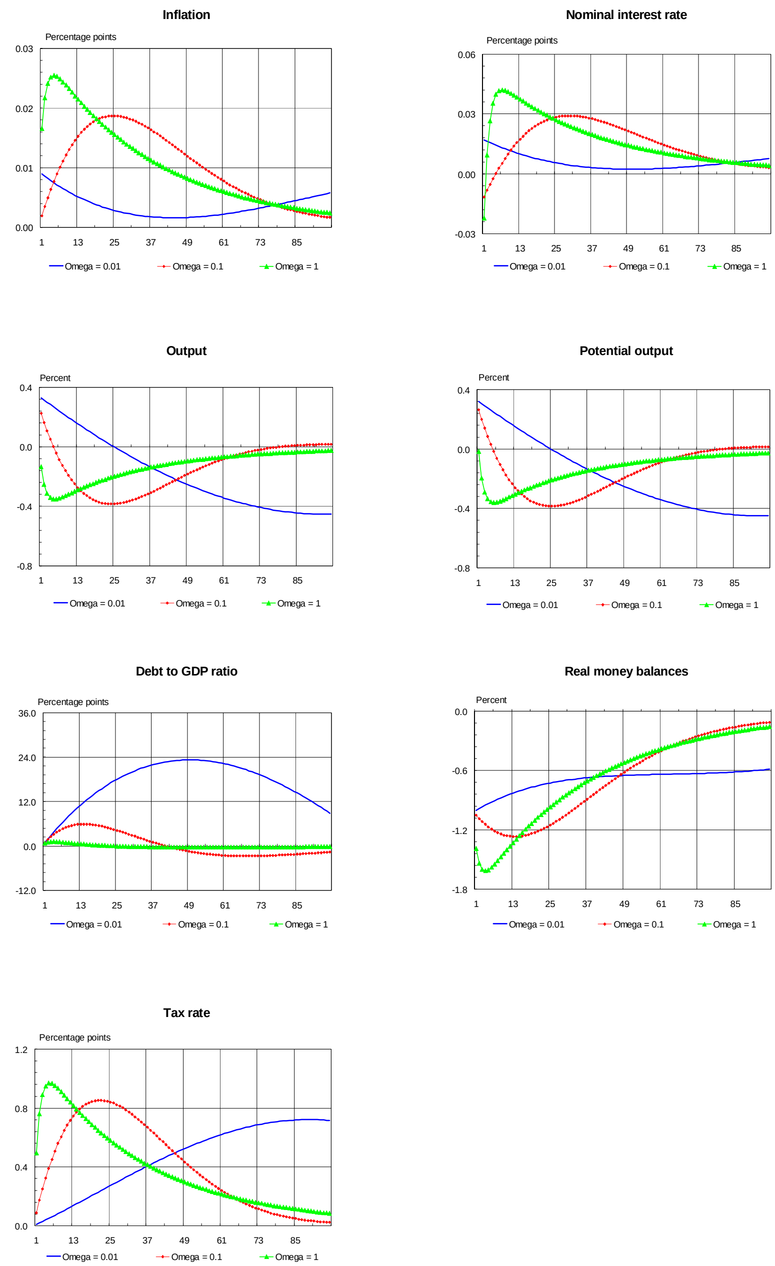
Some previous studies yield interesting results on the time profile of tax rates. Niepelt (2002) finds that with distortionary taxes and representative agent model, the optimal tax profile is required to be flat reflecting the tax smoothing properties in Barro (1979). Chari, Christiano and Kehoe (1991) suggest that as an outcome of optimal fiscal policy, the tax rate is roughly constant instead of being totally flat. Chari, Christiano and Kehoe (1994) state that labour tax inherits the persistence properties of the exogenous shock.

In figure 2 we show the responses to the government spending shock with the different fiscal policy parameter values. The government spending shock is transitory 1 % of GDP increase in public consumption. The shock is relative persistent, 95 percent of the shock still remains after two years. The solid line represents the case with $\Omega = 0.01$ which is active fiscal policy and tries to keep taxes virtually unchanged initially. The dotted line is more loose, but also active, fiscal policy with $\Omega = 0.1$ and the triangle line represents passive fiscal policy with $\Omega = 1$. The higher the weight on deficit in the fiscal policy rule the more interested the fiscal authority is keeping the budget balanced. The monetary policy parameter is set to be 0.5, which is the most common value for the Taylor rule parameter.

In the short run output response depends on the fiscal policy parameter. With low values of Ω output increase's about 0.3 percent initially, but high values result in a decrease in output even in the short run. In the long run resource constraint generates reduction of output due to growding out of private consumption. With high values of Ω the debt to GDP ratio is fairly constant and the tax rate reflects the pattern of the shock more closely than with low values of Ω. The higher the value of the fiscal policy parameter the more the tax rate chances initially and results in larger responses of inflation and nominal interest rate initially as the output gap tends to be positive. Low values of Ω put the economy through debt adjustment and we see that the initial debt finance of public consumption will be paid by increase in the tax rate in the future. Passive fiscal policy with large changes in taxation causes more inflation and less output in the short run than active fiscal policy.

Figure 3 shows the response to the 1 percent technology shock. The technology shock is also transitory, but less persistent than the government spending shock. 95 percent of the technology shock remains after one quarter. Again the solid line represents the case with $\Omega = 0.01$, the dotted line has $\Omega = 0.1$ and the triangle line shows responses with $\Omega = 1$. The technology shock has positive output and inflation effects. Differences in the fiscal policy parameter values have negligible small impacts on output, inflation and the interest rates. The technology shock increases output initially more than by one percent due to labour supply effects. As a result of the technology shock the tax rate drops initially and labour supply increases and output potential increases more than only due to the improvement in technology. The resulting negative output gap lowers inflation and the interest rate. The debt to GDP ratio decreases initially as the output increases. Passive fiscal policy with large values of Ω, tries to keep debt unchanged and taxes drop the most in the short run. As a result the debt to GDP ratio returns back to baseline values first. The ex-

Figure 2: Government consumption shock with the deficit rule. Deviations from baseline.



tremely small parameter of deficit rule keeps the tax rate stable but declining. The low parameter values generate very long swings in the tax rate.

3.3 Debt rule

An alternative to the deficit rule is to tie taxes to government liabilities. Leeper (1991) based his simple rule on debt, where the policy parameter is directly incorporated to the real government debt outstanding. In his model taxes were collected in lump sum and fluctuated around a constant. We use debt as an input in the error-correction type fiscal rule. The total government liabilities are $b_{t-1} + m_{t-1}$ and the debt rule is written

$$\tau_t = \tau_{t-1} + \phi \left[(b_{t-1} + m_{t-1}) - \psi_2 y_t \right] / y_t, \tag{42}$$

where $\psi_2 > 0$ can be interpreted to be the target level for the real government debt to the GDP ratio like in Woodford (2001).

The system is given by output equation 19, real money balances equation 20, potential output equation 30, government consumption 36, inflation equation 33, government budget constraint equation 34 and interest rate rule equation 37 with the tax rule being now the debt rule equation 42. The system has 2 non-predetermined variables, output and inflation $\{\widehat{y}, \widehat{\pi}\}$. We define

$$\widehat{X}_t' = \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array} \right], \tag{43}$$

$$\widehat{x}_t' = \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array} \right],$$

$$\epsilon = \left[\begin{array}{cc} \varepsilon_t^{y^*} & \varepsilon_t^{g} \end{array} \right],$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables. We write the reduced form as

$$A \left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = B \left[\begin{array}{c} E_t \widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right] + C\epsilon, \tag{44}$$

which is equal to

$$\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = M \left[\begin{array}{c} E_t \widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right] + N\epsilon, \tag{45}$$

where $M = A^{-1}B$. Matrix M is defined by suitable matrices A and B defined in Appendix B.

