Fiscal Rules and Sustainability of Public Finances in an Endogenous Growth Model*

Barbara Annicchiarico[†] and Nicola Giammarioli[‡]

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[†]University of Rome "Tor Vergata". *E-mail*: barbara.annicchiarico@uniroma2.it

[‡]ECB - Fiscal Policies Division. *E-mail*: nicola.giammarioli@ecb.int

Abstract

This paper presents a two period overlapping generations model with endogenous growth in the presence of a public sector with objectives of convergence for public debt and primary balance to GDP ratios. In order to ensure the existence of converging paths towards the target values of fiscal variables, we introduce a simple fiscal policy rule. According to this rule, the primary balance ratio is adjusted in function of the distance between the current and the target levels of the public debt and the primary surplus to GDP ratios. It is shown that the fiscal rule displaying time invariant parameters may produce non linear dynamic processes of adjustment of the public debt and of the primary balance ratios as well as endogenous fluctuations of the rate of growth of the economy. In addition the transitional process towards fiscal targets critically depends on the adjustment tool chosen by the fiscal authorities to implement the rule.

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Non-Technical Summary

Sustainability of public finances under specific economic conditions and fiscal convergence have acquired growing importance in the current policy debate in the Euro Area. The present paper studies the path of convergence of an economy towards well-defined fiscal targets in a simple framework of analysis with overlapping generations and endogenous growth.

We present a model in which the demand side is characterized by a two-period overlapping generations scheme *à la* Samuelson (1956) and Diamond (1965) with no bequest motive, while the supply side is described by a technology exhibiting positive externalities from capital accumulation of the learning-by-doing variety in the spirit of Arrow (1962), Sheshinsky (1967) and Romer (1986). The presence of positive externalities ensures that the social return of capital is larger than the private return so that the economy displays endogenous growth. The government levies taxes on the young generation, provides transfers to the old generation and issues one-period interest bearing bonds.

There are two channels through which public accounts affect the rate of growth of the economy. On the one hand there is a social security transfer scheme from the young to the old generation which reduces the level of savings. On the other hand public debt crowds out physical capital accumulation and negatively affects growth. In this simple set up we characterize a policy reaction rule ensuring convergence for different initial fiscal positions. In particular, we study the implications of adopting a fiscal policy rule according to which the primary balance is adjusted in function of the distance between the current and the target levels of public debt and of primary surplus to GDP ratios. The government has at its disposal two adjustment tools to implement the rule: the tax rate and the benefit rate. Alternatively, with the aim of fulfilling the rule and stabilising output growth at its steady state

level, the government may use both instruments in conjunction .

The simulation exercises show how fiscal variables affect the dynamics of the rate of growth of the economy along the adjustment process towards the long-run equilibrium. In particular, the operation of a fiscal convergence rule with time invariant parameters may trigger fluctuations of growth rates around the steady state and spiraling dynamics of the debt to GDP ratio around its target value. Moreover, it is shown that the alternative use of the benefit rate on pension or of the tax rate as fiscal policy instruments implies different patterns of adjustment to the fiscal targets and affects the velocity of convergence. From our simulations it emerges that the control of the benefit rate as instrument instead of the tax rate has a positive impact on growth and therefore on the speed of adjustment of the public debt towards its target.

1 Introduction

Sustainability of public finances and sound fiscal policies are at the core of the European Monetary Union (EMU). The Treaty of Maastricht and the Stability and Growth Pact set up precise objectives of fiscal convergence for public sectors of each member country. According to a protocol to the Treaty, the general government deficit to GDP ratio should in fact not exceed 3% and the public debt to GDP ratio should be lower than 60%. In addition, the Stability and Growth Pact requires member countries to reach a close to balance or in surplus position over the medium term. The question of whether a given level of public debt is sustainable under specific economic conditions has therefore acquired growing importance in the EMU, where governments can finance current deficits with higher taxes and/or lower public expenditures, without any recourse to seigniorage.

The paper illustrates the dynamics of debt and primary balance ratios under the hypothesis that the public sector presents well-defined objectives of convergence in a simple model of analysis with overlapping generations (OLG) and endogenous growth. The main aim of the study is to fully characterize a fiscal policy rule ensuring stability for different initial fiscal positions. Moreover, the analytical framework used in this paper allows us to analyze the interaction of public debt dynamics and the rate of growth of the economy.

The economy is assumed to be populated by three types of agents: consumers, firms and the government. The demand side of the model is described by a standard two-period overlapping generations model *à la* Samuelson (1956) and Diamond (1965), with no intergenerational altruism. Individuals consume in both periods, but work only when they are young. Savings of the young generation can be held in the form of physical capital and government interest bearing bonds. The supply side of the economy is characterized by a technology exhibiting positive externalities from capital accumulation of the learning-by-doing variety. It follows that the production of each firm is an increasing function of the average capital-labor ratio prevailing in the economy. Because of the positive externality the social return of capital is larger than the private return, and the economy displays endogenous growth of the type developed by Arrow (1962), Sheshinsky (1967) and Romer (1986).

The public sector is characterized by the existence of a positive stock of debt and by objectives of fiscal convergence. The government taxes the young generation, provides transfers to the old generation and issues one-period interest bearing bonds. This implies that the model takes into account two factors negatively affecting growth: the presence of public debt stock, which subtracts resources from the physical capital accumulation (crowding out effect), and a social security system which transfers resources from the young to the old generations. Therefore, as it is well known in the literature since Saint-Paul (1992), production externalities and constant social marginal productivity of capital determine an equilibrium rate of growth which is lower than it would be in the absence of public debt and of an unfunded social security scheme.

In the set up outlined above, we aim at exploring the implications of a simple fiscal rule performing some simulation exercises for alternative combinations of the parameters of the rule and for different initial fiscal positions. We consider a policy reaction rule according to which the primary balance is adjusted in function of the distance between the current and the target levels of public debt and of primary surplus to GDP ratios, as proposed by Marìn (2002). The government can use two fiscal instruments to implement the rule: the tax rate and the benefit rate.

This analysis provides interesting insights into the linkages between fiscal variables (i.e. debt and primary surplus ratios) and the dynamics of the rate of growth of the economy, along the adjustment process towards the steady state. Moreover the simulation exercise allows us to explore different path of adjustments depending on initial conditions and numerical values assigned to the parameters characterising the fiscal policy rule governing the public sector.

The paper is organized as follows. Section 2 relates the present paper to the economic literature, while Section 3 introduces the basic model and describes the market equilibrium conditions. In Section 4 we derive the dynamic equation regulating the rate of growth of the economy and introduce the fiscal policy rule used by the government in order to reach its objectives. In particular, we outline the equilibrium conditions of the model under alternative fiscal tools of adjustment. Section 5 is devoted to the calibration of parameters and Section 6 to perform some simulation exercises. Specifically, we derive the parameter space compatible with the sustainability of public finances, describe the path of adjustment of the debt and primary surplus towards the targets and illustrate the dynamics of all the other macro-variables of interest. Section 7 concludes summarising the main results of the paper.

2 Related Literature

This paper adopts a two-period overlapping generation model, where the young generation pays income taxes and the government provides transfers to the old generation (unfunded pension system). The model is characterised by endogenous growth and explicitly takes into consideration sustainability issues and fiscal convergence. This simple framework allows us to analyze the path of convergence of an economy towards fiscal targets and to characterise the influences of the public sector on the rate of growth of the economy via the effects on saving and investment decisions. In details, there are two channels through which public accounts affect the rate of growth of the economy. First, the distortive effects of the social security transfer scheme from the young to the old generations, which reduces the level of savings, and second, the crowding out effect of public debt on physical capital accumulation.

The single ingredients of the model are not new in the literature and are briefly discussed in this Section in turn. The effects produced by the government debt on growth and welfare is one of the most frequently discussed subject in the economic literature. The 'burden of the debt' issue and the fall of capital stock arising from the increase in the government debt (crowding out effect) are plainly illustrated by Domar (1944), Malinvaud (1953) and Modigliani (1961).

Overlapping generations models, in which the breakdown of the Ricardian equivalence makes the government debt net wealth for households, are the most appropriate tools of analysis to explore the influence of public debt on the economy. Diamond (1965) analyzes the effects of a positive stock of debt on the long-run competitive equilibrium of an economy with neoclassical technology. He shows that government debt causes a decline in the utility level when the equilibrium is dynamically efficient, but may increase the utility when the economy is dynamically inefficient. Ihori (1978) studies the effects of government debt on the long-run optimal conditions and analyzes the growth paths corresponding to alternative government policies in a life-cycle economy.