Figure 4 correspond to figure 1 in the case of the debt rule. In this figure the Taylor rule parameter η_1 for inflation runs from -1 to 1 and the debt rule parameter ϕ runs from -1 to 3.

Figure 3: Technology shock with the deficit rule. Deviations from baseline.

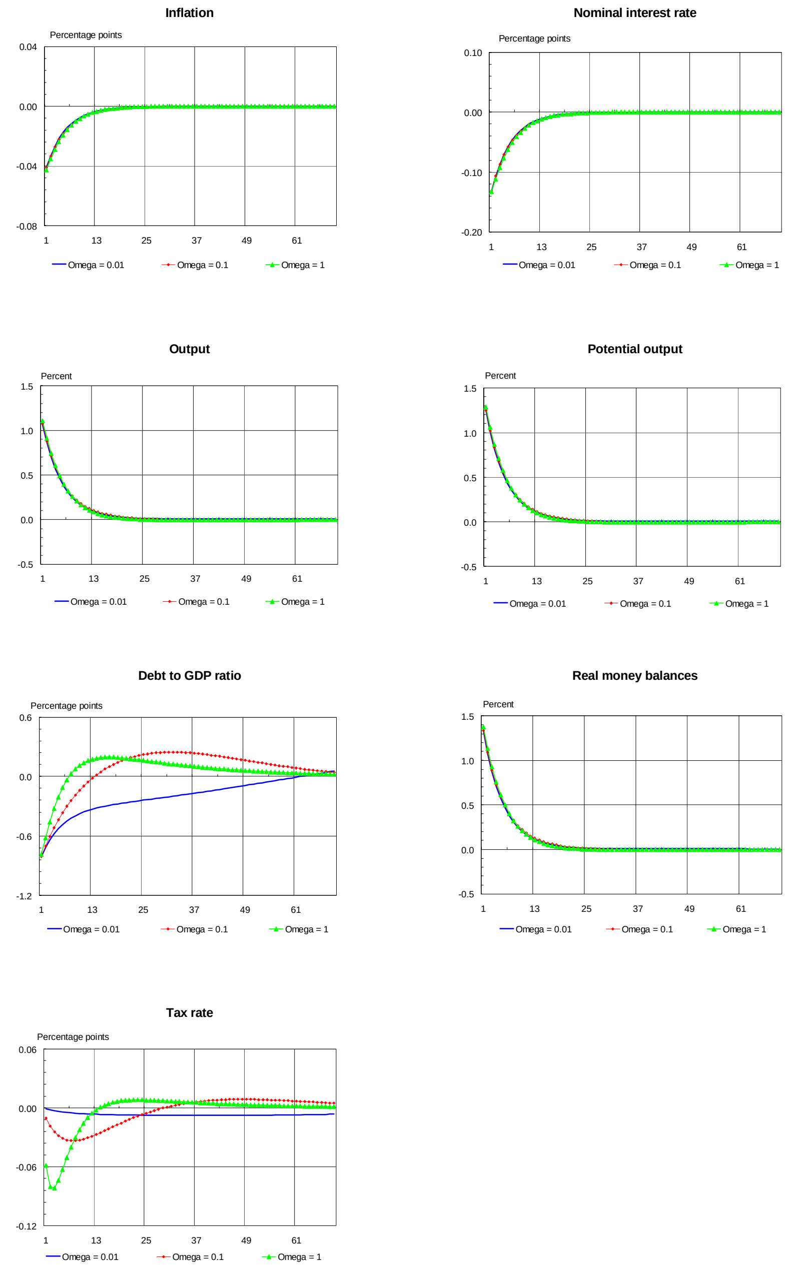
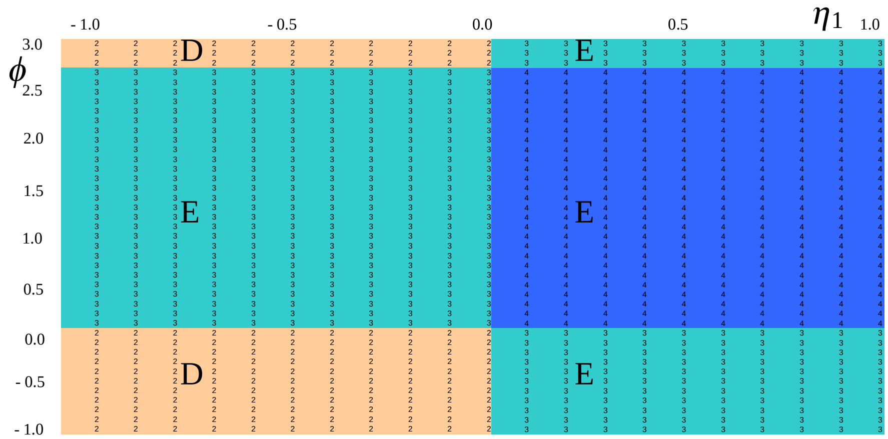


Figure 4: Determinate, indeterminate and explosive regions with the debt rule.



In figure 4 the determinate areas could be found in the upper and the lower left hand corners. The economy is always explosive with active monetary policy no matter what values the fiscal policy parameter gets. Railavo (2003) finds that the debt rule can be consistent with the dynamical stability of the economy with monetary policy that is consistent with the Taylor principle[10]. However, under tax smoothing assumption the debt rule turns out to result in unstable solutions with active monetary policy.

Interestingly the lower left hand side corner represents exactly same combination of policy parameters for determinate solutions as in Railavo (2003). The negative fiscal policy parameter values imply that increase in debt results in a lower tax rate, which is not very intuitive. Also in the upper left had corner there is a stable area if fiscal policy parameter is close to 3, which is extremely large since it means that for every 1 percentage point rise in the debt to GDP ratio the tax rate should increase by 3 percentage points. The debt rule results in a stable solution only if monetary policy do not fulfil the Taylor principle requirement, ie monetary policy is passive. We have defined

[10] Railavo (2003) finds that relative to the case of only demand side effects, introducing supply side effects of fiscal policy reduces the range of parameter values that result in determinate REE equilibria. The results were produced with the assumption that the tax rate evolves around a fixed tax rate like in Leeper (1991) not under random walk assumption in Barro (1979).

that the high values of ϕ mean passive fiscal policy. Hence, the determinate region consists of passive fiscal and monetary policy.

3.4 Composite rule

We also combine the previous rules. The composite fiscal policy rule follows the SGP convention about requirements for fiscal stability. Change in the tax rate response to the accounting budget deficit written in real terms and to the level of real debt outstanding. The composite fiscal policy rule is

$$\begin{aligned} \tau_t = \tau_{t-1} + \quad & \{\Omega\left[(g_t - \tau_t y_t + R_t b_{t-1}) - \psi_1 y_t\right] \\ & + \phi\left[(b_{t-1} + m_{t-1}) - \psi_2 y_t\right]\} / y_t, \end{aligned} \tag{46}$$

where $\psi_1 \geq 0$ and $\psi_2 > 0$ are the deficit and debt to GDP ratio targets respectively. The rule consists a systematic policy response to economic conditions. The fiscal authority responses to the debt by the magnitude ϕ and to the deficit by the magnitude Ω. Using the definion

$$\widehat{X}_t' = \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array}\right], \tag{47}$$

$$\widehat{x}_t' = \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array}\right],$$

$$\epsilon = \left[\begin{array}{cc} \varepsilon_t^{y^*} & \varepsilon_t^{g} \end{array}\right],$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables, we can write the reduced form as

$$A\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = B\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right] + C\epsilon, \tag{48}$$

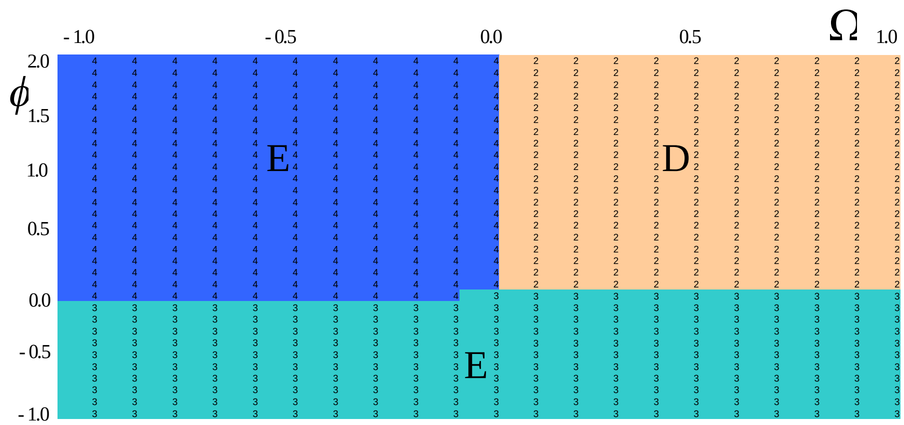
which is equal to

$$\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = M\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right] + N\epsilon, \tag{49}$$

where $M = A^{-1}B$. Matrix M is defined by suitable matrices A and B defined in Appendix C.

In figure 5 the parameter space in once again decomposed into determinate (D), indeterminate (I) and explosive (E) regions corresponding to RE solutions of the model. The deficit parameter Ω runs from -1 to 1 and the debt parameter ϕ runs from -1 to 2, while the Taylor rule parameter η_1 is hold constant at 0.5, ie the monetary policy is active.

Figure 5: Determinate, indeterminate and explosive regions with the composite rule when $\eta_1 = 0.5$.



We can see from figure 5 that the economy has a unique REE for the large range of positive values on debt and deficit when monetary policy is active. The same combination results in an explosive solution with passive monetary policy. Combination of the negative values on debt and deficit also results in an explosive solutions with both active and passive monetary policy.

Figure 6 fixes the debt parameter in the composite fiscal policy rule to $\phi = 0.1$ and shows the structure of the set of solutions of model, when the Taylor rule parameter runs from -1 to 1 and the deficit parameter Ω runs from -1 to 3.

Figure 7 repeats figure 6, but this time the deficit parameter is fixed at $\Omega = 0.1$ and the debt rule parameter ϕ runs from -1 to 3. If the deficit parameter is kept constant and low, the weight on debt can be set to high value with active monetary policy and the model is still determinate. In the opposite case when the weight on debt in the fiscal policy rule is kept constant and low, the weight on deficit has much more restriction for stability. The high weight on deficit together with the low weight on debt will destabilise the economy with active monetary policy, but actually stabilise it with passive monetary policy. The structure of the solutions is more complex with the composite rule than with previous two rules as we can see from the stripe pattern in figure 6. The solution has more complex roots than the other cases.

Figure 6: Determinate, indeterminate and explosive regions with the composite rule when $\phi = 0.1$.

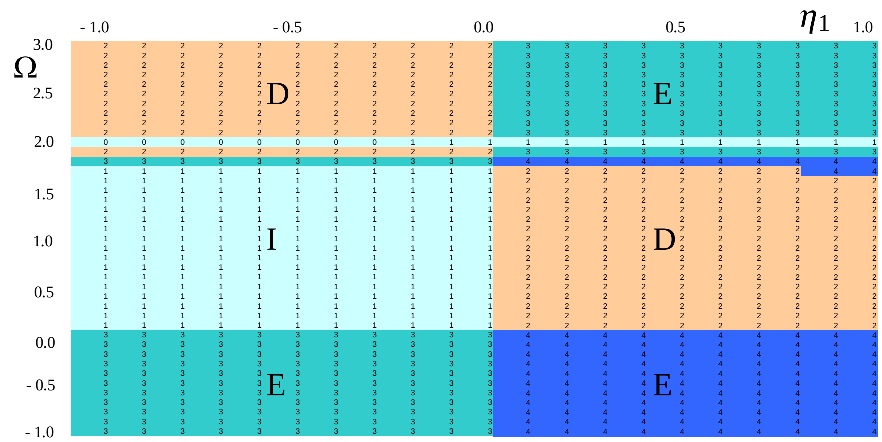
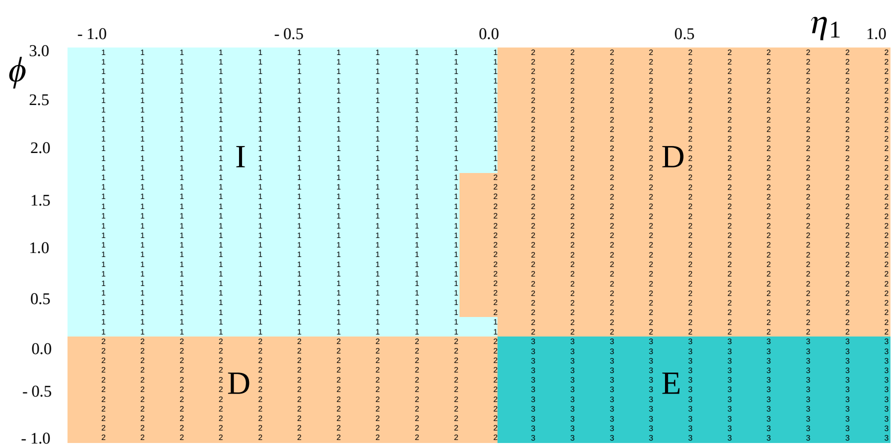


Figure 7: Determinate, indeterminate and explosive regions with the composite rule when $\Omega = 0.1$.



In figure 8 we show the dynamic responses to the government spending shock with the composite fiscal policy rule. The shock is similar than described above. The dotted line represents the case when there is more weight on deficit than on debt ie $\Omega = 0.5$ and $\phi = 0.1$. The triangle line is the opposite case with $\Omega = 0.1$ and $\phi = 0.5$. The responses to the shock show the feature that with more weight on debt than on deficit the economy is more likely to exhibit cycles. Koskela and Puhakka (2003) studied the effect of distortionary taxation on cycles in the OLG model. They found that there exist levels of the tax rate that changes the cyclical properties of the economy. Consequently, cyclicality is reduced under the tax rule with more weight on deficit than on debt. The total weight of deficit and debt is so high that increase in the government spending raises the tax rate so much that output actually declines also in the short run.

Figure 9 presents the responses to government spending shock. The solid line is the case with low weight on both deficit and debt ie $\Omega = 0.01$ and $\phi = 0.01$. The dotted line has higher weights, $\Omega = 0.1$ and $\phi = 0.1$ and the triangle line represent the passive fiscal policy with weights $\Omega = 1$ and $\phi = 1$. We see that reducing the weight on both deficit and debt makes the economy fluctuate with longer cycle. With the low weights the initial impact of the positive spending shock is positive for real values and negative for inflation. The responce of the tax rate is small, but the debt to GDP ratio increases heavily. With the higher weights the debt to GDP ratio remains quite unchanged, but the change in the tax rate reduces output. Inflation increases as well. The responses to technology shock has the same pattern. More weight on both deficit and debt reduces fluctuation. The impulse response functions of the technology shock with the composite fiscal policy rule are shown in figures 11 and 12.