In addition to the literature summarised above, there are several recent contributions in which the analysis of fiscal sustainability is shifted away from a present budget balance perspective and is instead conducted into a life-cycle model. Chalk (2000) analyzes the sustainability of permanent bond-financed deficits and shows the conditions under which a growth rate larger than the interest rate is a necessary but not a sufficient condition to ensure the sustainability of a permanent budget deficit. De la Croix and Michel (2002) investigate the effects of the introduction of public debt on the dynamic properties of a two-period overlapping generations model and derive the conditions for ensuring sustainability. Marín (2002) studies how a simple primary surplus budgetary rule can ensure the sustainability of public finances and provide automatic stabilization in a small open economy. Rankin and Roffia (2003) investigate the structural determinants of a maximum sustainable public debt and the conditions for its existence under non-degenerate values of the other economic variables in a Diamond two-period overlapping generations model.

The engine of growth considered in the present paper is based on capital externalities in the spirit of Arrow (1962), Sheshinsky (1967) and Romer (1986), as developed by Grossman and Yanagawa (1993). The basic idea is that the accumulation of capital at aggregate level is an index of knowledge and positively affects production at firm level. In his seminal contribution Arrow (1962) argues that the learning process is related to the acquisition of experience, which can be measured by the cumulative level of investments. In his words "each new machine produced and put into use is capable of changing the environment in which production takes place, so that learning is taking place with continually new stimuli" (p. 157).

The effects of an unfunded social security or of an increase in public debt have been studied in many contributions presenting endogenous growth settings with externalities from physical capital. Saint-Paul (1992) shows that an unfunded social security scheme or an increase in the level of public debt cannot be Pareto-improving in an overlapping generations model with endogenous growth[1]. However, his analysis is restricted to balanced growth path and does not address the issue of sustainability of public finances and of fiscal convergence. Azariadis and Reichlin (1996) analyze how public debt affects the accumulation of physical capital in an overlapping generations

[1] In two-period OLG model with endogenous growth model, Marchand et al. (1995) show that ascending intergenerational transfers can be welfare-improving when the population rate of growth falls unexpectedly.

model displaying production externalities. They show that in the presence of a positive initial value of the public debt the economy can converge to a low-level development trap. In particular, in their framework of analysis national debt is an asset with no intrinsic value and represents a liability of the government, which is assumed to have a zero expenditure and to levy no taxes. In other words, public debt is regarded as a pure bubble as in Tirole (1985).

3 The Model

The model of analysis is a two-period overlapping generations model *à la* Samuelson (1956) - Diamond (1965) extended to include a social security system. The supply side is described by a simple endogenous growth model with a positive externality generated by a learning by doing process in the production activity. The public sector presents objectives of convergence for public debt and primary balance. Individuals can hold their wealth in form of physical capital and of interest bearing public debt bonds.

3.1 Firms

The production side of the economy is described by a variant of the 'learning by doing' model of the type developed by Arrow (1962), Sheshinsky (1967) and Romer (1986) as in Grossman and Yanagawa (1992). Assume that there is a continuum of identical firms that employ labor force and physical capital in the production process. For simplicity the number of firms is normalized to one. The production function for each firm i at time t is given by

$$Y_{it} = F(K_{it}, E_{it}), \tag{1}$$

where K_{it} is the individual firm i level of capital at the beginning of period t, E_{it} is an efficiency variable defined as $E_{it} = \frac{K_t}{L_t} L_{it}$ and $F(\cdot)$ is an increasing function in both the two arguments and exhibits constant returns to scale. The rationale for a production function as equation (1) is that the production process generates positive externalities. The efficiency variable is in fact in an increasing function of the average capital intensity in the economy. The production function can be written in efficiency units as

$$Y_{it} = F(K_{it}, E_{it}) = E_{it} f(k_{it}), \tag{2}$$

where $f(\cdot)$ is a strictly concave function and $k_{it} = \frac{K_{it}}{E_{it}}$.

Under perfect competition the following optimality conditions must hold for each firm i

$$r_t = f'(k_{it}), \tag{3}$$

$$\widehat{w}_t = f(k_{it}) - k_{it} f'(k_{it}), \tag{4}$$

where r_t is the real interest rate and $\widehat{w}_t$ is the wage of efficiency units of labor. It follows that the wage of a unit raw labor w_t is

$$w_t = \frac{K_t}{L_t} \widehat{w}_t. \tag{5}$$

The aggregate production function is defined by summing up the production functions of all firms. Under the assumption that firms are identical $L_{it} = L_t$, $K_{it} = K_t$ and $k_{it} = 1$ for each firm i. It follows that in a symmetrical equilibrium total output is

$$Y_t = AK_t, \tag{6}$$

where $A \equiv f(1) > 0$. Clearly, at the aggregate level output is linear in capital and the production function exhibits constant returns to scale. The factor A can be interpreted as the social marginal product of capital, which incorporates the positive spillovers of capital accumulation, conversely r can be interpreted as the private marginal product of capital. In the symmetrical equilibrium factor prices are

$$r_t = f'(1) = r, \tag{7}$$

$$w_t = (A - r)\frac{K_t}{L_t}, \tag{8}$$

where we assume that $A > r$, that is that the social return of capital is larger than the real interest rate.

By combining the factor price equations (7) and (8) with the production function (6) we obtain

$$Y_t = AK_t = rK_t + L_t w_t, \tag{9}$$

which simply states that in the competitive equilibrium output is fully exhausted by factor payments.

3.2 Consumers

The demand side is described by a two period overlapping generations model. Total population is assumed to grow at a constant rate n. Agents work and earn wage income while young and consume in both periods. Labor is inelastically supplied by young agents, thus labor force also grows at rate n. By assumption, there is no bequest motive, young agents are all born without assets and a social security pension scheme is operative.

The representative young agent of generation born in t faces the following lifetime utility function

$$U(c_t^Y, c_{t+1}^O) = u(c_t^Y) + \frac{1}{1+\rho}u(c_{t+1}^O), \tag{10}$$

where ρ is the rate of time preference, c_t^Y denotes consumption at time t and c_{t+1}^o is consumption at time $t + 1$. We assume that preferences are logarithmic[2], so that $u(c) = \ln c$. Young consumers work and pay a tax T, while old consumers receive a transfer P. The representative agent born

[2]Logarithmic preferences imply that saving decisions do not depend on the interest rate in the absence of any transfer scheme. In this framework, however, since the interest rate is constant this special assumption about preferences can be made without loss of generality.

at time t chooses c_t^Y and c_{t+1}^o to maximize (10) subject to the flow budget constraint in the first and in the second period, respectively given by:

$$c_t^Y = w_t - T_t - \sigma_t^Y, \tag{11}$$

$$c_{t+1}^o = (1+r)\sigma_t^Y + P_{t+1}, \tag{12}$$

where σ_t^Y denotes savings. At the optimum the standard Euler equation must hold:

$$c_t^Y = \frac{1+\rho}{1+r} c_{t+1}^o. \tag{13}$$

Combining the above condition with the constraints (11) and (12) yields the individual saving function

$$\sigma_t^Y = w_t - T_t - c_t^Y = \frac{1}{2+\rho}\left(w_t - T_t - \frac{1+\rho}{1+r}P_{t+1}\right), \tag{14}$$

Taxes and benefits are assumed to be proportional to the wage level

$$T_t = \tau_t w_t, \tag{15}$$

$$P_{t+1} = \pi_{t+1} w_t, \tag{16}$$

where $0 < \tau_t < 1$ is the tax rate and $\pi_{t+1} > 0$ is the benefit rate. It follows that the individual saving function (14) can be re-written as

$$\sigma_t^Y = \frac{1}{2+\rho}\left(1 - \tau_t - \frac{1+\rho}{1+r}\pi_{t+1}\right) w_t. \tag{17}$$

Finally, young consumers are assumed to accumulate their wealth in form of capital K_t and of interest bearing-bonds B_t issued by the public sector. The condition of no-arbitrage between the two assets guarantees that their rate of return is the same.

3.3 Public sector

The public sector is described by an infinite lived government whose flow budget constraint is

$$B_{t+1} = (1+r)B_t - S_t, \tag{18}$$

where S_t is the primary budget surplus and is defined as the difference between the total amount of contributions paid by young consumers and the total amount of benefits paid to the old generation

$$S_t = L_t T_t - L_{t-1} P_t. \tag{19}$$

The fiscal policy regime is characterized by the sequences $\{B_t,\ S_t\}_{t=0}^{\infty}$ and only one of these sequences can be chosen independently, since one is implied by the other[3]. Recalling that taxes and pension benefits are set according to (15) and (16), the government has two instruments to control the dynamics of the primary surplus and, therefore, the public debt: the tax rate, τ, and the benefit rate, π.