3.5 Real deficit rule

Instead of relying on the SGP definition of deficit, we formulate the tax rule using the economic theory definition of deficit. The real deficit rule is derived from the government budged constraint. We rewrite the government real flow budged constraint as

$$b_t - b_{t-1} + m_t - m_{t-1} = g_t - \tau_t y_t + r_{t-1} b_{t-1} - \pi_t m_{t-1}, \qquad (50)$$

where the right hand side defines the real deficit. Now the fiscal policy rule for real deficit can be written as

$$\tau_t = \tau_{t-1} + \Omega \left[(g_t - \tau_t y_t + r_{t-1} b_{t-1} - \pi_t m_{t-1}) - \psi_3 y_t \right] / y_t, \qquad (51)$$

Figure 8: Government consumption shock with the composite rule. Deviatins from baseline.

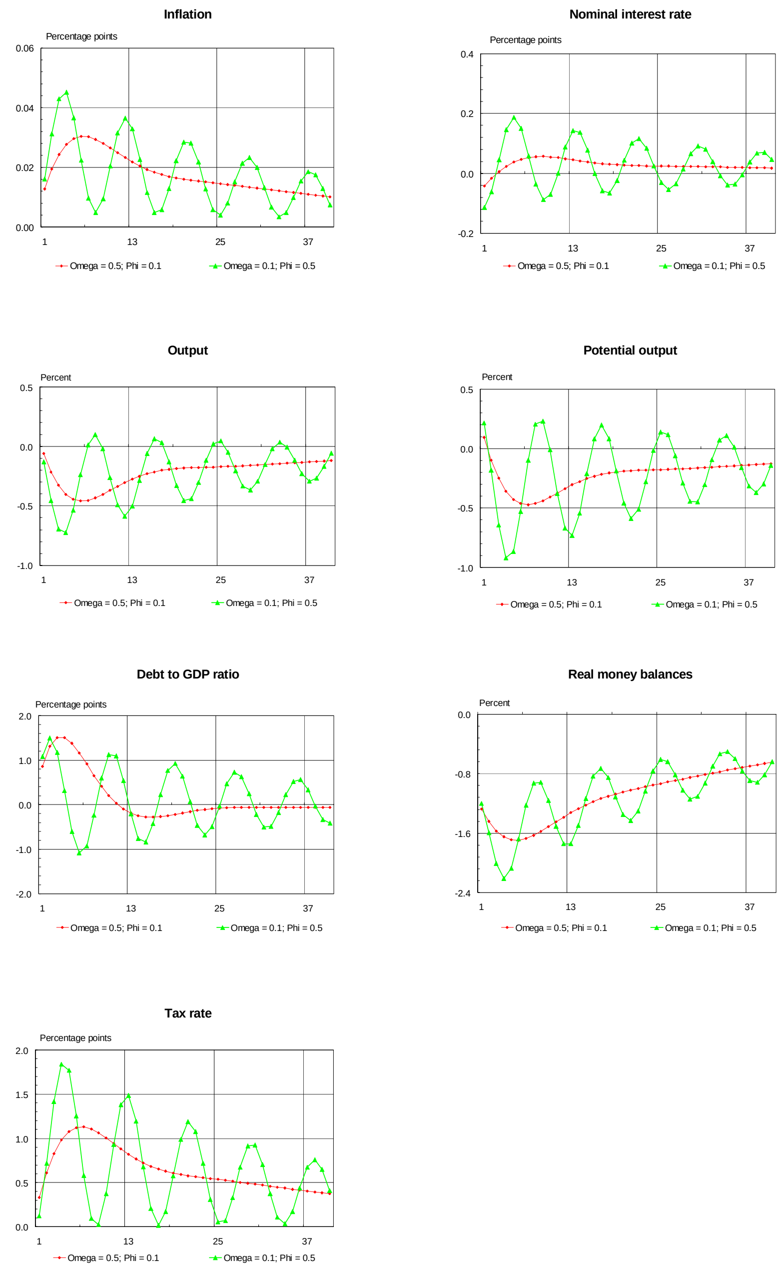
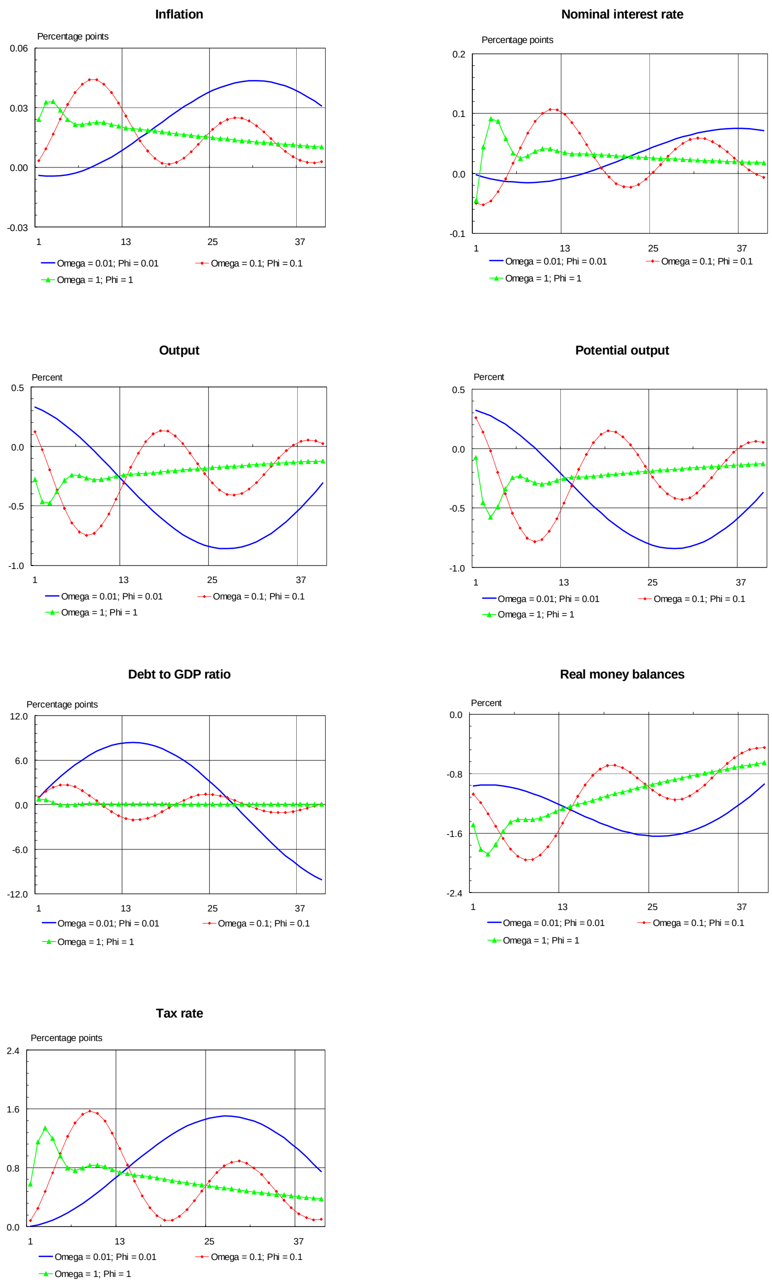


Figure 9: Government consumption shock with the composite rule. Deviations from baseline.



where $\psi_3 \geq 0$ is the target level for the real deficit to GDP ratio. Now, in addition to the primary deficit, the growth of the tax rate is affected by the real interest payments of debt and the real inflation tax on the money stock.

Figure 10 show impulse responses of government spending shock. The dotted line represents deficit rule with $\Omega = 0.1$ and the triangle line is the real deficit rule with the same policy parameter value. With the real deficit the tax rate reacts slightly less and hence the debt to GDP ratio adjust back to the baseline slower than with the accounting deficit definition. Therefore inflation responses with the real deficit definition are slightly smaller and the nominal interest rate rises less than with the accounting deficit rule. The difference of the impulse responses is relatively small between the different definitions of deficit. From figure 13 can be seen that the difference between the impulse responses are even smaller with the technology shock than they were with the government spending shock. It could be said that the SGP definition of deficit performs as well as the economic definition based on the government budget constraint. For simulation we have used the real deficit to GDP target $\psi_3 = 0.015$, which results in the same debt to GDP ratio than accounting deficit rule in the steady state.

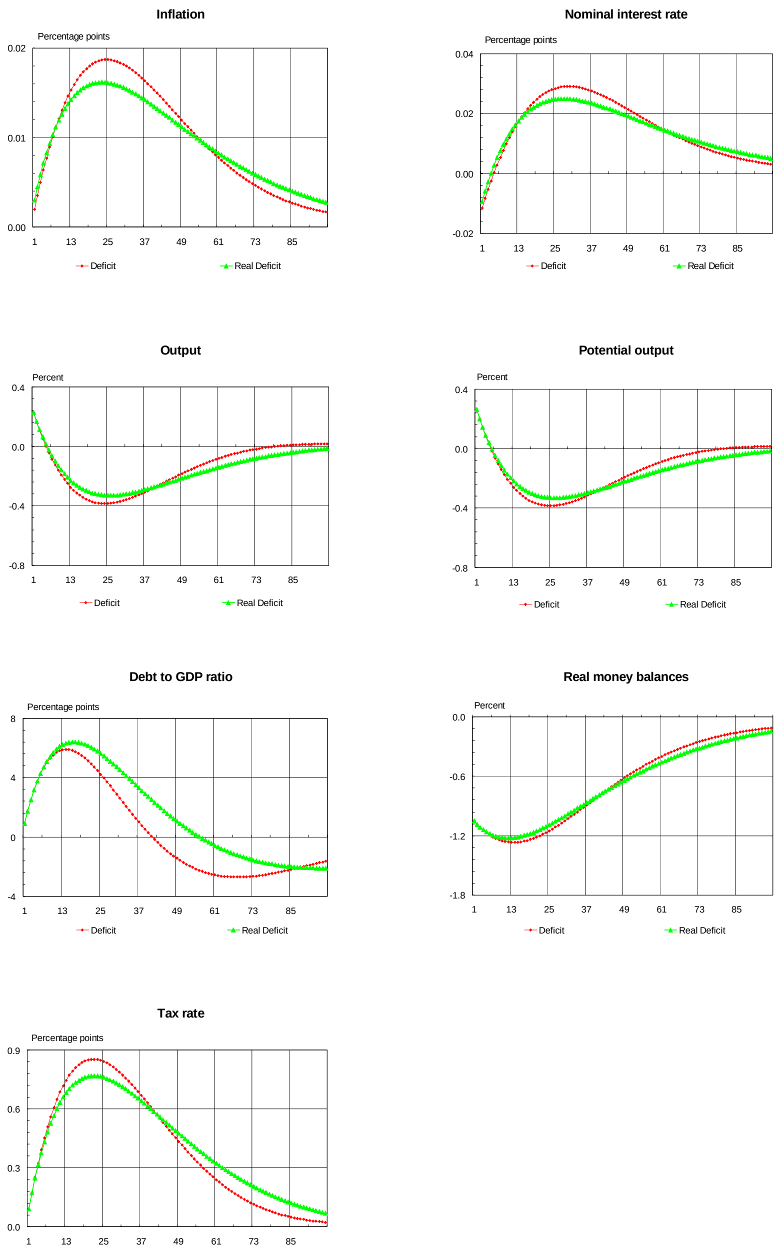
4 Conclusions

We have studied alternative fiscal rules in a New Keynesian model with distortionary taxes. In our gradualist, realistic specification the fiscal policy rule based on debt results in unstable solution with monetary policy that is consistent with the Taylor principle. The debt rule results in a stable solution only if monetary policy does not fulfil the Taylor principle requirement ie if monetary policy is passive and if the fiscal policy parameter gets high positive or negative values. Hence, we conclude that the debt rule results in a determinate solution for passive fiscal and monetary policy.

The fiscal policy rule based on the SGP definition of budget deficit results in a stable solution for a wide range of positive parameter values consistent with active monetary policy. We claim that fiscal policy can even be active together with active monetary policy and still be consistent with the dynamic stability of the economy. Hence, we conclude that the distortionary tax rate creating a supply side channel for policy changes the interpretation of active and passive monetary-fiscal policy regimes, and it is possible to have an active monetary-fiscal policy regime with the government deficit based fiscal policy rule.

The SGP sets requirements for both the debt to GDP and deficit to GDP

Figure 10: Government consumption shock. The deficit rule versus the real deficit rule. Deviations from baseline.



ratios. By forming the fiscal policy rule which combines the two, we can say that by setting more weight on deficit than debt tends to reduce the cyclicality of the dynamic response of the economy to the shocks to the government spending and technology. Cyclicality decreases also when the sum of weights on debt and deficit gets larger. At the same time the tax rate response to the government expenditure shock becomes so large that it reduces output also in the short run. This also happens with the deficit and real deficit rules when the value of the fiscal policy rule parameter gets large enough values, ie if the fiscal policy is passive and cares only about the stable debt to GDP ratio. With passive fiscal policy, the expansionary government spending shock actually decreases output and causes more inflation than active fiscal policy with same shock. On the other hand, the low values of fiscal policy parameter increases debt initially causing the tax rate to rise in the future and results in a debt driven cycle to the economy. The larger the fiscal policy parameter the more closely tax rate reflects the pattern of the shocks.

The SGP definition of deficit performs as well as the real deficit based on the real government flow budget constraint. The responses to the government spending and technology shocks are almost identical when we look output, inflation and the nominal interest rates. The only differences are in the debt to GDP ratio and in the tax rate responses. The difference of the two is, nevertheless, insignificant.