3.4 Market equilibrium

The resource constraint for the economy describing the capital accumulation process can be expressed as

$$Y_t - C_t = K_{t+1} - K_t, \tag{20}$$

where C_t is total consumption at time t defined as the sum of consumption by the young and the old generations in period t:

$$C_t = L_t c_t^Y + L_{t-1} c_t^O, \tag{21}$$

which can be re-written as

$$C_t = L_t(w_t - \sigma_t^Y - T_t) + (1 + r)(B_t + K_t) + L_{t-1} P_t. \tag{22}$$

The old agents, as a group, own the entire stock of capital and of public debt bonds. Their aggregate level of consumption is then equal to the total amount of benefits plus the whole wealth and the interest payments that they receive from the firms and the government.

[3] In the following Section we will characterise the fiscal regime.

Recalling that total income is given by (9), equation (22) becomes

$$C_t = Y_t - L_t(\sigma_t^Y + T_t) + (1+r)B_t + K_t + L_{t-1}P_t, \tag{23}$$

that combined with the government budget constraint (18) yields

$$C_t = Y_t - L_t\sigma_t^Y + B_{t+1} + K_t. \tag{24}$$

Combining this result with resource constraint of the economy (20) we obtain the following equilibrium condition linking the current level of saving to the next period stock of wealth

$$B_{t+1} + K_{t+1} = L_t\sigma_t^Y. \tag{25}$$

This condition simply states that savings of period t become wealth in period $t+1$ and guarantees the equilibrium on capital market. Recalling the saving function (17), condition (25) becomes

$$B_{t+1} + K_{t+1} = \frac{A-r}{2+\rho}\left(1 - \tau_t - \frac{1+\rho}{1+r}\pi_{t+1}\right)K_t. \tag{26}$$

In the absence of any bequest motive the old generation want to end up with no wealth when they pass away. It follows that all the stock of wealth owned by the old is purchased by the young with their savings. In addition, from the above equilibrium condition is clear that a positive stock of public debt crowds out physical capital[4].

[4]It should be noted that the resource transfer from the young generation to the government in the form of taxes and purchase of government bond, $B_{t+1} + L_tT_t$, is equal to the resource transfer from the government to the old generation, consisting of debt services and transfers, $(1+r)B_t + L_{t-1}P_t$. This property of the model is at the basis of the so-called 'equivalence result' illustrated by Buiter and Kletzer (1992a, 1992b), which can be stated as follows. A given sequence of variables, denoted by $\{c_t^Y, c_t^o, K_t\}_{t=0}^{\infty}$, can always be generated by an appropriate sequence for taxes and transfers $\{T_t, P_t\}_{t=0}^{\infty}$ for any given sequence for debt $\{B_t\}_{t=0}^{\infty}$. In other words, any sequence of variables can be replicated, whatever the sequence for debt, by an appropriate choice for the path of taxes and transfers.

4 The Control of Primary Balance and Economic Growth

The whole economy can be described by equations (19), (18) and (26) which can be expressed, respectively, in output terms as follows

$$s_t = \left(\tau_t - \frac{\pi_t}{G_t}\right)\frac{A-r}{A}, \tag{27}$$

$$b_{t+1}G_{t+1} = (1+r)b_t - s_t, \tag{28}$$

$$G_{t+1} = \frac{A-r}{2+\rho}\left(1-\tau_t - \frac{1+\rho}{1+r}\pi_{t+1}\right) - A\left[(1+r)b_t - s_t\right], \tag{1}$$

where $G_{t+1} = \frac{Y_{t+1}}{Y_t}$ is the growth factor, $b_t = \frac{B_t}{Y_t}$ and $s_t = \frac{S_t}{Y_t}$ are the public debt and the primary surplus to GDP ratios, respectively.

To close the model we consider a general policy reaction function of the type proposed by Marín (2002). The behavior of the primary balance surplus ratio s_t is described by the following equation

$$s_t = s_{t-1} + u(b_t - b^*) - v(s_{t-1} - s^*), \tag{30}$$

where b^* and s^* are the long run target ratios for the public debt and the primary surplus, respectively. The fiscal rule represented in (30) implies two convergence parameters, u and v. Henceforth, we will denote this rule as the '$u - v$ rule'.

The economy displays different dynamics, depending on the use of the tax rate τ or of the benefit rate π as fiscal adjustment instrument to control the primary surplus and implement the '$u - v$ rule'. The government may use, in fact, one of the two instruments, keeping the other one as fixed in order to reach its fiscal targets. Alternatively, the government may elect to combine the two instruments in order to pursue fiscal convergence and stabilize the rate of growth of the economy at the same time.

4.1 Tax rate as fiscal instrument

Consider the case in which the tax rate is the endogenously determined to implement the '$u - v$ rule', while the benefit rate is kept constant, $\pi_{t+1} = \pi_t = \pi$. In such circumstances the dynamic equation describing the time path of the growth factor can be obtained by solving equation (27) for τ_t and substituting the result into (29)

$$G_{t+1} = -\left(\frac{\pi}{G_t} - 1 + \frac{1+\rho}{1+r}\pi\right)\frac{A-r}{2+\rho} + \left[\frac{1+\rho}{2+\rho}s_t - (1+r)b_t\right]A. \tag{31}$$

Equation (31) is a nonlinear difference equation in G_t. For every value of G_t and the initial level of b_t, the above equation determines the equilibrium value of G_{t+1} corresponding to the level of the primary surplus s_t[5].

Under this fiscal regime the macroeconomic equilibrium is thus defined as a set of sequences $\{G_{t+1}, b_{t+1}, s_t, \tau_t\}_{t=0}^{\infty}$, satisfying (27), (28), (30) and (31), given the initial conditions $\{G_0, b_0, s_{-1}\}$. It should be noted that the equations describing the economy are recursive, thus given the initial conditions one can determine the sequences $\{G_{t+1}, b_{t+1}, s_t\}_{t=0}^{\infty}$ for each combination of the parameters (u, v).

4.2 Benefit rate as fiscal instrument

We now examine the case in which the benefit rate is the is used as adjustment tool to fulfill the '$u - v$ rule', while the tax rate is kept constant, $\tau_t = \tau$. Moving (27) one period ahead and combining the result with (29) give the

[5]It should be noted that in the absence of public debt and of a pension system, the growth factor is $\frac{A-r}{2+\rho}$.

equation governing the growth factor

$$G_{t+1} = \frac{\frac{A-r}{2+\rho}(1-\tau) - A\left[(1+r)b_t - s_t\right]}{1 + \frac{1+\rho}{2+\rho}\frac{1}{1+r}\left[(A-r)\tau - As_{t+1}\right]}. \tag{32}$$

In such circumstances the macroeconomic equilibrium is described by the set of sequences $\{G_{t+1}, b_{t+1}, s_t, \pi_t\}_{t=0}^{\infty}$, satisfying (27), (28), (30) and (32), given the initial conditions $\{G_0, b_0, s_{-1}\}$.

4.3 A mixed approach

Consider the case in which the government control the tax and the benefit rates in order to implement the '$u - v$ rule' and keep the output growth constant at its long-run level, G. Substituting (27) into (29) gives

$$G_{t+1} = \frac{A-r}{2+\rho}\left(1 - \tau_t - \frac{1+\rho}{1+r}\pi_{t+1}\right) - A(1+r)b_t + (A-r)\left(\tau_t - \frac{\pi_t}{G_t}\right), \tag{33}$$

which describes the time path of the growth factor as function of the two fiscal instruments π_t and τ_t.

In this policy regime the government is assumed to choose sequences $\{\tau_t, \pi_{t+1}\}_{t=0}^{\infty}$ in order stabilise growth $G_{t+1} = G_t = G$ in (33) and to satisfy (27). In particular the tax and the benefit rates evolve as follows

$$\tau_t = \frac{A}{A-r}s_t + \frac{\pi_t}{G}, \tag{34}$$

$$\pi_{t+1} = \frac{1+r}{1+\rho}\left\{(1 - \frac{\pi_t}{G}) - \frac{2+\rho}{A-r}\left[G + A(1+r)b_t\right] + \frac{1+\rho}{A-r}As_t\right\}, \tag{35}$$

where the primary surplus dynamics is governed by the'$u - v$ rule'. The macroeconomic equilibrium can be then defined as set of sequences $\{\tau_t, \pi_{t+1}, b_{t+1}, s_t\}_{t=0}^{\infty}$, satisfying (28), (30), (34) and (35), given the initial conditions $\{\pi_0, b_0, s_{-1}\}$[6].

[6]Note that for simplicity we have assumed that the government takes as given the benefit rate paid to the old generation at time t.

5 Calibration

In order to calibrate the model and perform some simulation exercises, we interpret each period as being composed by 30 years[7]. This means that according to the equilibrium condition (26) the old generation own the total stock of capital and of public debt acquired with the flow of savings generated in a period of 30 years. In other words, savings become productive only after 30 years. This is an unfortunate feature of the two period OLG model. The time structure of the model also implies that the various rates should be interpreted as 30 years based. In order to analyze the dynamics and the adjustment path of the economy towards the fiscal policy objectives, we calibrate the model as follows.