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A Appendix. Deficit rule

The system given by output equation 52, real money balances equation 53, inflation equation 54, potential output equation 56, government consumption equation 55, interest rate rule equation 57, government budget constraint equation 58 and deficit rule for tax rate equation 59. We use the log-linearisation techniques[11] in Uhlig (1999) to centre the government budget constraint 34 and tax rule for deficit 38 around constant steady state, and move them one period forward. We also write Taylor rule, potential output, government consumption and Phillips curve equations as deviations from the steady state. The system can be written as

$$\widehat{y}_t = E_t\widehat{y}_{t+1} + \frac{\overline{g}}{\overline{y}}\left[\widehat{g}_t - E_t\widehat{g}_{t+1}\right] - \frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}\left(\widehat{R}_t - \widehat{\pi}_{t+1}\right), \tag{52}$$

$$\widehat{m}_t = \frac{\overline{y}}{\overline{c}}\widehat{y}_t - \frac{\overline{g}}{\overline{c}}\widehat{g}_t - \frac{1}{\sigma}\widehat{R}_t, \tag{53}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + a\left[\left(\sigma\frac{\overline{y}}{\overline{c}} + \lambda\right)\left(\widehat{y}_t - \widehat{y}_t^*\right)\right], \tag{54}$$

$$\rho^g\widehat{g}_t - \rho^g\widehat{y}_t = \widehat{g}_{t+1} - \rho^g\widehat{y}_{t+1} + \widehat{\varepsilon}_t^g, \tag{55}$$

$$\widehat{y}_t^* = \frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}} + \lambda}\widehat{g}_t - \frac{1}{\sigma\frac{\overline{y}}{\overline{c}} + \lambda}\widehat{\tau}_t + \widehat{\varepsilon}_t^{y^*}, \tag{56}$$

$$\widehat{R}_t = (1+\eta_1)\,\widehat{\pi}_t + \eta_2\left(\widehat{y}_t - \widehat{y}_t^*\right), \tag{57}$$

$$\begin{aligned} & \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\left(\frac{1}{\overline{R}-\overline{\pi}} + 1\right)\right]\widehat{b}_t + \frac{\overline{m}}{\overline{y}}\left(\frac{1}{\overline{\tau}} - \frac{\overline{\pi}}{\overline{\tau}}\right)\widehat{m}_t \\ + \quad & \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\right]\widehat{R}_t = \\ & \widehat{y}_{t+1} + \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\frac{\overline{\pi}}{\overline{R}-\overline{\pi}} + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\right]\widehat{\pi}_{t+1} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\widehat{g}_{t+1} \\ & + \frac{1}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\widehat{m}_{t+1} - + \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\frac{1}{\overline{R}-\overline{\pi}}\right]\widehat{b}_{t+1} + \widehat{\tau}_{t+1}, \end{aligned} \tag{58}$$

[11] In log-linearisation we use notations $c_t = \overline{c}e^{\widehat{c}_t} \approx \overline{c}\,(1+\widehat{c}_t)$ and $\tau_t y_t = \overline{\tau}\overline{y}e^{\widehat{\tau}_t+\widehat{y}_t} \approx \overline{\tau}\overline{y}\,(1+\widehat{\tau}_t+\widehat{y}_t)$. By using the steady state conditions, the coefficients can be eliminated.

$$\frac{1}{\Omega}\widehat{\tau}_t + \left(1 + \frac{\psi_1}{\overline{\tau}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\widehat{b}_t = \tag{59}$$
$$\left(1 + \frac{\psi_1}{\overline{\tau}}\right)\widehat{y}_{t+1} - \left(\frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\widehat{g}_{t+1} - \left(1 + \frac{\psi_1}{\overline{\tau}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\widehat{R}_{t+1}$$
$$+ \left(\frac{1+\Omega}{\Omega}\right)\widehat{\tau}_{t+1}.$$

We solve the steady state tax rate by setting the steady state government budget constraint and steady state debt rule to be equal. Then solve for tax rate

$$\overline{\tau} = \frac{\overline{R} - \overline{\pi}}{\overline{\pi}}\psi_1 - \frac{\overline{R}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)}{\overline{\pi}} - \frac{\overline{R} - \overline{\pi}}{\overline{\pi}}\frac{\overline{g}}{\overline{y}}. \tag{60}$$

After some substitutions we can write the 8 equation system with 5 equations. Then we write the system in state-space form. Defining

$$\widehat{X}_t' = \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array}\right], \tag{61}$$
$$\widehat{x}_t' = \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array}\right],$$
$$\epsilon = \left[\begin{array}{cc} \varepsilon_t^{y^*} & \varepsilon_t^{g} \end{array}\right],$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables. The reduced form can be written

$$A\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = B\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right] + C\epsilon, \tag{62}$$

which is equal to

$$\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = M\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right] + N\epsilon, \tag{63}$$

where $M = A^{-1}B$. We omit the matrix N. The matrices A and B can be written

$$A = \left[\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & 0 & a_{15} \\ a_{21} & a_{22} & a_{23} & 0 & a_{25} \\ a_{31} & 0 & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{array}\right], B = \left[\begin{array}{ccccc} b_{11} & b_{12} & b_{13} & 0 & 0 \\ 0 & b_{22} & 0 & 0 & 0 \\ b_{31} & 0 & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & 0 & b_{55} \end{array}\right],$$

where

$$a_{11} = \left[1 + \frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}\eta_2\right], a_{12} = \left[\frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}\left(1+\eta_1\right)\right], a_{13} = -\left[\frac{\overline{g}}{\overline{y}} + \frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right],$$
$$a_{15} = \left[\frac{\overline{c}}{\overline{y}}\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right], a_{21} = \left[-a\left(\sigma\frac{\overline{y}}{\overline{c}}+\lambda\right)\right], a_{22} = 1,$$
$$a_{23} = \left[a\left(\sigma\frac{\overline{y}}{\overline{c}}+\lambda\right)\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right], a_{25} = -\left[a\left(\sigma\frac{\overline{y}}{\overline{c}}+\lambda\right)\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right],$$
$$a_{31} = -\rho^g, a_{33} = \rho^g,$$
$$a_{41} = \left[\frac{\overline{m}}{\overline{y}}\left(\frac{1-\overline{\pi}}{\overline{\tau}}\right)\left(\frac{\overline{y}}{\overline{c}} - \frac{1}{\sigma}\eta_2\right) + \left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\eta_2\right],$$
$$a_{42} = \left[-\frac{\overline{m}}{\overline{y}}\left(\frac{1-\overline{\pi}}{\overline{\tau}}\right)\frac{1}{\sigma}\left(1+\eta_1\right) + \left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\left(1+\eta_1\right)\right],$$
$$a_{43} = \left[\frac{\overline{m}}{\overline{y}}\left(\frac{1-\overline{\pi}}{\overline{\tau}}\right)\left(\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda} - \frac{\overline{g}_t}{\overline{c}_t}\right) - \left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right],$$
$$a_{44} = \left[\left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\left(\frac{1}{\overline{R}-\overline{\pi}} + 1\right)\right],$$
$$a_{45} = \left[\frac{\overline{m}}{\overline{y}}\left(\frac{1}{\overline{\tau}} - \frac{\overline{\pi}}{\overline{\tau}}\right)\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda} + \left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right],$$
$$a_{54} = \left(1 + \frac{\psi_1}{\overline{\tau}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right), a_{55} = \frac{1}{\Omega},$$
$$b_{11} = 1, b_{12} = \frac{\overline{c}_t}{\overline{y}_t}\frac{1}{\sigma}, b_{13} = -\frac{\overline{g}_t}{\overline{y}_t}, b_{22} = \beta, b_{31} = -1, b_{33} = 1,$$
$$b_{41} = \left[1 + \frac{1}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{\overline{y}_t}{\overline{c}_t} - \frac{1}{\sigma}\eta_2\right)\right],$$
$$b_{42} = \left[\left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{\pi}}{\overline{R}-\overline{\pi}} + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\left(1+\eta_1\right)\right)\right],$$
$$b_{43} = \left[\frac{1}{\overline{\tau}}\left(\frac{\overline{m}}{\overline{y}}\left(\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda} - \frac{\overline{g}_t}{\overline{c}_t}\right) - \frac{\overline{g}}{\overline{y}}\right)\right],$$
$$b_{44} = \left[\left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{1}{\overline{R}-\overline{\pi}}\right],$$
$$b_{45} = \left[1 - \frac{1}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda} - \right],$$
$$b_{51} = \left[1 + \frac{\psi_1}{\overline{\tau}} - \left(1 + \frac{\psi_1}{\overline{\tau}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\eta_2\right],$$
$$b_{52} = -\left[\left(1 + \frac{\psi_1}{\overline{\tau}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\left(1+\eta_1\right)\right],$$
$$b_{53} = \left[\left(1 + \frac{\psi_1}{\overline{\tau}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda} - \left(\frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\right],$$
$$b_{55} = \left[\left(\frac{1+\Omega}{\Omega}\right) - \left(1 + \frac{\psi_1}{\overline{\tau}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right].$$

B Appendix. Debt rule

Centre debt rule 42 around constant steady state to get

$$\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\widehat{m}_t+\frac{\phi}{\overline{\tau}}\left(\psi_2-\frac{\overline{m}}{\overline{y}}\right)\widehat{b}_t+\widehat{\tau}_t=\frac{\phi}{\overline{\tau}}\psi_2\widehat{y}_{t+1}+\widehat{\tau}_{t+1}. \tag{64}$$

We solve the steady state tax rate by setting the steady state government budget constraint and steady state debt rule to be equal. Then solve for tax rate

$$\overline{\tau}=\left(\overline{R}-\overline{\pi}\right)\psi_2-\overline{R}\frac{\overline{m}}{\overline{y}}+\frac{\overline{g}}{\overline{y}}. \tag{65}$$

Defining

$$\widehat{X}_t'=\begin{bmatrix}\widehat{y}_t & \widehat{\pi}_t\end{bmatrix}, \tag{66}$$

$$\widehat{x}_t'=\begin{bmatrix}\widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t\end{bmatrix},$$

$$\epsilon=\begin{bmatrix}\varepsilon_t^{y^*} & \varepsilon_t^{g}\end{bmatrix},$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables. The reduced form can be written

$$A\begin{bmatrix}\widehat{X}_t\\ \widehat{x}_t\end{bmatrix}=B\begin{bmatrix}E_t\widehat{X}_{t+1}\\ \widehat{x}_{t+1}\end{bmatrix}+C\epsilon, \tag{67}$$

which is equal to

$$\begin{bmatrix}\widehat{X}_t\\ \widehat{x}_t\end{bmatrix}=M\begin{bmatrix}E_t\widehat{X}_{t+1}\\ \widehat{x}_{t+1}\end{bmatrix}+N\epsilon, \tag{68}$$

where $M=A^{-1}B$. We omit the matrix N. The matrices A and B can be written

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13} & 0 & a_{15}\\ a_{21} & a_{22} & a_{23} & 0 & a_{25}\\ a_{31} & 0 & a_{33} & 0 & 0\\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55}\end{bmatrix}, B=\begin{bmatrix}b_{11} & b_{12} & b_{13} & 0 & 0\\ 0 & b_{22} & 0 & 0 & 0\\ b_{31} & 0 & b_{33} & 0 & 0\\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45}\\ b_{51} & 0 & 0 & 0 & b_{55}\end{bmatrix},$$

where $a_{11}-a_{45}$ and $b_{11}-b_{45}$ are like above and

$a_{51}=\left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{\overline{y}}{\overline{c}}-\frac{1}{\sigma}\eta_2\right)\right], a_{52}=-\left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\left(1+\eta_1\right)\right],$

$a_{53}=\left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}-\frac{\overline{g}_t}{\overline{c}_t}\right)\right], a_{54}=\left[\frac{\phi}{\overline{\tau}}\left(\psi_2-\frac{\overline{m}}{\overline{y}}\right)\right],$

$a_{55}=\left[1-\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right],$

$b_{51}=\frac{\phi}{\overline{\tau}}\psi_2, b_{55}=1.$

C Appendix. Composite rule

Centre composite rule 46 around constant steady state to get

$$\begin{aligned}
& \frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\widehat{m}_t + \left[\left(\frac{\frac{\overline{g}}{\overline{y}} - \overline{\tau} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}}\right)\right. \\
& \left.\left(\frac{\Omega}{\overline{\tau}}\overline{R} + \frac{\phi}{\overline{\tau}}\right)\right]\widehat{b}_t + \widehat{\tau}_t \\
= \quad & \left[\Omega + \Omega\frac{\psi_1}{\overline{\tau}} + \phi\frac{\psi_2}{\overline{\tau}}\right]\widehat{y}_{t+1} - \left[\frac{\Omega}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right]\widehat{g}_{t+1} - \\
& \left[\left(\frac{\frac{\overline{g}}{\overline{y}} - \overline{\tau} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}}\right)\frac{\Omega}{\overline{\tau}}\overline{R}\right]\widehat{R}_{t+1} + (1+\Omega)\,\widehat{\tau}_{t+1}.
\end{aligned} \tag{69}$$

We solve the steady state tax rate by setting the steady state government budget constraint and steady state debt rule to be equal. Then solve for tax rate

$$\overline{\tau} = \left[\frac{\frac{\overline{g}}{\overline{y}} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}} - \frac{\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}}{\overline{R} - \overline{\pi}}\right]\left[\frac{1}{\overline{R} - \frac{\phi}{\Omega}} + \frac{1}{\overline{R} - \overline{\pi}}\right]^{-1}. \tag{70}$$

Defining

$$\begin{aligned}
\widehat{X}_t' &= \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array}\right], \\
\widehat{x}_t' &= \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array}\right], \\
\epsilon &= \left[\begin{array}{cc} \varepsilon_t^{y^*} & \varepsilon_t^{g} \end{array}\right],
\end{aligned} \tag{71}$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables. The reduced form can be written

$$A\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = B\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right] + C\epsilon, \tag{72}$$

which is equal to

$$\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = M\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right] + N\epsilon, \tag{73}$$

where $M = A^{-1}B$. We omit the matrix N. The matrices A and B can be written

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & a_{15} \\ a_{21} & a_{22} & a_{23} & 0 & a_{25} \\ a_{31} & 0 & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 \\ 0 & b_{22} & 0 & 0 & 0 \\ b_{31} & 0 & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & 0 & b_{55} \end{bmatrix},$$

where a_{11} to a_{45} and b_{11} to b_{45} are like above and

$$a_{51} = \left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{\overline{y}}{\overline{c}} - \frac{1}{\sigma}\eta_2\right)\right], a_{52} = -\left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}(1+\eta_1)\right],$$

$$a_{53} = \left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda} - \frac{\overline{g}}{\overline{c}}\right)\right], a_{54} = \left[\left(\frac{\frac{\overline{g}}{\overline{y}} - \overline{\tau} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}}\right)\left(\frac{\Omega}{\overline{\tau}}\overline{R} + \frac{\phi}{\overline{\tau}}\right)\right],$$

$$a_{55} = \left[1 - \frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right],$$

$$b_{51} = \left[\Omega + \Omega\frac{\psi_1}{\overline{\tau}} + \phi\frac{\psi_2}{\overline{\tau}} - \left(\frac{\frac{\overline{g}}{\overline{y}} - \overline{\tau} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}}\right)\frac{\Omega}{\overline{\tau}}\overline{R}\eta_2\right],$$

$$b_{52} = -\left[\left(\frac{\frac{\overline{g}}{\overline{y}} - \overline{\tau} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}}\right)\frac{\Omega}{\overline{\tau}}\overline{R}(1+\eta_1)\right],$$

$$b_{53} = \left[\left(\frac{\frac{\overline{g}}{\overline{y}} - \overline{\tau} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}}\right)\frac{\Omega}{\overline{\tau}}\overline{R}\eta_2\frac{\sigma\frac{\overline{g}}{\overline{c}}}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda} - \left(\frac{\Omega}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\right],$$

$$b_{55} = \left[(1+\Omega) - \left(\frac{\frac{\overline{g}}{\overline{y}} - \overline{\tau} + \frac{\phi}{\Omega}\frac{\overline{m}}{\overline{y}} - \left(\psi_1 + \frac{\phi}{\Omega}\psi_2\right)}{\overline{R} - \frac{\phi}{\Omega}}\right)\frac{\Omega}{\overline{\tau}}\overline{R}\eta_2\frac{1}{\sigma\frac{\overline{y}}{\overline{c}}+\lambda}\right].$$

Figure 11: Technology shock with the composite rule. Deviations from baseline.