The real interest rate r is set equal to 3% per annum; consumers are assumed to be relatively patient and the annual rate of time preference ρ is set equal to 1%. We set the steady state rate of growth of the economy to 2.5% per year, consistently with De la Croix and Michel (2002). The target level of the public debt is equal to 60% of a year's GDP, that is that is equal to 2% of the GDP produced in 30 years[8]. The long-run benefit rate π is set equal to 0.50, while the implied tax rate τ is equal to 0.24, which in the absence of public expenditure could be considered a rough approximation of the young agents average tax burden. Finally, the implied value for the social return of capital A is 10% on an annual basis. Table 1 reports the underlying parameter values, the target fiscal ratios and the long-run growth factor of the economy. It should be noted that the calibration is only for illustrative

[7]This choice is standard in the literature calibrating two-period models. See for example Feldestein (1985), Barro and Sala-i-Martin (1992) and De la Croix and Michel (2002).

[8]This is another unfortunate feature of this theoretical scheme. However, the old generation owns the entire stock of wealth through the savings of 30 years. We cannot imagine that they are able to purchase the entire stock of existing wealth with the savings of only one year.

purposes and is not meant to fully reflect reality.

6 Dynamics of Public Debt and Primary Balance Ratios

In this Section we study the dynamics of the public debt and of the primary balance ratios under three alternative adjustment options to implement the '$u-v$ rule', as outlined in the previous Section. The analysis will proceed as follows. We first identify the combinations of policy reaction parameters (u, v) which ensure convergence of the public debt and of the primary balance towards the government targets. Then we simulate the model taking into account different combinations of the parameters which guarantee fiscal solvency, given different initial levels of the debt to GDP ratio. In the simulations, we take the initial conditions $\{G_0, b_0, s_{-1}\}$ as given and we assume that the '$u-v$ rule' becomes operative at time $t = 1$. For simplicity, we hypothesize that the growth factor is initially at its long-run level, $G_0 = G$ and the primary balance surplus is equal to zero, $s_{-1} = 0$.

6.1 Tax rate as fiscal instrument

Consider the case in which the government uses the tax rate τ as instrument to control the dynamics of the primary surplus, while the benefit rate is constant over time, $\pi_t = \pi$. In order to characterize the '$u-v$ rule', we simulate the model with the aim of describing the combinations of the parameters u and v which ensure fiscal convergence under these circumstances. The result is presented in Figure 1. The shadow area represents the combinations of u and v which guarantee fiscal convergence, while any other combinations of those parameters determine an explosive path for the debt[9]. In details, the parameter governing the adjustment of the primary deficit towards its targets v should be greater than 0.4 but less than 1.4 at maximum. On the other hand the parameter regulating the speed of adjustment of public debt u towards its target should be greater than one. The darker area indicates the subset of parameter combinations which rule out the possibility of overshooting the target of the public debt to GDP ratio during the convergence process.

Once the parameter space guaranteeing convergence has been determined, we perform some simulations aiming at discovering the path of debt adjustment towards the government targets (60% of GDP) from different initial positions. Moreover, it is our intention to show the resulting time paths for the fiscal instrument (tax rate) and the effects on growth. Finally we replicate the simulation, fixing the u parameter at 2.5 and moving the level of the parameter v from 1 to 0.5.

Figures 2-4 report the results of the simulation (u=2.5 and v=1). We observe that in order to reach the target level of a debt ratio of 60% the tax rate should be increased by a large size (the greater the initial debt position the higher the increase in the tax rate). This increase guarantees sufficient primary surpluses for determining a reduction in the debt to GDP ratio. At a certain point in time the tax rate can be decreased determining a reduction of the primary surplus, but without prejudice for the adjustment process. As for growth, the fiscal policy tightening implies a reduction in the economic activity in the first periods and a recovery thereafter.

[9]In the numerical simulation a set of parameter values lead to fiscal convergence if after 200 periods the public debt to GDP ratio is in a small neighborhood of its target, ($|b_{200} - b^*| < 0.001\%$). All parameter combinations which lead the economy to the fiscal targets, but produce negative values for the debt ratios at early stages, are not considered in the fiscal convergence space. The initial level for the public debt to GDP ratio b_0 is set to 100% at annual level.

In the following simulation we calibrate the fiscal policy rule with the same value for u, but with a reduced value for v (0.5). Figures 5-7 shows that, the results do not change in the first periods. However the impact of the tax changes and the operation of the fiscal policy rule determine fluctuations in the growth factor such that the level of debt targeted by the government is overshooted. The graphs show a long-lasting dynamics around the target level mainly driven by growth fluctuations around the steady state.

Table 2 reports the number of periods necessary to reduce the public debt to GDP ratio to 60% for different starting levels for the debt ratio and various combinations of the fiscal parameters, u and v. This exercise shows that the relationships among the parameters of the fiscal rule and the length of the convergence period are not straightforward. As observed above, in fact, the mechanical application of the primary balance rule may cause cyclical dynamics of the public debt and of the primary balance ratios and be source of endogenous fluctuations of the rate of growth of the economy. The observed non-linearities would explain the absence of clear relationships among the parameters of the rule and the fiscal convergence periods. Nonetheless, from Table 2 it emerges that the rule performs relatively better for intermediate values of v and large u values. Not surprisingly, the higher the initial debt ratio, the longer the period necessary to reach the target ratio.

6.2 Benefit rate as fiscal instrument

When the adjustment instrument used by the fiscal authorities is the benefit rate and the tax rate is invariant, the families of parameters (u, v) that guarantee fiscal convergence are represented by the shadow area in Figure 8. It clearly emerges that in this case fiscal convergence is also possible for lower level of the parameters. In particular, the parameter governing the adjustment of the primary balance v from its target should be smaller than 1.7, but greater than 0.3 in correspondence of low levels of the fiscal parameter u. Conversely, for larger level of the velocity of reaction of the primary balance to the divergence of the debt ratio from its target, u, the parameter v can be lower. The darker area represents the combinations of policy parameters ensuring a non-increasing path for the debt ratio. As observed in the previous case, for low values of u and large of v it is possible to reach the fiscal targets without overshooting the targeted debt ratio.

Figures 9-11 display the simulation results for u=2.5 and v=1. Figure 9 shows the paths of the debt and of the primary surplus ratios. As expected the debt ratio follows a decreasing path toward its long-run value. Figure 10 shows that the implementation of the rule determines a temporary increase in the growth factor on impact, while according to Figure 11 the benefit rate is initially reduced. The improved economic conditions reduce the critical fiscal ratios, it follows that in the following period the mechanical application of the fiscal rule requires an increase in the benefit rate, which determines an increase in the growth factor.

Figures 12-14 show the simulation results for u=2.5 and v=0.5. Figure 12 represents the dynamics of the debt and of the primary surplus ratios showing the alternative paths of convergence starting from different initial debt ratios. Again we observe endogenous fluctuations around the fiscal targets. Figure 14 shows that the benefit rate is initially reduced, determining an increase in the growth factor (see Figure 13). During the process of adjustment the endogenous fluctuations become smaller and the economy converges to its long-run equilibrium.

Table 3 reports the number of periods necessary to bring the public debt to GDP ratio to 60% for different initial conditions for the debt ratio and alternative combinations of the parameters, u and v. As observed for the tax rate regime case, there is no clear relationship among the parameter values of the fiscal rule and the length of the convergence period. In such circumstances, however the rule implies a shorter convergence period for large values of v and a lower level of u, independently of the starting level of the public debt to GDP ratios. In general, when the benefit rate is used as fiscal instrument, it takes less time to reach the 60% target ratio than when the tax rate is endogenously set.

6.3 A mixed approach

We now analyze the fiscal regime under which the policies authorities are assumed to set the tax and the benefit rate in order to implement the '$u - v$ rule' and keep the output growth constant at its long-run level, G. Figure 15 illustrates the region of the parameter space associated with fiscal convergence. The region of convergence expands, relative to the previous cases. For values of u close to zero and for low levels of v in fact much of the pictured space is associated with fiscal convergence. The dark area, referring to the parameter combinations ensuring a never increasing convergence path is also larger than in the previous cases.

Figures 16-18 represent the dynamics of the economy for u=2.5 and v=1. Clearly, the benefit ratio must initially fall, while the tax rate must increase so as to respect the rule and to keep the growth factor as fixed.

Figure 19 shows that dynamics of the fiscal ratios along the adjustment path towards their long-run levels. We notice once again endogenous fluctuations of the critical fiscal ratios all along the convergence process towards the steady state. In this case the observed fluctuations are determined by the combined use of the two fiscal instruments which are continuously adjusted such that the fiscal rule is fulfilled and the growth factor of the economy is maintained at its long-run level (see Figures 20 and 21).

Finally, Table 4 reports the number of periods necessary for the economy to reach the fiscal targets and shows that the combined use of both instruments tends to perform relatively better for large levels of the parameter v combined with a low u. Comparing Table 4 with Table 3 we observe that under the mixed approach the economy converges slightly quickly for various combinations of the parameters.

7 Conclusion

In order to evaluate the performance of a simple fiscal policy rule in terms of its capability of guaranteeing convergence and sustainability of public finances, we derived a simple two-period overlapping generation model in an endogenous growth framework. The policy reaction rule considered in the paper linked the primary balance ratio to the distance between the current and the target levels of public debt and of primary surplus to GDP ratios. With the objective of satisfying the rule, the government has two fiscal instruments available: either the tax rate or the benefit rate. Furthermore, with the aim of stabilising output growth at its steady state level, the government can choose to use the two instruments in conjunction.

This analysis provided interesting insights into the linkages between fiscal variables (i.e. debt and primary deficit ratios) and the dynamics of the rate of growth of the economy, along the adjustment process towards the steady state. The simulation exercise allowed us to explore different path of adjustments depending on initial conditions and numerical values assigned to the parameters characterising the fiscal policy rule governing the public sector.

The paper showed that fiscal policy decisions may eventually determine fluctuations of growth rates around the steady state and converging cycles of the debt to GDP ratios around target values. It emerged that the implementation of a fiscal rule characterised by time invariant parameters may trigger a non linear process of adjustment towards the objective of convergence. In particular, the larger the initial public debt GDP ratio, the larger the fluctuations of the rate of growth that the economy will experience.

The velocity of fiscal convergence of economies pursuing the same targets and implementing the same primary balance rule is showed to depend on the adjustment tools used by the fiscal authorities. Reducing the benefit rate instead of increasing the tax rate has a beneficial impact on growth and therefore on the speed of adjustment of the public debt towards its target. Noticeably, if the government aims at stabilising output growth while fulfilling the fiscal rule requirements for convergence, it should use both available tools at the same time.

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Table 1-Calibration

	30 Years	**Annual**
Rate of time preferences ρ	0.35	0.01
Interest rate r	1.43	0.03
Benefit rate π	0.5	0.5
Tax Rate τ	0.24	0.24
Public debt-GDP b^*	0.02	0.6
Primary balance-GDP s^*	0.006	0.006
Long-run growth factor G	2.10	1.025
Social return of capital A	14.82	0.10

Table 2-Number of Periods to Bring the Public Debt-GDP Ratio to 60%

(Tax Rate as Fiscal Instrument)

	$u = 2.5$			$u = 3$			$u = 3.5$		
v	*0.5*	*0.8*	*1*	*0.5*	*0.8*	*1*	*0.5*	*0.8*	*1*
$b_0 = 1.1$	23	7	17	25	7	9	26	8	7
$b_0 = 1$	23	7	15	22	7	8	24	6	7
$b_0 = 0.9$	23	6	13	22	7	7	22	6	7
$b_0 = 0.8$	20	6	11	20	5	5	20	6	7
$b_0 = 0.7$	18	6	8	18	5	3	18	4	5

Table 3-Number of Periods to Bring the Public Debt-GDP Ratio to 60%

(Benefit Rate as Fiscal Instrument)

	$u = 2.5$			$u = 3$			$u = 3.5$		
v	*0.5*	*0.8*	*1*	*0.5*	*0.8*	*1*	*0.5*	*0.8*	*1*
$b_0 = 1.1$	20	7	2	20	8	5	20	7	8
$b_0 = 1$	20	7	2	20	7	4	20	7	8
$b_0 = 0.9$	18	7	2	18	6	4	18	7	8
$b_0 = 0.8$	18	6	2	16	6	4	16	6	7
$b_0 = 0.7$	13	5	2	14	5	3	13	4	6

Table 4-Number of Periods to Bring the Public Debt-GDP Ratio to 60%

(Mixed Approach)

	$u = 2.5$			$u = 3$			$u = 3.5$		
v	*0.5*	*0.8*	*1*	*0.5*	*0.8*	*1*	*0.5*	*0.8*	*1*
$b_0 = 1.1$	18	7	2	18	7	4	19	7	8
$b_0 = 1$	18	7	2	18	7	4	17	7	8
$b_0 = 0.9$	16	6	2	16	6	4	17	7	7
$b_0 = 0.8$	16	6	2	14	6	4	15	6	7
$b_0 = 0.7$	11	5	2	12	5	3	13	4	6