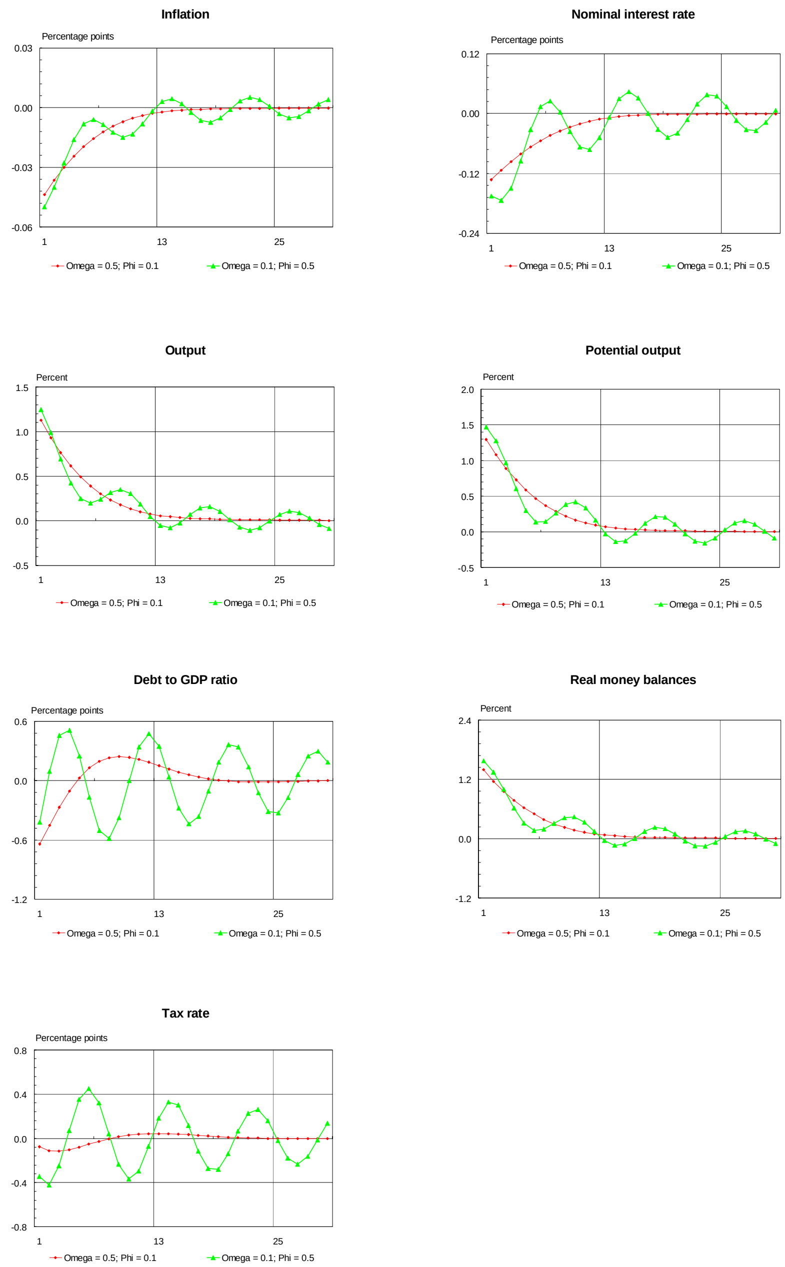

Figure 12: Technology shock with the composite rule. Deviations from baseline.

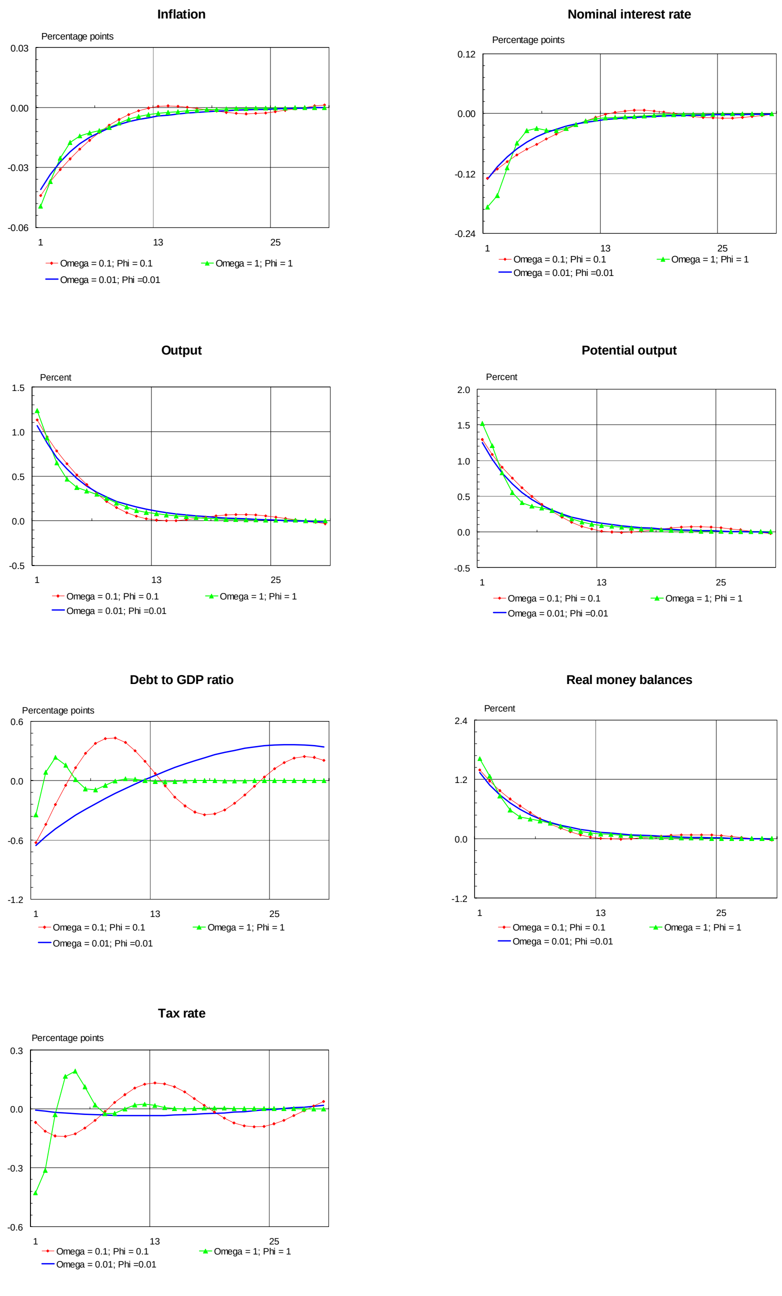

Figure 13: Technology shock. The deficit rule versus the real deficit rule. Deviations from baseline.