Figure 1: Tax Rate as Fiscal Instrument

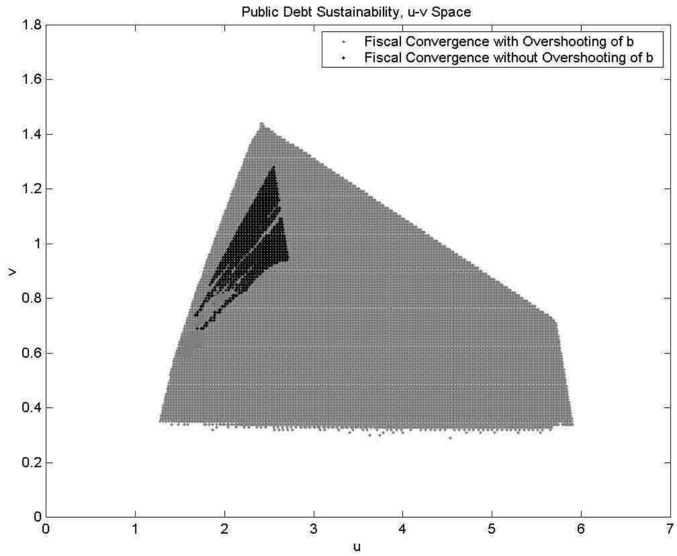

Figure 2: Tax Rate as Fiscal Instrument

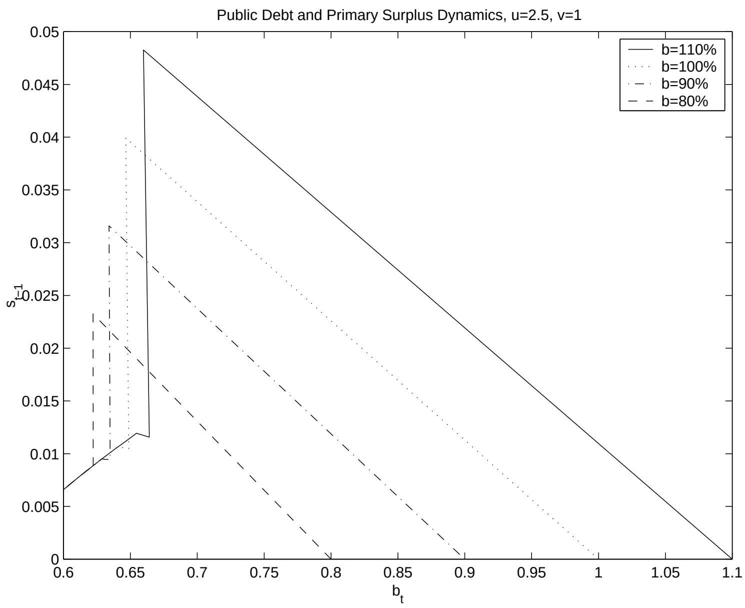

Figure 3: Tax Rate as Fiscal Instrument

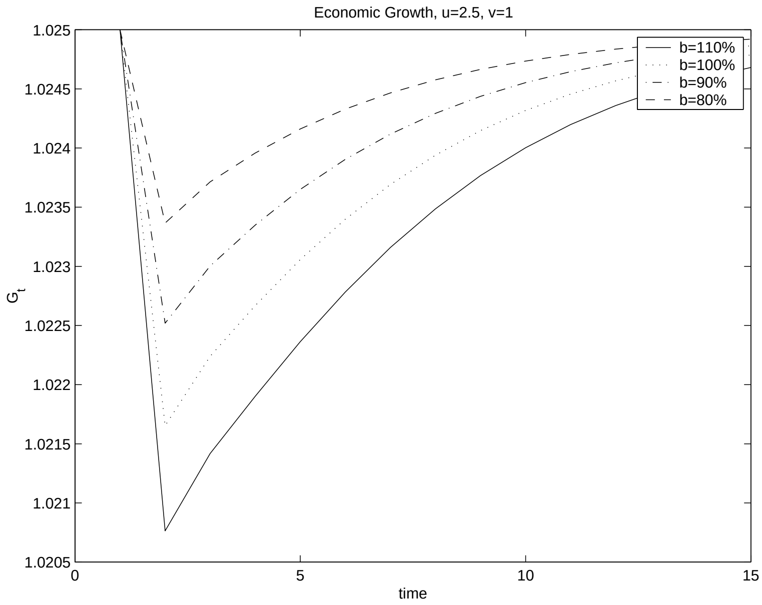

Figure 4: Tax Rate as Fiscal Instrument

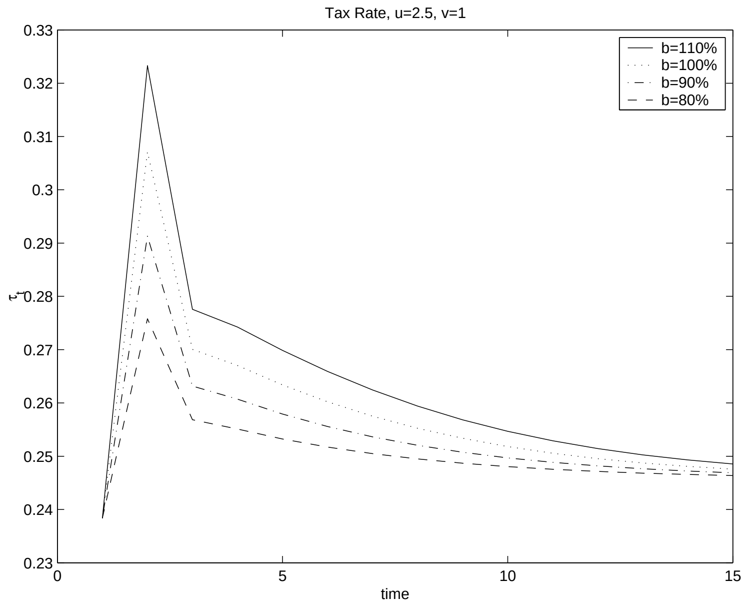

Figure 5: Tax Rate as Fiscal Instrument

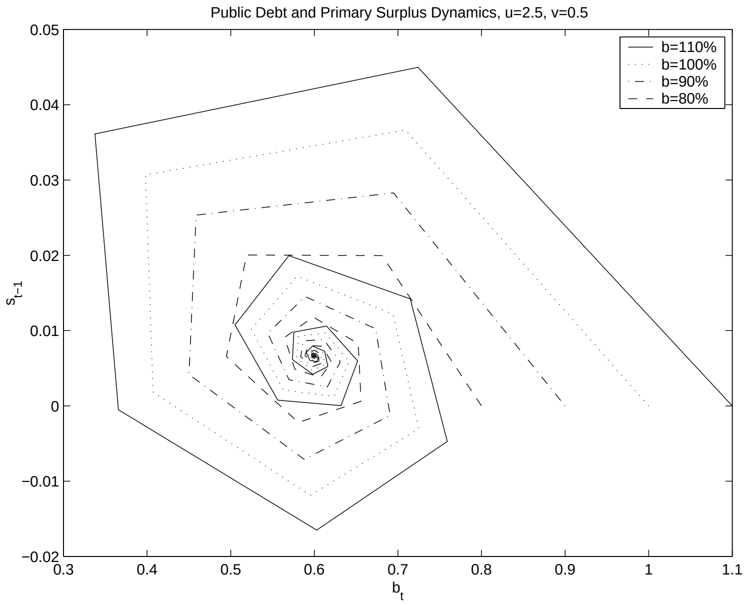

Figure 6: Tax Rate as Fiscal Instrument

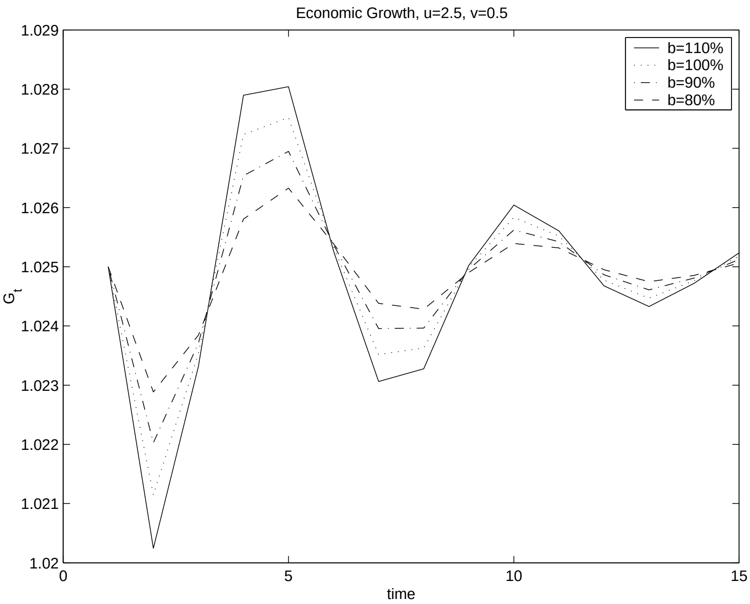

Figure 7: Tax Rate as Fiscal Instrument

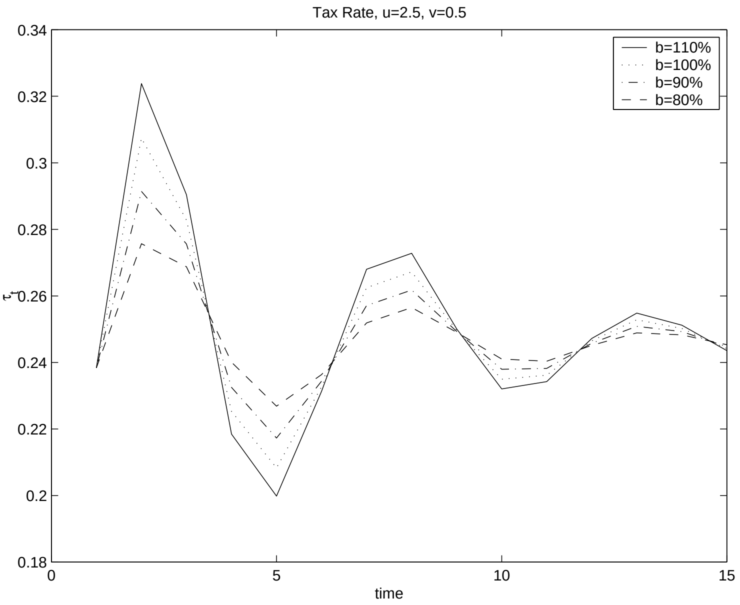

Figure 8: Benefit Rate as Fiscal Instrument

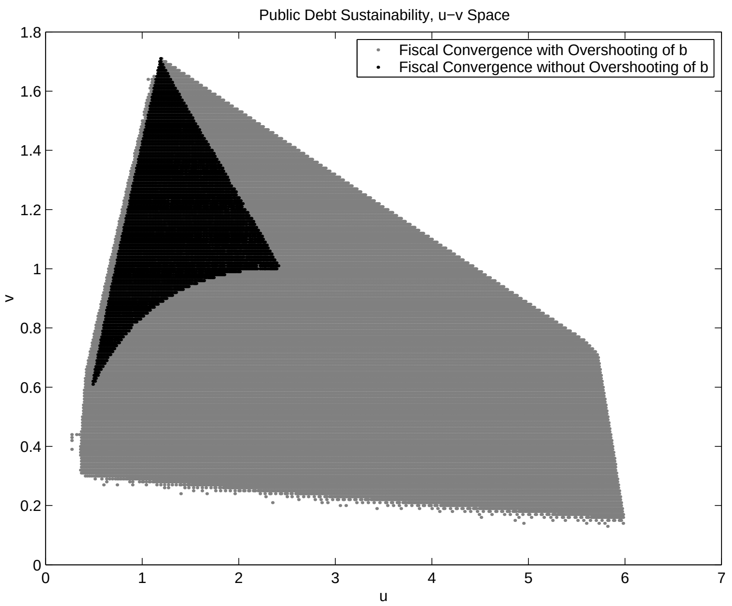

Figure 9: Benefit Rate as Fiscal Instrument

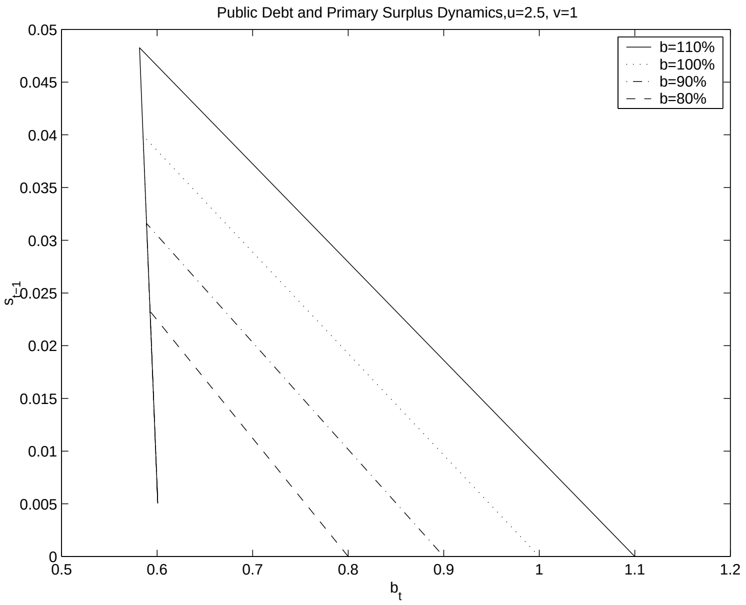

Figure 10: Benefit Rate as Fiscal Instrument

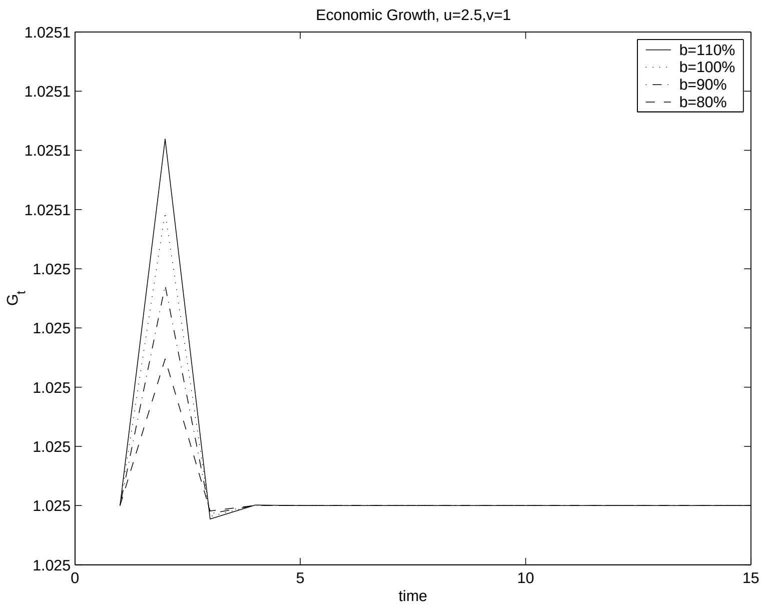

Figure 11: Benefit Rate as Fiscal Instrument

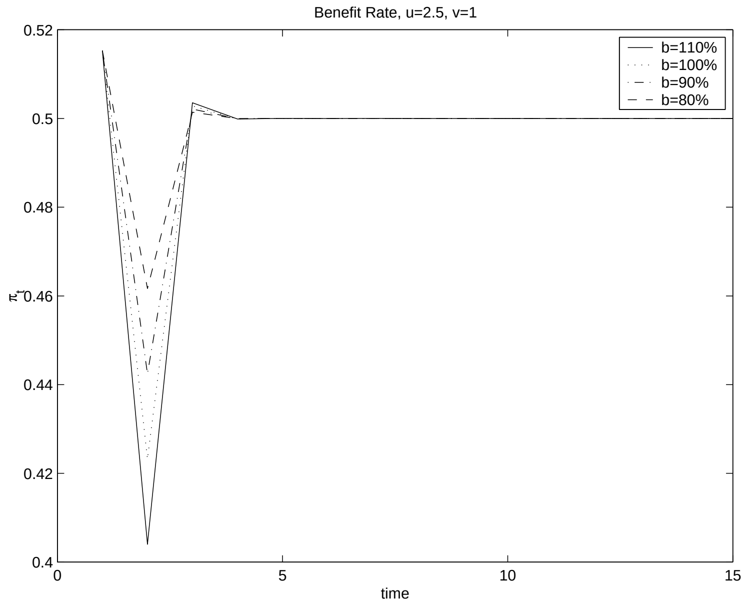

Figure 12: Benefit Rate as Fiscal Instrument

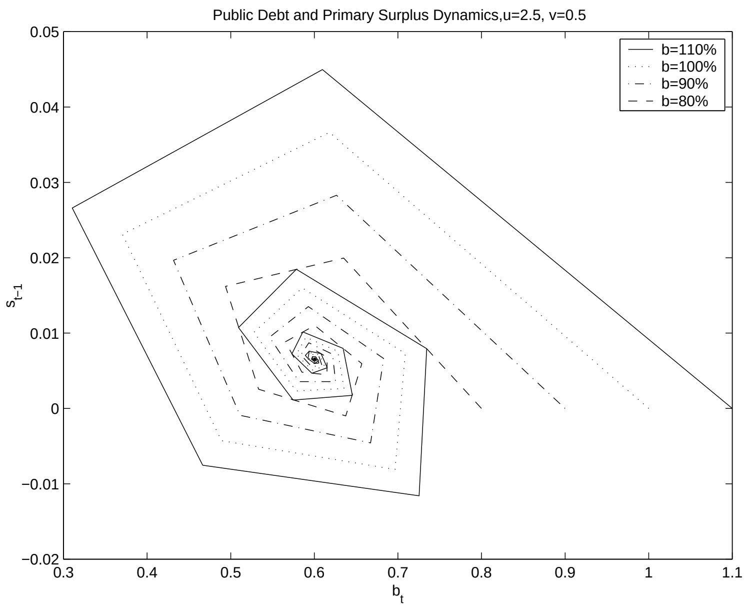

Figure 13: Benefit Rate as Fiscal Instrument

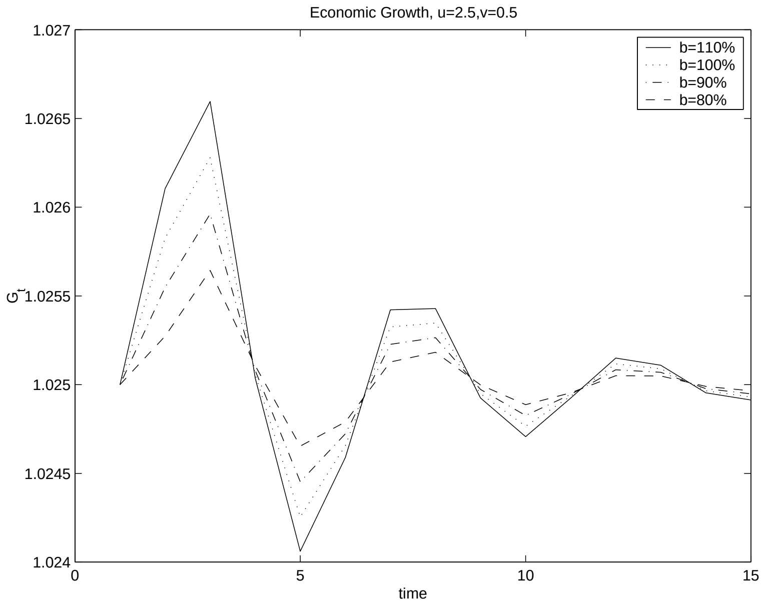

Figure 14: Benefit Rate as Fiscal Instrument

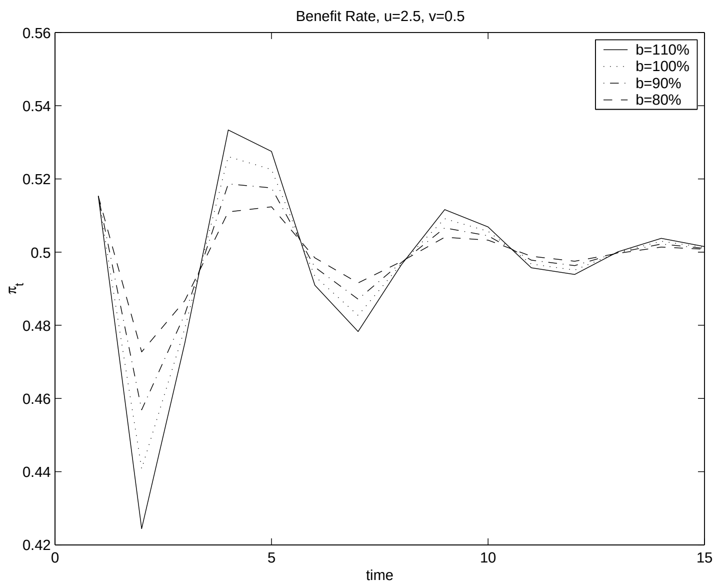

Figure 15: Mixed Approach

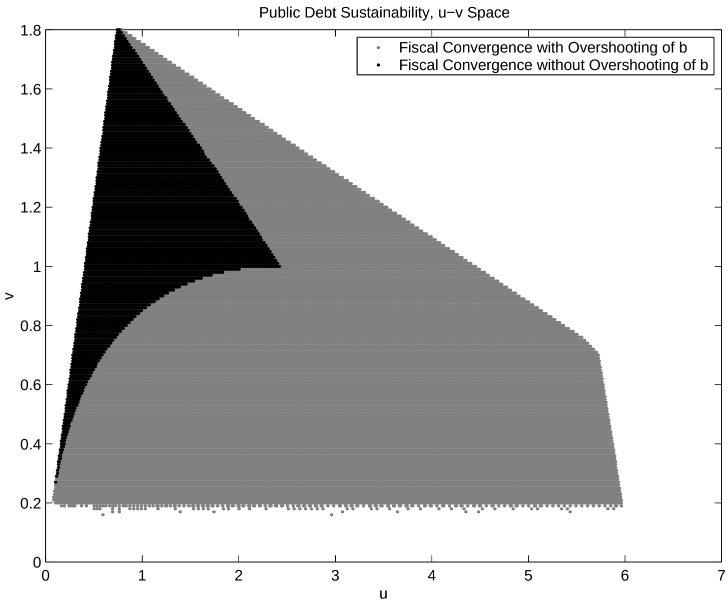

Figure 16: Mixed Approach

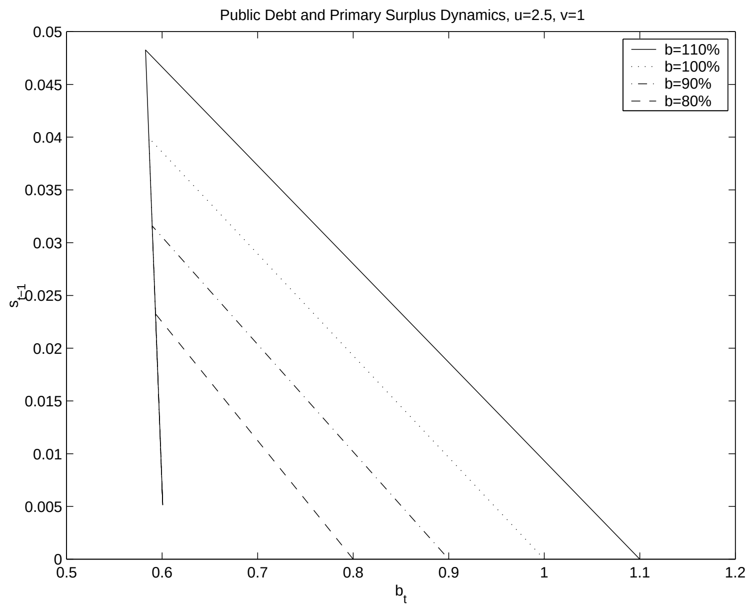

Figure 17: Mixed Approach

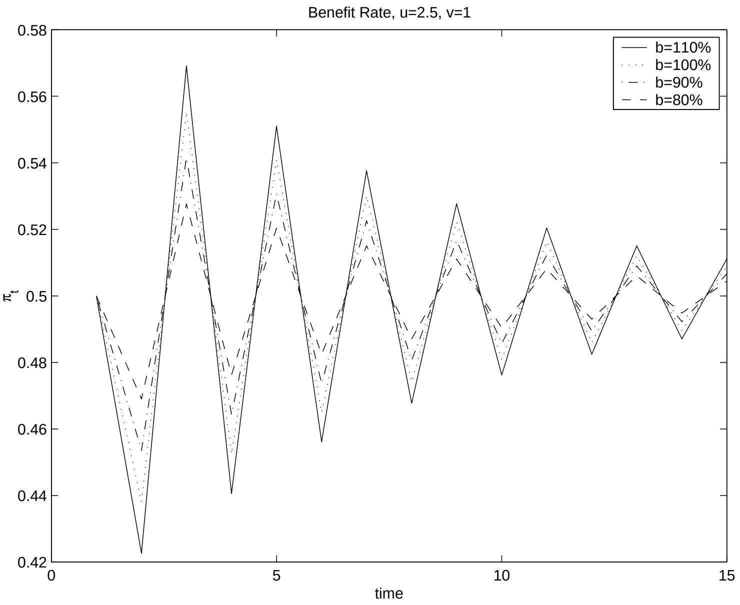

Figure 18: Mixed Approach

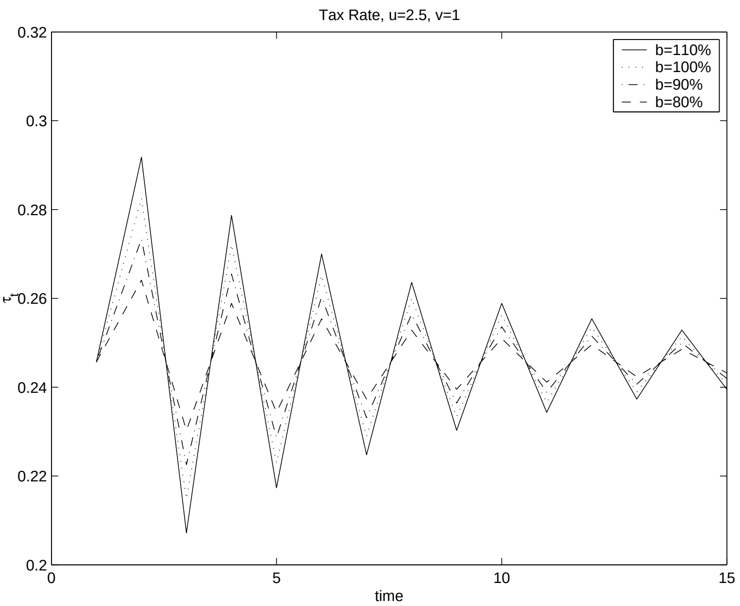

Figure 19: Mixed Approach

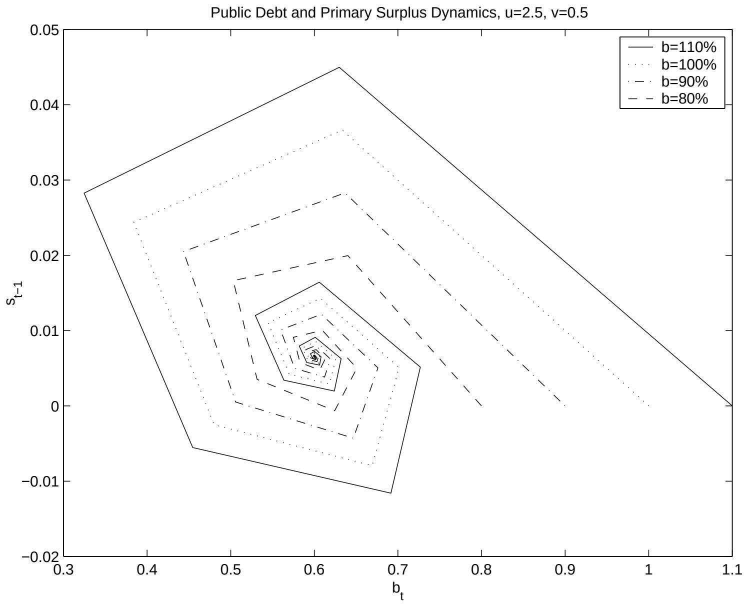

Figure 20: Mixed Approach

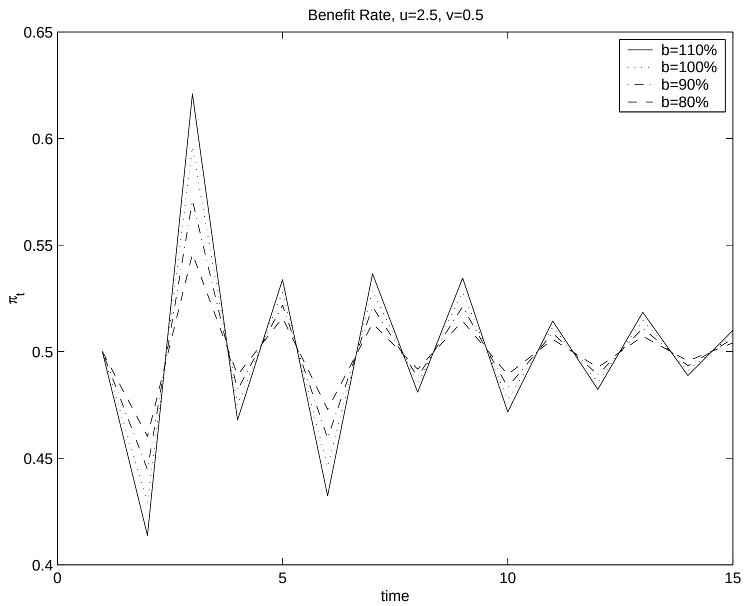

Figure 21: Mixed Approach